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FROM THE

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VOLUME VIII

NOS. 148-168

WITH THIRTEEN PLATES

FEBRUARY, 1918—MARCH, 1919

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THE GENERAL MAGNETIC FIELD OF THE SUN

APPARENT VARIATION OF FIELD-STRENGTH WITH LEVEL IN THE SOLAR ATMOSPHERE

By G. E. HALE, F. H. SEARES, A. VAN MAANEN, AND F. ELLERMAN

The preliminary results of a study of the Zeeman effect due to the general magnetic field of the sun have been given in a previous paper.[1] With the aid of suitable polarizing apparatus, used in conjunction with the 75-foot spectrograph of the 150-foot tower telescope, four lines in the third-order spectrum of an excellent Michelson grating were found to show displacements corresponding in sign and agreeing closely in magnitude with theoretical values calculated for a uniformly magnetized sphere. The extreme minuteness of the displacements, usually less than a thousandth of an angstrom, led us to defer final acceptance of the provisional conclusions until they could be rigorously tested by additional measures. The present paper contains results which amply confirm those previously published, and reveal a relation between the intensity of the sun's general field and the character of the spectral lines used for its determination. The stronger lines give smaller values of the field-strength, and, since the intensity of a line depends upon the level in the solar atmosphere at which it originates, it is natural to interpret these differences in field-strength as a consequence of differences in level.

In the original paper only four lines were shown to have displacements attributable to the general field of the sun. A number of others, mainly stronger lines known from laboratory investigations to have large Zeeman separations, showed no corresponding solar displacements. Fortunately an explanation of this apparent contradiction was offered in the circumstance that the displaced lines probably originate at a low level in the solar atmosphere, while the others correspond to higher levels where the field is too weak to be detected.

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[1] Hale, *Mt. Wilson Contr.*, No. 71; *Astrophysical Journal*, 38, 27, 1913. See also Hale, *Terrestrial Magnetism*, 17, 173, 1912.

Although the hypothesis that the general field decreases rapidly in intensity with increasing elevation has usually proved a reliable guide in the choice of additional lines, caution must be exercised in accepting this interpretation of the differences in field-strength, inasmuch as the measures may perhaps be affected by systematic errors depending upon line-intensity. The question is one of much complexity, and the relevant evidence now available is presented in a later section. Although a final answer cannot now be given, this uncertainty in no wise affects the two main results of the investigation. So far as there may be doubt, it is associated with the interpretation of the second result—the relation between field-strength and line-intensity—and, perhaps involved with this, is the puzzling circumstance that certain lines having large displacements in the third-order spectrum show little or no displacement in the second order.

Before proceeding to a discussion of the observations, certain details concerning the measurement of the displacements require consideration.

I. CONFIRMATION OF DISPLACEMENTS BY OTHER OBSERVERS

The observations and measures have always been arranged so as to avoid vitiating influences that might arise from a knowledge of the observing conditions. Nevertheless we have considered it of the utmost importance to obtain all possible confirmation of the measured displacements, which are so small that it is difficult to obtain definite evidence of their reality. The lines are wide in comparison with their shifts, and when measures are first undertaken the accidental errors are usually so large as to mask completely the quantities to be observed, which even after much practice remain for many observers below the limit of perception. Thus five members of the Observatory's staff have made more or less extensive series of measures without obtaining a positive result. On the other hand an equal number of other members of our staff have produced evidence of the objectivity of the displacements and of their agreement with the hypothesis that the sun behaves approximately as a magnetized sphere.

Besides those whose measures were described in *Mount Wilson Contribution*, No. 71, Miss Richmond and Miss Felker of the Computing Division have made trial series which confirm the general character of the results. These measures, which were on λ 5856.312 (Fe, 2), are illustrated in Fig. 1, and a summary is given in Table I.

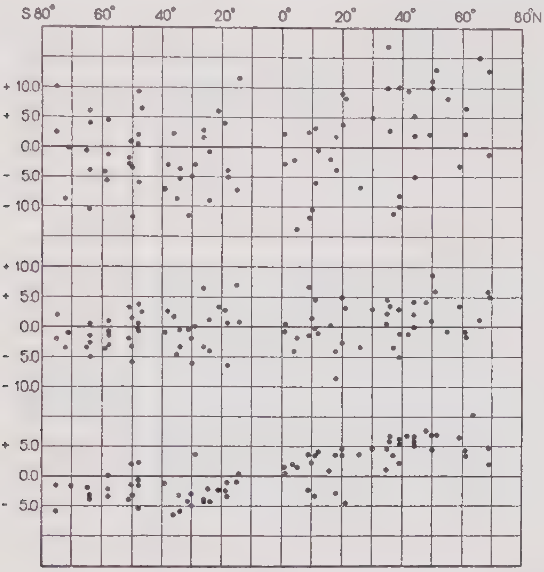

FIG. 1.—Measures by Miss Richmond (upper series), Miss Felker (middle series), and van Maanen on λ 5856.312. Abscissae are heliographic latitudes; ordinates are values of the displacements, the unit being 0.001 mm. The number of settings of the micrometer in the middle series is twice that for the upper series.

TABLE I

SUMMARY OF MEASURES ON λ 5856.312

LIMITS OF LATITUDE	MEAN LATITUDE	MEAN Δ		
		Richmond	Felker	van Maanen
69° N–30° N. . . .	47° N	+3.6±1.1 (24)	+1.8±0.5 (24)	+5.2±0.3 (23)
26 N–29 S. . . .	1 S	−1.2±0.8 (28)	0.0±0.5 (28)	0.0±0.4 (28)
30 S–65 S. . . .	49 S	−2.4±0.7 (29)	−1.2±0.3 (29)	−2.7±0.3 (21)

Appended to each mean displacement in the table is its probable error, the unit being 0.001 mm; the number of values included in the mean is added in parentheses. The accidental errors are large (Miss Richmond's series includes half as many settings as that by Miss Felker) and the mean displacements are systematically smaller than those by van Maanen; but for the 45° regions the algebraic signs are all correct, and the means themselves are sufficiently in excess of their probable errors to make the results of significance.

2. MEASUREMENT OF DISPLACEMENTS WITH KOCH'S REGISTERING MICROPHOTOMETER

In the earlier work two forms of measuring machine were employed: a comparator of the ordinary type, with fixed cross-hair, and a parallel-plate micrometer, with which adjacent sections of the displaced line can be brought into alinement by inclining a plane-parallel strip of glass. At the outset the results of different observers with the ordinary micrometer were frequently in disagreement, and it was in the hope of avoiding such discrepancies that the parallel-plate machine was tried. This has proved so satisfactory that all of van Maanen's measures have been made with it. His results for the spectra measured by Miss Lasby with the comparator agree well with hers in sign, although there is a marked systematic difference in magnitude.[1]

A promising means of avoiding systematic errors of measurement is offered by Koch's registering microphotometer, which automatically records the distribution of density in the photographic image of a spectral line. The photograph of the line is moved at a uniform rate across a narrow slit, through which light from a constant source falls upon the sensitive electrode of a photo-electric cell. The variations in electromotive force, caused by changes in the intensity of the transmitted light, produce horizontal displacements of the filament of a string electrometer connected with the other electrode. The image of a small section of the filament projected on a photographic plate, moved vertically by the same clock that carries the negative across the slit, traces a record of the intensity-curve of the line (see Fig. 2). The records may be used to determine the relative positions of spectral lines, as well as the distribution of intensities within them, and thus afford a means of measurement independent of the personal errors that affect observations made with a comparator.

---

[1] *Mt. Wilson Contr.*, No. 71, p. 63; *Astrophysical Journal*, **38**, 87, 1913.

The instrument employed, which was kindly loaned by Professor Röntgen from the collection of the University of Munich for use in Pasadena during Professor Koch's visit in 1913, was not designed for the measurement of extremely small displacements, and it was doubtful whether the necessary degree of precision in the relative motion of the negative and the recording plate could be counted upon.

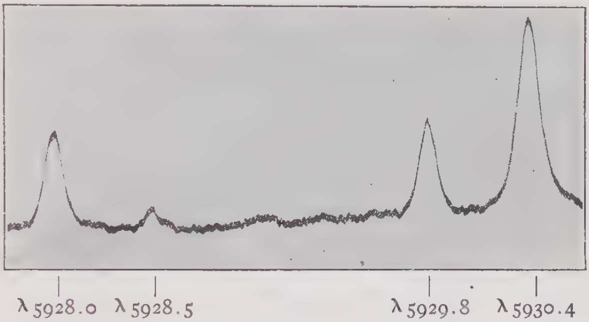

FIG. 2.—Intensity-curve registered by the Koch microphotometer used in measuring the displacements of λ 5928.013 and λ 5929.898 with respect to λ 5930.406, which is not displaced by the sun's general field. The atmospheric line λ 5928.510 is not suitable as a reference line because of its low intensity. Cut is 0.72 original curve.

Three solar lines were first studied, two of which (λ 5928.013 and λ 5929.898) had been found to be subject to displacement, while one (λ 5930.406) was not appreciably affected and thus furnished a reference mark to which the other lines could be referred.[1] Although the range of spectrum is only 2.4 A, Professor Koch was compelled to limit the ratio between the movements of the negative and the recording plate to 7.65, instead of using the more advantageous value of 46.4 applicable to very small fields. Curves were measured for two general-field plates representing direct and reverse records of each of four adjoining strips of the

[1] The atmospheric line λ 5928.510 would be preferable for this purpose, but its intensity-curve cannot be satisfactorily measured.

quarter-wave plate. One of these curves is reproduced in Fig. 2. Pairs of abscissae corresponding to ordinates increasing by one millimeter were measured to hundredths of a millimeter. Two reductions were made. In the first all the measures were used, averaging fifteen pairs for λ 5928, thirteen for λ 5929, and thirty-one for λ 5930; in the second, only the ten pairs corresponding to ordinates common to all three lines were employed. The means of the abscissae give the positions of the lines, from which the displacements in Table II were derived.

TABLE II

COMPARISON OF MEASURES

LINE	PLATE	KOCH'S REGISTERING PHOTOMETER		VAN MAANEN PARALLEL-PLATE MICROMETER
		First Reduction	Second Reduction	
λ 5928.013.......	T′193*a*	−2.7 μ	−5.9 μ	− 0.7 μ
	193*b*	+9.3	+8.4	+ 2.6
λ 5929.898.......	193*a*	−7.3	−7.9	−10.3
	193*b*	+6.6	+7.2	+ 6.2

The differences between the two reductions indicate the uncertainty of the results obtained with the registering photometer. For the shorter interval (λ 5929.898) the measures are in approximate agreement with each other and with those of van Maanen. For the longer interval the inaccuracies of the driving mechanism are naturally reflected in the results.

As a further test, measures were made with the Koch photometer on λ 6302.709 (Fe, 5), which was referred to the neighboring atmospheric line λ 6302.975. Here it was possible to use the larger magnification of 46.4, although owing to the inherent uncertainties it is doubtful if any greater precision was obtained.

Twenty density-curves covering four adjacent spectra were registered from plate T′337*a*. As before, two reductions were made, one including all the abscissae for each curve, the other only those for ordinates common to the two curves. The results are

First reduction, $\Delta = +1.9\ \mu$
Second reduction, $\Delta = +3.0$

From the original negative van Maanen had previously found $+2.8\,\mu$. The uncertainty of the results from the Koch machine is rather large in this case, and the agreement with van Maanen's value, though probably of real significance, is better than might have been expected.

These preliminary results indicate that a specially designed microphotometer should afford measures of great value, particularly for the study of systematic personal errors. As soon as opportunity permits, the instrument recently constructed in our shops will be used for a study of this question.

3. THE OBSERVATIONS

The measures considered in the present discussion depend on the groups of plates designed as Series IV, V, VI, VII, and VIII. The details, as well as the general method of observation, were the same as those previously described. The photographs of the third-order spectrum were made by Ellerman personally or else under his immediate supervision, with the 75-foot spectrograph of the 150-foot tower telescope, the 43-cm solar image being used throughout. In view of the high excellence required for successful use, plates have frequently been rejected without measurement; insufficient or excessive contrast and irregularities in density were the most common reasons for rejection.

The record of observations for Series IV was published in *Mount Wilson Contribution*, No. 71, the original investigation having been partly based upon this series. Since Series V–VIII usually include only two days (λ 5247, Series VII, and λ 4406 and λ 4418, Series VIII, were observed on three days, and λ 4421, Series VIII, on four days), the chronological list of plates for these series is omitted, the essential data being collected in Tables III–VIII.

The observations given in this paper were not intended for an exhaustive study of the sun's magnetic field, but rather as a means of checking the earlier results and of selecting lines suitable for investigating the position of the magnetic axis and other related questions. The measures have therefore been restricted to the regions of maximum displacement near 45° N. and S. latitude. Most of the lines combine the characteristics of low or moderate solar level and large laboratory separation, although several showing displacements in the sun have been included whose laboratory separations have not yet been determined.

TABLE III

LINES SHOWING MAGNETIC DISPLACEMENT

SERIES	ROWLAND			PLATES	DATES OF OBSERVATION	*D*	ALGEBRAIC SIGN OF Δ		ZERO VALUES
	λ	El.	Int.				Right	Wrong	
IV......	5831.821	Ni	1	T′255–290	1913 Jan. 29–Feb. 18	−6°4	94	28	3
	5856.312	Fe	2	264–290	Jan. 31–Feb. 18	−6.6	66	8	1
	5928.013	Fe	2	258–290	Jan. 30–Feb. 18	−6.5	75	18	2
V.......	6007.540	Ni	1	318–333	June 19–20	+1.7	54	17	1
	6039.953	V	0	"	"	"	79	6	0
	6079.227	Fe	2	"	"	"	94	12	2
	6111.290	Ni	2	324–333	"	"	53	13	0
	6119.740	V	1	318–333	"	"	74	9	0
	6129.190	Ni	1	"	"	"	67	13	0
	6149.458	Fe*	2	"	"	"	65	10	1
	6173.553	Fe	5	"	"	"	85	5	0
VII.....	5247.737	Cr	2	424–440	Sept. 9–11	+7.2	171	12	1
	5250.817	Fe	3	431–436	Sept. 9–10	"	48	6	0
	5253.633	Fe	2	"	"	"	49	5	0
	5263.486	Fe	4	"	"	"	47	5	2
	5300.929	Cr	2	430–435	"	"	45	4	1
	5304.355	Cr	0	"	"	"	36	8	3
	5328.515	Cr	2	"	"	"	46	4	1
	5329.329	Cr	3	"	"	"	42	8	2
	5329.975	Cr	0	"	"	"	40	8	0
	5340.639	Cr	0	"	"	"	37	8	0
	5348.511	Cr	4	"	"	"	46	6	1
VIII....	4406.810	V	2	463–464	Nov. 10	+3.3	30	3	1
	4418.499	Ti	1	479–480	Dec. 1, 5	+0.5	36	0	0
	4421.733	V	0	463–480	Nov. 10–Dec. 5	......	49	19	0
	4430.785	Fe	3	463–480	Nov. 10–Dec. 5	......	23	2	1
	4438.006	V	0	463–464	Nov. 10	+3.3	14	4	1

* Unidentified by Rowland.

The twenty-six new lines for which appreciable displacements have been detected are listed in Table III, with dates of observation, limiting plate numbers, etc. The table also contains λ5831.821; the general behavior of this line was indicated in *Mount Wilson Contribution*, No. 71, but the individual displacements are given in this paper for the first time. The last three columns of Table III show that a large majority of the algebraic signs accord with the hypothesis that the displacements are caused by a general magnetic field. Indeed, but few of the disagreeing cases can be said to present contradictions, for frequently the real displacements are so small that a measured value may show a wrong sign without deviating excessively from the truth.

TABLE IV

LINES FROM SERIES IV–VIII WHICH SHOW NO DISPLACEMENT

SERIES	ROWLAND			ALGEBRAIC SIGN OF Δ		ZERO VALUES
	λ	El.	Int.	Right	Wrong	
IV..........	5804.681	Fe	0	18	20	0
	5838.592	Fe	1	19	19	0
	5848.342	Fe	3	15	16	1
	5892.920	Fe	00	7	9	0
	5905.895	Fe	4	15	18	6
	5916.475	Fe	3	25	31	3
V...........	5991.600	....	2	22	25	4
	5998.002	Fe	2	23	26	3
	6005.770	Fe	1	31	38	4
	6012.450	Ni	1	22	16	1
	6042.315	Fe	3	31	26	3
	6081.665	V	0	21	24	1
	6142.700	....	1	24	25	8
VI..........	6455.820	Ca	2	32	38	2
	6496.688	Fe	2	41	34	3
	6597.807	Cr	1	32	44	0
VII.........	5224.471	Ti	0	22	21	3
VIII........	4404.433	Ti	1N	10	9	0

All the measures discussed have been made by van Maanen with the parallel-plate micrometer. The results, collected for each line according to latitude, are given in Tables V–VIII. Those for Series VIII are less numerous than those for the other series, but the photographs are generally of excellent quality, and compensate in some degree for the smaller number of displacements measured. The results for λ4421 from three plates of this series (sixteen values of Δ) were, however, rejected after measurement because of discordances due probably to lack of proper density. The line is weak and generally difficult of measurement. The rejected values do not appear in Table VIII.

TABLE V

Displacements for Lines of Series IV

Lat.	Plate	Δ
λ 5831.821		
N. 71°...	276*a*	+0.7
70....	274*a*	+1.3
68....	272*a*	−1.2
68....	273*b*	+2.0
64....	271*b*	+4.0
63....	274*a*	+3.0
60....	273*b*	+4.0
59....	276*a*	+1.0
58....	271*b*	+1.3
58....	272*a*	−0.7
50....	273*b*	+0.5
50....	274*a*	+1.5
49....	276*a*	(−4.1)
47....	271*b*	(+7.8)
47....	272*a*	+0.7
45....	274*a*	+0.3
44....	273*b*	+1.5
44....	276*a*	+1.7
42....	271*b*	+1.2
41....	272*a*	+5.0
40....	273*b*	+0.5
40....	274*a*	+2.6
38....	276*a*	+1.8
37....	271*b*	−1.0
36....	272*a*	−0.7
36....	273*b*	+3.3
36....	274*a*	+2.8
35....	271*b*	−0.3
30....	275*b*	+2.0
27....	275*a*	−2.0
26....	275*b*	+2.0
21....	288*a*	+0.8
21....	288*b*	+1.7
20....	275*a*	+2.3
20....	275*b*	+1.0
17....	288*a*	−3.0
17....	288*b*	0.0
16....	275*a*	+1.2
16....	275*b*	+1.3
14....	288*a*	+0.7
14....	288*b*	+3.6
12....	275*b*	+4.0
10....	275*a*	+1.3
10....	288*a*	−0.3
10....	288*b*	+1.5
9....	275*b*	(+6.3)
5....	288*a*	+1.3
5....	288*b*	+2.1
1....	288*a*	+1.3
N. 1....	288*b*	+1.7
λ 5831.821—*Cont.*		
0°...	290*a*	−1.0
0....	290*b*	0.0
S. 3....	290*a*	+0.7
3....	290*b*	+2.0
7....	290*a*	+4.3
7....	290*b*	+3.6
12....	290*a*	+2.3
12....	290*b*	(+6.9)
17....	290*a*	+2.6
17....	290*b*	(+4.3)
19....	283*a*	−1.0
21....	290*a*	+0.8
21....	290*b*	−1.0
22....	283*a*	−0.3
28....	283*a*	−1.0
31....	283*a*	−2.0
35....	283*a*	−0.2
37....	283*b*	−0.7
39....	261*b*	−0.3
41....	283*b*	−0.3
42....	258*a*	−2.0
42....	258*b*	0.0
42....	261*a*	(+5.3)
44....	261*a*	(+6.3)
44....	261*b*	−3.0
46....	258*a*	−2.6
46....	258*b*	−4.1
47....	255*b*	+0.8
47....	264*b*	−3.1
47....	265*b*	+1.0
48....	265*a*	−4.0
48....	283*b*	−0.7
49....	255*a*	−2.0
51....	264*a*	(−7.3)
51....	264*b*	−1.8
51....	265*a*	−3.6
51....	265*b*	−1.0
51....	283*b*	+2.6
52....	255*a*	−3.0
52....	255*b*	−5.6
52....	258*a*	−5.6
52....	258*b*	−5.9
52....	261*a*	−1.5
52....	261*b*	−3.1
56....	258*a*	+2.6
56....	258*b*	−1.2
57....	261*a*	−0.3
57....	261*b*	+0.5
59....	265*a*	(+5.0)
S. 59....	265*b*	−4.6
λ 5831.821—*Cont.*		
S. 60°...	255*a*	−3.0
60....	255*b*	−4.3
60....	264*a*	−1.7
60....	264*b*	−2.3
62....	261*a*	−0.7
62....	261*b*	−1.7
63....	258*a*	−3.3
63....	258*b*	−3.6
65....	265*a*	−4.6
65....	265*b*	−1.7
66....	255*a*	−5.8
66....	255*b*	−1.8
66....	264*a*	−5.3
68....	264*b*	−2.0
72....	255*b*	−3.8
72....	265*a*	−2.6
72....	265*b*	−2.1
73....	255*a*	(−6.3)
73....	264*a*	+2.3
73....	264*b*	−1.7
79....	255*b*	−2.6
79....	264*a*	−4.1
79....	264*b*	+0.3
82....	265*a*	(−6.9)
S. 82....	265*b*	−2.3
λ 5856.312		
N. 69°...	273*b*	+4.8
69....	274*a*	+2.0
66....	272*a*	(+10.2)
61....	273*b*	+3.6
61....	274*a*	+4.0
59....	272*a*	+6.4
51....	274*a*	+6.9
50....	273*b*	+6.9
50....	276*a*	+4.3
48....	272*a*	+7.4
44....	273*b*	+6.3
44....	274*a*	+5.8
44....	276*a*	+5.3
42....	272*a*	+6.6
39....	273*b*	+5.9
39....	274*a*	+2.1
39....	276*a*	+5.6
37....	272*a*	+3.3
36....	273*b*	+6.3
36....	276*a*	+5.9
35....	272*a*	+1.0
35....	274*a*	+4.5
N. 30....	275*b*	+4.5

TABLE V—*Continued*

Lat.	Plate	Δ
λ 5856.312—*Cont.*		
N. 26° . . .	275*b*	+3.6
21. . . .	288*a*	(−4.8)
20. . . .	275*b*	+3.5
20. . . .	288*b*	+4.3
18. . . .	288*a*	(−3.0)
18. . . .	288*b*	+3.6
16. . . .	275*b*	+0.7
12. . . .	275*b*	+4.0
11. . . .	288*a*	−3.3
11. . . .	288*b*	+3.3
10. . . .	275*b*	+2.0
9. . . .	288*a*	−2.3
9. . . .	288*b*	+3.5
5. . . .	288*b*	+1.5
4. . . .	288*a*	+1.8
1. . . .	288*a*	+1.5
N. 1. . . .	288*b*	+0.3
S. 14. . . .	290*b*	+0.2
15. . . .	290*a*	−1.0
18. . . .	290*a*	−1.2
18. . . .	290*b*	−3.3
19. . . .	283*a*	−2.6
21. . . .	283*a*	−2.3
24. . . .	290*a*	−4.3
24. . . .	290*b*	−2.3
26. . . .	290*a*	−4.1
26. . . .	290*b*	−4.3
29. . . .	283*a*	(+3.6)
30. . . .	290*a*	−5.0
30. . . .	290*b*	−3.0
31. . . .	283*a*	−4.3
34. . . .	290*a*	−3.3
34. . . .	290*b*	−5.9
36. . . .	283*b*	−6.4
39. . . .	283*b*	−1.3
48. . . .	264*a*	−1.3
48. . . .	264*b*	−0.7
48. . . .	265*a*	−5.6
48. . . .	283*b*	(+2.1)
50. . . .	264*a*	(+1.8)
50. . . .	264*b*	−1.7
50. . . .	265*a*	−3.3
51. . . .	283*b*	−4.0
58. . . .	264*a*	0.0
58. . . .	264*b*	−2.3
58. . . .	265*a*	−3.5
64. . . .	264*a*	−4.0
64. . . .	264*b*	−3.3
64. . . .	265*a*	−2.0
71. . . .	265*a*	−1.8
75. . . .	264*a*	−1.7
S. 75. . . .	264*b*	−5.9
λ 5928.013		
N. 69° . . .	276*a*	+3.6
68. . . .	273*b*	+4.3
67. . . .	271*b*	+5.3
66. . . .	272*a*	+6.8
62. . . .	273*b*	+4.0
61. . . .	276*a*	+4.0
59. . . .	272*a*	+5.6
57. . . .	271*b*	+3.6
50. . . .	273*b*	+3.6
50. . . .	276*a*	+5.9
48. . . .	272*a*	+3.6
44. . . .	273*b*	+4.3
44. . . .	276*a*	+4.0
42. . . .	271*b*	+1.5
42. . . .	272*a*	+4.6
39. . . .	273*b*	+1.8
39. . . .	276*a*	+2.1
37. . . .	271*b*	+2.6
37. . . .	272*a*	+1.0
36. . . .	273*b*	+3.6
36. . . .	276*a*	+3.5
35. . . .	271*b*	+2.3
35. . . .	272*a*	+1.3
30. . . .	275*b*	+4.3
26. . . .	275*b*	+1.3
21. . . .	288*a*	+1.8
21. . . .	288*b*	(−5.6)
20. . . .	275*b*	+0.8
17. . . .	288*a*	+5.4
17. . . .	288*b*	(−6.6)
16. . . .	275*b*	0.0
12. . . .	275*b*	+2.3
12. . . .	288*a*	+4.3
12. . . .	288*b*	(−5.0)
10. . . .	275*b*	+1.7
10. . . .	288*a*	−3.6
10. . . .	288*b*	(−8.4)
5. . . .	288*a*	+6.3
5. . . .	288*b*	−4.3
1. . . .	288*a*	+3.0
N. 1. . . .	288*b*	−5.0
S. 15. . . .	290*a*	−2.0
15. . . .	290*b*	−1.7
18. . . .	283*a*	−1.8
18. . . .	290*a*	−4.6
18. . . .	290*b*	−2.3
21. . . .	283*a*	−1.3
24. . . .	290*a*	−3.3
24. . . .	290*b*	−3.0
26. . . .	290*a*	−4.0
26. . . .	290*b*	−3.3
S. 29. . . .	283*a*	−0.3
λ 5928.013—*Cont.*		
S. 30° . . .	290*a*	−1.3
30. . . .	290*b*	−2.8
31. . . .	283*a*	−2.3
34. . . .	290*a*	−6.3
34. . . .	290*b*	−5.4
35. . . .	283*a*	−4.8
35. . . .	283*b*	−1.0
39. . . .	283*b*	+2.3
42. . . .	258*a*	−6.6
42. . . .	258*b*	+1.5
42. . . .	261*a*	−3.6
42. . . .	261*b*	−2.3
44. . . .	261*a*	−0.3
44. . . .	261*b*	−1.7
46. . . .	258*a*	+0.3
46. . . .	258*b*	+2.5
48. . . .	264*a*	−2.6
48. . . .	264*b*	+0.7
48. . . .	283*b*	(+4.0)
50. . . .	264*a*	−1.7
50. . . .	264*b*	−2.5
50. . . .	283*b*	(+3.3)
52. . . .	258*a*	−4.0
52. . . .	258*b*	+1.7
52. . . .	261*a*	−2.3
52. . . .	261*b*	0.0
56. . . .	283*b*	+2.8
57. . . .	258*a*	(+5.1)
57. . . .	258*b*	−1.5
57. . . .	261*a*	−5.9
57. . . .	261*b*	(+3.3)
58. . . .	264*a*	−5.4
58. . . .	264*b*	−1.3
63. . . .	258*b*	−2.3
63. . . .	261*a*	−0.5
63. . . .	261*b*	−4.3
64. . . .	258*a*	−0.3
64. . . .	264*a*	−4.3
64. . . .	264*b*	−2.0
68. . . .	261*b*	−1.0
70. . . .	264*a*	−0.7
70. . . .	264*b*	−3.0
S. 76. . . .	264*a*	−4.3

TABLE VI

Displacements for Lines of Series V

Lat.	Plate	Δ
	λ 6007.540	
N. 64°...	333*a*	+5.0
64....	333*b*	+7.6
62....	332*a*	−1.7
62....	332*b*	+8.6
61....	318*a*	+3.3
61....	318*b*	+3.6
58....	318*a*	+0.3
58....	318*b*	+4.3
58....	332*a*	−1.8
58....	332*b*	−3.6
57....	333*a*	+4.1
57....	333*b*	+3.5
50....	333*a*	+2.8
50....	333*b*	+1.3
49....	318*a*	+4.6
49....	332*a*	+0.3
49....	332*b*	−3.1
45....	332*a*	−4.0
45....	332*b*	−1.0
45....	333*a*	+5.0
45....	333*b*	+4.8
44....	318*a*	+3.3
44....	318*b*	+4.0
39....	318*a*	+3.0
39....	318*b*	+5.6
39....	332*a*	−2.6
39....	332*b*	+4.5
39....	333*a*	+2.6
39....	333*b*	+5.0
35....	318*a*	+4.6
35....	318*b*	+3.5
35....	332*a*	−4.6
35....	332*b*	+4.6
35....	333*a*	+2.3
N. 35....	333*b*	+3.6
S. 30....	330*a*	−5.9
30....	330*b*	−3.3
31....	319*a*	+2.1
31....	319*b*	−4.0
31....	327*a*	+5.0
31....	327*b*	−4.6
34....	330*a*	−3.0
34....	330*b*	−5.0
35....	319*a*	+5.4
35....	319*b*	−1.3
35....	327*a*	+4.0
35....	327*b*	−3.1
37....	326*a*	(+5.9)
37....	326*b*	−4.6
40....	327*a*	−3.6
S. 40....	327*b*	−6.9
	λ 6007.540—*Cont.*	
S. 41°...	319*a*	−3.0
41....	319*b*	−6.4
41....	326*a*	+4.0
41....	326*b*	−1.8
41....	330*a*	−5.6
41....	330*b*	−3.1
45....	326*a*	(+5.6)
45....	327*b*	−3.6
46....	330*a*	−3.3
46....	330*b*	−2.1
51....	327*a*	−6.8
52....	319*a*	−3.6
52....	326*a*	+5.0
52....	326*b*	−4.6
52....	330*a*	−2.8
52....	330*b*	−5.3
54....	327*b*	−3.3
57....	319*a*	0.0
57....	326*a*	+3.3
57....	326*b*	−3.6
S. 57....	327*a*	−2.6
	λ 6039.953	
N. 65....	333*b*	+7.4
62....	318*a*	(+11.9)
62....	332*a*	+4.0
62....	332*b*	+2.8
60....	324*a*	(−8.6)
60....	324*b*	+3.0
59....	318*a*	+7.3
59....	318*b*	+3.3
59....	333*a*	+4.8
59....	333*b*	+9.2
58....	332*a*	+6.8
58....	332*b*	+6.3
57....	324*a*	(−5.0)
57....	324*b*	+4.0
50....	318*a*	+7.3
50....	318*b*	+6.4
50....	324*a*	+6.6
50....	324*b*	+5.3
49....	332*a*	+7.9
49....	332*b*	+4.3
49....	333*a*	+4.0
49....	333*b*	+3.8
45....	324*a*	+9.2
45....	324*b*	+3.6
45....	332*b*	+7.9
45....	333*a*	+10.1
45....	333*b*	+7.1
N. 39....	318*b*	+3.5
	λ 6039.953—*Cont.*	
N. 39°...	324*a*	+5.6
39....	324*b*	+1.7
39....	332*a*	+5.1
39....	332*b*	+4.6
39....	333*a*	+7.1
39....	333*b*	+9.2
37....	333*a*	+6.4
37....	333*b*	+6.4
36....	318*b*	+6.4
35....	324*a*	(−5.3)
35....	324*b*	+6.3
34....	318*a*	+6.4
34....	318*b*	+5.0
N. 34....	332*a*	+4.1
S. 30....	330*a*	−3.6
30....	330*b*	−3.5
31....	319*a*	−6.9
31....	319*b*	−6.6
32....	327*b*	−6.1
35....	319*a*	−6.9
35....	319*b*	−2.3
35....	326*b*	−6.9
35....	327*a*	−4.0
35....	327*b*	−5.9
35....	330*a*	+0.5
35....	330*b*	−7.3
41....	319*a*	−5.8
41....	319*b*	−9.9
41....	326*a*	−4.5
41....	326*b*	−4.6
41....	327*a*	−5.3
41....	327*b*	−7.9
41....	330*a*	(+10.4)
41....	330*b*	−1.3
45....	326*a*	−6.3
45....	326*b*	−5.6
45....	327*a*	−4.3
45....	327*b*	−1.3
45....	330*a*	(+8.6)
45....	330*b*	−5.9
46....	319*a*	−7.6
46....	319*b*	−2.0
51....	327*a*	−6.9
51....	327*b*	−6.8
52....	319*a*	−7.6
52....	319*b*	−6.9
52....	326*a*	−7.6
52....	326*b*	−5.6
52....	330*a*	−6.6
52....	330*b*	−7.6
S. 57....	326*a*	−5.9

TABLE VI—*Continued*

Lat.	Plate	Δ
λ 6039.953—*Cont.*		
S. 57°	326*b*	−7.3
57	330*a*	−8.9
58	319*a*	−4.0
58	319*b*	−6.3
58	327*a*	−0.2
S. 58	327*b*	−6.8
λ 6079.227		
N. 64	333*a*	+5.6
64	333*b*	+5.4
63	318*a*	+4.3
63	318*b*	+5.0
62	332*a*	+4.0
62	332*b*	+2.6
61	324*b*	+2.0
59	332*a*	+3.5
59	332*b*	+1.7
57	318*a*	+4.3
57	324*a*	+4.6
57	324*b*	+4.3
57	333*a*	+6.8
57	333*b*	(+7.6)
50	318*a*	+6.1
50	318*b*	+5.6
50	324*a*	+5.6
50	324*b*	+5.6
50	333*a*	+7.4
50	333*b*	+5.9
49	332*a*	+4.3
49	332*b*	+3.5
45	318*a*	+4.0
45	318*b*	+5.6
45	324*a*	+5.3
45	324*b*	+4.6
45	332*a*	+4.1
45	332*b*	+2.3
45	333*a*	+5.6
45	333*b*	+5.8
40	324*a*	+2.6
40	324*b*	+5.3
39	318*b*	+4.0
39	332*b*	+4.1
39	333*a*	+4.0
39	333*b*	+5.9
36	318*a*	+5.1
36	318*b*	+5.9
35	324*a*	+4.6
35	324*b*	+3.0
35	332*a*	+4.6
35	332*b*	+2.6
35	333*a*	+3.3
N. 35	333*b*	+5.0
λ 6079.227—*Cont.*		
S. 30°	319*a*	−4.0
30	319*b*	−4.6
31	327*a*	−2.6
31	327*b*	−5.4
34	326*a*	−3.5
34	326*b*	−6.9
35	319*a*	−5.3
35	327*a*	(−0.2)
35	327*b*	−4.6
40	327*a*	−3.6
41	319*a*	−6.4
41	319*b*	−5.6
41	326*a*	−4.0
41	326*b*	−7.6
45	319*a*	−5.3
45	326*a*	−6.4
45	327*a*	−4.6
45	327*b*	−6.3
52	319*a*	(−7.9)
52	319*b*	−3.5
52	326*a*	−3.0
52	326*b*	−4.0
52	327*a*	−5.4
52	327*b*	−3.6
57	319*a*	−3.6
57	319*b*	−5.6
57	327*a*	−5.0
57	327*b*	−3.0
58	326*a*	−5.8
S. 58	326*b*	−5.9
λ 6111.290		
N. 64	332*a*	+0.7
64	333*a*	+3.0
64	333*b*	+4.3
62	332*b*	+2.3
59	332*a*	+3.3
59	332*b*	+0.2
57	324*a*	+4.3
57	324*b*	+0.5
57	333*a*	+2.1
57	333*b*	−1.3
50	324*a*	+1.0
50	324*b*	−1.7
50	333*a*	+2.3
50	333*b*	+3.5
49	332*a*	+4.0
49	332*b*	+3.0
45	324*a*	+0.7
45	324*b*	+4.0
45	332*a*	−1.0
N. 45	332*b*	+0.2
λ 6111.290—*Cont.*		
N. 45°	333*a*	+3.3
45	333*b*	+1.8
39	324*a*	+3.3
39	332*a*	+3.6
39	332*b*	+4.3
39	333*a*	+2.6
39	333*b*	+2.3
35	324*a*	−0.3
35	324*b*	+0.2
35	332*a*	+0.7
35	332*b*	+1.0
35	333*a*	(−3.1)
N. 35	333*b*	+2.3
S. 30	330*a*	(+3.6)
30	330*b*	−1.8
31	327*a*	−3.6
31	327*b*	−2.6
34	326*a*	−1.8
34	326*b*	−5.0
35	327*a*	−0.3
35	327*b*	−1.3
35	330*a*	+0.3
35	330*b*	−5.4
40	327*a*	−2.3
40	327*b*	−1.7
40	330*a*	(+4.0)
40	330*b*	+0.3
41	326*a*	−0.3
41	326*b*	−2.8
45	326*a*	−1.3
45	327*a*	−1.3
45	327*b*	−4.0
45	330*a*	(+4.0)
45	330*b*	+0.7
51	330*a*	+2.3
51	330*b*	−5.9
52	326*a*	−1.7
52	326*b*	−3.0
52	327*a*	−3.0
52	327*b*	−1.7
57	327*a*	−2.5
57	327*b*	−4.5
57	330*a*	+0.2
57	330*b*	−3.6
58	326*a*	−1.3
S. 58	326*b*	−3.3
λ 6119.740		
N. 64	332*a*	+4.3
64	333*a*	(−4.0)
64	333*b*	+1.0
N. 63	318*b*	+6.4

TABLE VI—*Continued*

Lat.	Plate	Δ
λ 6119.740—*Cont.*		
N. 62°	332*b*	+4.6
60	324*b*	+5.6
59	332*a*	+6.9
59	332*b*	+3.8
57	318*b*	+2.6
57	324*a*	(−8.3)
57	324*b*	+2.6
57	333*a*	+5.3
57	333*b*	+4.3
50	318*b*	+4.0
50	324*a*	+6.4
50	324*b*	+3.3
50	333*a*	+2.1
50	333*b*	+5.3
49	332*a*	+4.5
49	332*b*	+5.8
45	318*b*	+2.3
45	324*a*	+4.3
45	324*b*	+2.0
45	332*a*	+4.0
45	332*b*	+5.8
45	333*a*	+4.0
45	333*b*	+6.9
39	318*a*	+8.3
39	318*b*	+2.8
39	324*b*	+5.4
39	332*a*	+6.3
39	332*b*	+5.4
39	333*a*	+7.3
39	333*b*	+4.0
36	318*a*	+6.9
36	318*b*	+6.4
35	324*a*	+5.3
35	324*b*	+5.6
35	332*a*	+3.3
35	332*b*	+2.3
35	333*a*	+2.3
N. 35	333*b*	+6.9
S. 30	319*a*	−3.3
30	319*b*	−5.3
30	330*b*	−3.0
31	327*a*	−1.0
31	327*b*	−5.4
34	326*a*	(+3.0)
34	326*b*	−3.0
35	319*a*	(+8.9)
35	319*b*	−5.0
35	327*a*	−4.6
35	327*b*	+1.5
35	330*a*	−3.0
35	330*b*	−3.1
S. 40	327*a*	−5.1
λ 6119.740—*Cont.*		
S. 40°	327*b*	−6.3
40	330*a*	−5.9
40	330*b*	−3.3
41	319*a*	(+7.6)
41	319*b*	−3.6
41	326*a*	−3.3
41	326*b*	−5.0
45	319*a*	(+6.3)
45	319*b*	−3.0
45	326*a*	−5.1
45	327*b*	−3.3
45	330*a*	−3.6
45	330*b*	−6.9
51	330*a*	−6.1
51	330*b*	−5.9
52	327*b*	−3.6
52	326*b*	−4.6
52	326*a*	−6.3
52	319*b*	−1.2
52	319*a*	(+4.0)
57	319*a*	(+7.3)
57	319*b*	−6.3
57	327*a*	−5.9
57	327*b*	−6.3
57	330*a*	−5.0
58	326*a*	−4.1
S. 58	326*b*	−3.6
λ 6129.190		
N. 64	332*a*	+3.0
64	332*b*	+5.1
63	324*a*	+6.1
63	333*a*	+8.6
63	333*b*	+5.1
62	318*a*	(−6.3)
58	332*a*	+4.3
58	332*b*	+4.3
57	324*a*	+4.0
57	324*b*	+5.3
57	333*a*	−1.7
57	333*b*	+2.0
56	318*a*	(−4.6)
50	332*a*	+7.3
50	332*b*	+2.3
50	333*a*	−1.7
50	333*b*	+3.8
49	318*a*	(−3.3)
48	324*b*	+6.8
45	332*a*	+2.6
45	333*a*	+3.6
45	333*b*	+2.3
N. 44	318*a*	−1.8
λ 6129.190—*Cont.*		
N. 44°	324*a*	+3.6
44	324*b*	+3.6
40	332*a*	+3.1
39	324*a*	+1.3
39	324*b*	+5.6
39	333*a*	+4.6
39	333*b*	+1.7
38	318*a*	−0.3
35	332*a*	+2.6
35	332*b*	+5.6
35	333*a*	+2.1
34	318*a*	(−4.8)
34	324*a*	+0.3
N. 34	324*b*	+5.4
S. 31	319*a*	+3.0
31	319*b*	−5.3
31	330*a*	−8.3
31	330*b*	−1.7
32	326*a*	−3.8
32	326*b*	−3.6
35	330*a*	−5.0
35	330*b*	−4.6
36	319*a*	−4.0
36	319*b*	−1.5
36	326*a*	−4.5
36	326*b*	−5.1
36	327*b*	−2.5
41	319*a*	−4.8
41	319*b*	−5.3
41	327*a*	+3.0
41	327*b*	−3.6
41	330*a*	+2.6
41	330*b*	−4.3
42	326*a*	−3.6
42	326*b*	−1.5
46	319*a*	−5.6
46	319*b*	−3.0
46	326*a*	−3.0
46	326*b*	−7.3
46	327*a*	−1.0
46	327*b*	−5.3
52	319*a*	−4.3
52	319*b*	(+3.5)
52	326*a*	−4.3
52	326*b*	−2.5
52	330*a*	−4.8
52	330*b*	−3.5
53	327*a*	−7.1
53	327*b*	+1.0
58	319*a*	−5.4
58	319*b*	−4.0
S. 58	326*a*	−2.1

TABLE VI—*Continued*

Lat.	Plate	Δ	Lat.	Plate	Δ	Lat.	Plate	Δ
λ 6129.190—*Cont.*			λ 6149.458—*Cont.*			λ 6173.553—*Cont.*		
S. 58°...	326*b*	−5.0	S. 35°...	330*b*	−2.8	N. 49°...	318*b*	+9.2
58....	330*a*	−4.6	36....	319*a*	−3.3	48....	324*a*	+6.8
58....	330*b*	−6.3	36....	319*b*	−4.6	48....	324*b*	+6.9
59....	327*a*	−5.3	36....	326*a*	−5.8	45....	332*a*	+4.0
S. 59....	327*b*	−5.6	36....	326*b*	−6.3	45....	332*b*	+6.3
λ 6149.458			36....	327*b*	+0.3	45....	333*a*	+6.8
N. 64....	332*a*	+4.0	41....	319*a*	−4.3	45....	333*b*	+5.8
64....	332*b*	+5.1	41....	327*a*	−4.6	44....	318*a*	+6.6
63....	324*a*	+5.3	41....	327*b*	−5.0	44....	318*b*	+5.1
63....	333*a*	+5.1	41....	330*a*	−4.1	44....	324*a*	(−0.7)
63....	333*b*	+6.8	41....	330*b*	−3.3	44....	324*b*	+5.0
58....	332*a*	+2.6	42....	326*a*	−3.6	40....	332*a*	+5.1
58....	332*b*	+2.0	42....	326*b*	−4.6	39....	324*a*	+4.6
57....	324*a*	+4.1	45....	330*a*	−3.0	39....	324*b*	+6.6
57....	324*b*	+5.9	45....	330*b*	0.0	39....	333*a*	+0.2
57....	333*a*	+1.3	46....	319*a*	−6.6	39....	333*b*	+4.0
57....	333*b*	+3.3	46....	319*b*	−2.6	38....	318*b*	+6.4
56....	318*a*	(−3.8)	46....	326*a*	−3.0	35....	332*a*	+6.1
50....	332*a*	+4.1	52....	319*b*	−1.2	35....	332*b*	+5.0
50....	332*b*	+5.3	52....	326*a*	−4.3	35....	333*a*	+0.7
50....	333*a*	−1.7	52....	330*a*	−3.0	35....	333*b*	+4.8
50....	333*b*	+5.1	52....	330*b*	+2.3	34....	318*a*	+5.3
49....	318*a*	(−4.0)	53....	327*b*	−0.5	34....	318*b*	+4.6
48....	324*a*	+2.6	58....	319*a*	−4.3	34....	324*a*	+4.6
48....	324*b*	+0.3	58....	319*b*	−3.0	N. 34....	324*b*	+7.9
45....	332*b*	+3.5	58....	326*a*	−2.3	S. 31....	319*a*	−4.0
45....	333*a*	−1.0	58....	326*b*	−3.3	31....	319*b*	−4.3
45....	333*b*	+5.3	58....	330*a*	−4.0	31....	330*a*	(+1.6)
44....	318*a*	(−5.0)	58....	330*b*	−7.9	31....	330*b*	−3.3
44....	324*a*	+3.6	59....	327*a*	−4.5	32....	326*a*	−5.9
44....	324*b*	+5.0	S. 59....	327*b*	−4.3	32....	326*b*	−5.4
40....	332*a*	+5.3	λ 6173.553			35....	330*a*	(+0.6)
40....	332*b*	+7.3	N. 64....	332*a*	+3.0	35....	330*b*	−2.8
39....	324*a*	+3.6	64....	332*b*	+5.3	36....	319*a*	−4.8
39....	324*b*	+3.8	63....	333*a*	+6.6	36....	319*b*	−6.4
39....	333*a*	+3.3	63....	333*b*	+8.2	36....	326*a*	−6.3
39....	333*b*	+2.6	62....	318*a*	+6.3	36....	326*b*	−5.3
38....	318*a*	(−4.6)	62....	318*b*	+7.3	36....	327*a*	−1.2
35....	332*a*	+6.3	58....	332*a*	+5.6	36....	327*b*	−5.6
35....	332*b*	−0.3	58....	332*b*	+4.3	41....	319*a*	−4.8
35....	333*a*	+2.0	57....	324*a*	+1.0	41....	319*b*	−4.3
35....	333*b*	+5.0	57....	324*b*	+2.1	41....	327*a*	−8.1
34....	318*a*	(−3.3)	57....	333*a*	+4.3	41....	327*b*	−3.8
N. 34....	324*a*	+0.8	57....	333*b*	+3.3	41....	330*a*	−0.1
S. 31....	319*a*	−1.5	56....	318*a*	+0.2	41....	330*b*	−4.3
31....	319*b*	−2.6	56....	318*b*	+5.8	42....	326*a*	−5.9
31....	330*a*	−5.3	50....	332*a*	+6.6	42....	326*b*	−6.1
31....	330*b*	−1.5	50....	332*b*	+3.8	45....	330*a*	(+5.1)
32....	326*a*	−4.1	50....	333*a*	+0.5	45....	330*b*	−2.6
32....	326*b*	−2.3	50....	333*b*	+5.3	46....	319*a*	−5.3
S. 35....	330*a*	−6.9	N. 49....	318*a*	+4.0	46....	319*b*	−5.3
						S. 46....	326*a*	−5.9

TABLE VI—*Continued*

Lat.	Plate	Δ	Lat.	Plate	Δ	Lat.	Plate	Δ
λ 6173.553—*Cont.*			λ 6173.553—*Cont.*			λ 6173.553—*Cont.*		
S. 46°...	326*b*	−7.3	S. 52°...	330*a*	(+2.3)	S. 58°...	326*b*	−7.6
46....	327*a*	−6.9	52....	330*b*	−4.6	58....	330*a*	−2.2
46....	327*b*	−3.8	53....	327*a*	−5.3	58....	330*b*	−6.4
52....	319*a*	−1.7	53....	327*b*	−4.3	59....	327*a*	−3.6
52....	319*b*	−3.6	58....	319*a*	−6.3	S. 59....	327*b*	−6.9
52....	326*a*	−5.0	58....	319*b*	−5.0			
S. 52....	326*b*	−6.1	S. 58....	326*a*	−7.1			

Since the compound quarter-wave plate was sometimes used in the normal (+) and sometimes in the inverted (−) position, and since in nearly all cases the measurer was ignorant of its position and also of the hemisphere under observation, personal bias has been eliminated. The signs of displacements observed with the inverted quarter-wave plate have been changed, so that all results refer to the normal position of the plate.

Besides the lines given in Table III, others have also been measured. A list of these, which show no sensible displacement, is in Table IV. The close equality in the numbers of right and wrong signs is in striking contrast with the preponderance of correct signs in Table III.

4. DETERMINATION OF MAXIMUM DISPLACEMENT FOR DIFFERENT LINES

The first step in the calculation of the sun's general field is the determination of the displacement at $\phi=45°$ from the measures in Tables V–VIII. The displacements of a normal Zeeman triplet produced by the sun's field, assumed to be that of a uniformly magnetized sphere, may be represented by[1]

$$k\Delta=A\cos i+B\sin i\cos\lambda \qquad (1)$$

in which

Δ = displacement of spectral line,

k = a constant depending upon the strength of field, the magnetic separation of the line, and the units employed,

i = inclination of sun's magnetic axis to axis of rotation, and

λ = heliographic longitude of magnetic axis.

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[1] Seares, *Mt. Wilson Contr.*, No. 72; *Astrophysical Journal*, **38**, 99, 1913.

TABLE VII

Displacements for Lines of Series VII—λ5247.737

Lat.	Plate	Δ	Lat.	Plate	Δ	Lat.	Plate	Δ
N. 70°...	424*a*	+4.0	N. 50°...	429*a*	+5.9	S. 25°...	440*b*	−5.3
70....	424*b*	+6.3	50....	429*b*	+3.6	29....	433*a*	−5.8
70....	426*a*	+5.1	50....	430*a*	+5.6	29....	433*b*	(+4.3)
70....	426*b*	+4.3	50....	430*b*	+6.3	29....	434*a*	−4.3
70....	427*a*	+4.1	50....	431*a*	+5.9	29....	434*b*	−2.8
70....	427*b*	+5.3	50....	431*b*	+3.6	29....	435*a*	−2.6
70....	428*a*	(−4.3)	50....	432*a*	+4.1	29....	435*b*	−5.6
70....	428*b*	+5.1	50....	432*b*	+2.6	29....	436*a*	−6.1
69....	429*a*	+5.6	46....	424*a*	+3.3	29....	436*b*	−4.0
69....	430*a*	+3.6	46....	424*b*	+5.0	29....	437*a*	−5.3
69....	430*b*	+2.0	46....	426*a*	+3.3	29....	437*b*	−5.6
69....	431*a*	+4.3	46....	427*a*	+0.3	29....	438*a*	−5.0
69....	431*b*	+4.8	46....	427*b*	+2.6	29....	438*b*	−7.3
69....	432*a*	(+8.3)	46....	428*a*	(−5.6)	29....	439*a*	−4.3
69....	432*b*	+5.9	46....	428*b*	+2.6	29....	439*b*	−4.0
63....	424*a*	+2.6	45....	429*a*	+5.3	30....	440*a*	−7.1
63....	424*b*	+5.1	45....	429*b*	+3.6	35....	433*a*	−6.6
63....	426*a*	+5.3	45....	430*a*	+5.9	35....	433*b*	(+6.3)
63....	426*b*	+3.6	45....	430*b*	+5.0	35....	434*a*	−5.0
63....	427*a*	+2.5	45....	431*a*	+4.3	35....	434*b*	−6.6
63....	427*b*	+7.6	45....	431*b*	+3.6	35....	435*a*	−5.6
63....	428*a*	(−3.6)	45....	432*a*	+4.8	35....	435*b*	−4.3
63....	428*b*	+3.6	45....	432*b*	+6.3	35....	436*a*	−4.0
62....	429*a*	+4.3	41....	424*a*	+4.0	35....	436*b*	−5.3
62....	430*a*	+3.3	41....	424*b*	+3.0	35....	437*a*	−4.0
62....	430*b*	+6.1	41....	426*a*	+1.7	35....	437*b*	−5.1
62....	431*a*	+6.6	41....	426*b*	+6.3	35....	438*a*	−4.6
62....	431*b*	+5.4	41....	427*a*	+4.6	35....	438*b*	−4.5
62....	432*a*	+4.5	41....	427*b*	+4.6	35....	439*a*	−3.8
62....	432*b*	+6.9	41....	428*b*	+4.3	35....	439*b*	−3.3
56....	424*a*	+4.3	40....	429*a*	+2.1	35....	440*a*	−5.0
56....	424*b*	+2.6	40....	429*b*	+4.0	35....	440*b*	−6.1
56....	426*a*	+3.3	40....	430*a*	+3.5	39....	433*a*	−6.3
56....	426*b*	+2.6	40....	430*b*	+4.8	39....	433*b*	(+4.1)
56....	427*a*	+4.6	40....	431*a*	+5.6	39....	434*a*	−4.6
56....	427*b*	+4.6	40....	431*b*	+4.0	39....	434*b*	−6.3
56....	428*a*	(−5.4)	40....	432*a*	+5.3	39....	435*a*	−3.6
55....	429*a*	+5.4	N. 40....	432*b*	+3.8	39....	435*b*	−5.6
55....	429*b*	+5.3	S. 25....	433*a*	−3.0	39....	436*a*	−5.6
55....	430*a*	+4.1	25....	433*b*	(+5.3)	39....	436*b*	−5.6
55....	430*b*	+2.6	25....	434*a*	−5.1	39....	437*a*	−1.0
55....	431*a*	+3.3	25....	434*b*	−1.8	39....	437*b*	−3.3
55....	431*b*	+6.4	25....	435*a*	−5.4	39....	438*a*	−4.3
55....	432*a*	+5.6	25....	435*b*	−3.3	39....	438*b*	−5.6
55....	432*b*	+5.4	25....	436*a*	−2.6	39....	439*a*	−4.0
51....	424*a*	+2.3	25....	436*b*	−6.3	39....	439*b*	−7.6
51....	424*b*	+4.5	25....	437*a*	−1.7	40....	440*a*	−4.6
51....	426*a*	+3.3	25....	437*b*	−5.3	40....	440*b*	−3.5
51....	426*b*	+3.5	25....	438*a*	−4.8	47....	433*a*	−2.3
51....	427*a*	+5.4	25....	438*b*	−4.1	47....	433*b*	(+2.8)
51....	427*b*	+5.9	25....	439*a*	−2.0	47....	434*a*	−4.8
51....	428*a*	(−3.0)	25....	439*b*	−4.8	47....	434*b*	−6.4
N. 51....	428*b*	+4.6	S. 25....	440*a*	−4.0	S. 47....	435*a*	−3.6

TABLE VII—*Continued*

Lat.	Plate	Δ	Lat.	Plate	Δ	Lat.	Plate	Δ
S. 47°...	435*b*	−3.0	S. 48°...	440*a*	−6.3	S. 51°...	437*b*	−6.6
47....	436*a*	−3.3	48....	440*b*	−4.6	51....	438*a*	−5.6
47....	436*b*	−5.0	51....	433*a*	−6.3	51....	438*b*	−3.6
47....	437*a*	−2.3	51....	433*b*	(+4.0)	51....	439*a*	−5.6
47....	437*b*	−6.6	51....	434*a*	−4.6	51....	439*b*	−5.0
47....	438*a*	−4.6	51....	434*b*	−7.3	52....	440*a*	−3.1
47....	438*b*	−5.4	51....	435*a*	−6.6	S. 52....	440*b*	0.0
47....	439*a*	−4.3	51....	435*b*	−7.1			
S. 47....	439*b*	−5.6	S. 51....	437*a*	+0.3			

Lat.	Plate	Δ			Lat.	Plate	Δ		
		λ5250	λ5253	λ5263			λ5250	λ5253	λ5263
N. 69°..	431*a*	+2.0	+5.1	+1.7	S. 25°..	435*b*	−1.0	+0.3	−2.3
69...	431*b*	+4.5	(−3.1)	+2.6	25...	436*a*	−0.7	−0.3	−2.3
69...	432*a*	+1.8	+2.3	+3.8	29...	434*a*	−3.0	−3.0	+0.3
69...	432*b*	−1.3	+1.3	+0.3	29...	434*b*	(+3.6)	+1.3	−0.2
62...	431*a*	+3.6	+2.0	(+5.0)	29...	435*a*	+0.3	−0.3	−1.5
62...	431*b*	+4.0	+4.3	+1.3	29...	435*b*	−3.6	−3.6	−2.3
62...	432*a*	+1.3	+1.7	(−1.3)	29...	436*a*	+0.7	−3.5	−2.8
62...	432*b*	+1.8	+4.1	+1.0	35...	434*a*	−3.6	−3.8	−2.1
55...	431*a*	+0.7	+0.7	0.0	35...	434*b*	−2.3	−1.7	−3.3
55...	431*b*	+2.0	+3.5	+1.7	35...	435*a*	−3.6	−3.1	−1.3
55...	432*a*	+3.3	+2.6	+3.1	35...	435*b*	−2.6	−0.7	(+1.2)
55...	432*b*	+1.3	+4.3	+2.5	35...	436*a*	−2.6	−1.3	+0.7
50...	431*a*	+1.7	+0.3	+2.3	39...	434*a*	−3.0	−1.2	−3.6
50...	431*b*	+4.3	+2.6	0.0	39...	434*b*	−2.0	−3.5	−1.0
50...	432*a*	+4.3	+3.6	+2.3	39...	435*a*	−2.6	(+1.3)	−2.0
50...	432*b*	+1.0	+3.1	+3.3	39...	435*b*	−2.1	−5.3	−1.7
45...	431*a*	+2.3	+2.0	+1.3	39...	436*a*	−3.6	−3.3	−4.0
45...	431*b*	+2.8	+3.3	+2.3	47...	434*a*	−2.3	−4.0	−1.3
45...	432*a*	+3.3	+2.3	+1.2	47...	434*b*	−4.0	−0.7	−0.3
45...	432*b*	+5.3	+2.6	+2.3	47...	435*a*	+0.2	−3.6	−4.1
40...	431*a*	+3.6	+2.1	+1.0	47...	435*b*	−0.3	−3.6	−5.0
40...	431*b*	+3.0	+1.0	+2.0	47...	436*a*	−4.3	−2.3	−3.0
40...	432*a*	+1.8	+2.6	+2.1	51...	434*a*	−0.8	−2.6	−1.0
N. 40...	432*b*	+0.2	+1.8	+3.6	51...	434*b*	−5.6	(−6.3)	(−6.3)
S. 25...	434*a*	(+2.6)	+0.3	+1.0	51...	435*a*	−1.7	−2.3	−3.6
25...	434*b*	−2.6	−1.0	−1.2	51..	435*b*	−3.0	−2.6	−1.0
S. 25...	435*a*	−2.3	−4.0	−1.8	S. 51...	436*a*	−3.6	−1.0	−3.3

TABLE VII—*Continued*

Lat.	Plate	Δ						
		λ5300	λ5304	λ5328	λ5329.3	λ5329.9	λ5340	λ5348
N. 69°	430*a*	....	+8.3	+2.3	+1.3	+3.6	+4.8	+1.3
69	430*b*	+3.3	....	....	....	....	....	....
69	431*a*	+2.0	−0.2	+3.6	−1.0	+0.8	+4.6	−0.5
69	431*b*	+3.0	0.0	(+5.3)	+2.0	+5.8	+4.3	(+4.6)
69	432*a*	(+7.6)	(+14.2)	(+5.3)	+2.3	+5.9	+3.0	+1.5
69	432*b*	+4.6	+4.1	+1.8	+3.1	−2.3	+7.8	0.0
62	430*a*	+5.0	0.0	+2.6	−0.7	+2.1	+2.3	+0.7
62	430*b*	+0.7	....	....	....	....	....	....
62	431*a*	+4.8	+4.1	+3.0	+3.0	+5.0	+4.3	+3.8
62	431*b*	+5.3	+1.3	+2.8	(−2.3)	+1.3	+2.3	+1.7
62	432*a*	+2.6	+5.3	+1.3	+2.6	+3.6	+9.6	+2.0
62	432*b*	+5.0	+7.4	+2.6	−0.7	+6.9	+1.0	+3.6
55	430*a*	+1.7	(−4.6)	−0.7	+0.7	+2.6	−2.3	+0.7
55	430*b*	+0.8	....	....	....	....	....	....
55	431*a*	+3.6	+7.3	−0.5	−0.2	+2.6	−1.0	+1.7
55	431*b*	+3.8	+5.3	+2.0	+2.3	+1.2	−1.0	+1.5
55	432*a*	+4.3	+3.5	+4.0	+1.5	+4.0	+2.1	+3.0
55	432*b*	+1.3	+6.9	+3.6	+2.6	−2.6	+7.6	+1.3
50	430*a*	+3.3	+2.0	+2.6	+3.6	+5.3	+8.7	+2.8
50	430*b*	+3.6	....	....	....	....	....	....
50	431*a*	−0.3	−1.7	+1.8	+2.6	+4.3	+5.3	+0.8
50	431*b*	+2.8	+8.7	+2.3	+1.7	+0.8	+8.3	+2.3
50	432*a*	+2.1	+9.2	+0.7	+1.3	−0.3	+2.6	−0.3
50	432*b*	+5.0	+1.3	+1.2	+2.6	+8.6	+4.0	+2.3
45	430*a*	0.0	+6.9	+2.0	+1.7	+6.1	+4.6	+4.1
45	431*a*	+2.0	+4.6	+3.3	+2.6	−2.0	+2.5	+2.6
45	431*b*	+0.8	+2.0	+1.0	+2.0	+0.5	+8.4	+0.7
45	432*a*	+3.0	+5.6	+2.1	+1.3	+3.3	(−4.3)	+1.2
45	432*b*	+2.8	+7.8	+2.3	+2.3	+4.1	+7.9	+0.7
40	430*a*	+4.0	+5.9	+2.5	+5.3	+1.8	....	(+4.5)
40	431*a*	+3.0	0.0	+1.0	+3.0	+4.0	+5.0	+2.8
40	431*b*	−0.3	+6.3	+1.8	(−2.3)	+5.6	(−4.6)	+2.3
40	432*a*	+5.3	+3.0	+1.7	+1.8	−2.3	+8.7	+1.0
N. 40	432*b*	(−1.3)	+1.8	+2.0	0.0	−1.3	....	(−2.3)
S. 24	434*b*	....	(+10.9)	....	....	....	....	....
25	434*a*	−3.0	+2.5	0.0	−2.0	+0.7	−5.0	−1.7
25	434*b*	−3.6	....	....	0.0	−1.0	+0.7	−1.8
S. 25	435*a*	−2.0	....	−3.0	−3.6	−5.3	+1.3	−1.3

TABLE VII—*Continued*

Lat.	Plate	Δ						
		λ5300	λ5304	λ5328	λ5329.3	λ5329.9	λ5340	λ5348
S. 25	435*b*	....	....	−2.0	−3.8	....	....	(+1.7)
28	434*b*	....	+0.8	....	....	....	....	....
29	434*a*	....	+4.6	−3.3	−2.6	−4.8	−5.0	−1.3
29	434*b*	−0.7	....	−0.7	(+2.0)	−0.3	−5.9	−0.3
29	435*a*	−3.6	−6.6	−2.0	−2.3	−5.6	−10.2	−1.0
29	435*b*	....	....	−2.1	−2.3	....	....	−2.1
35	434*a*	−4.6	−4.5	(−5.3)	−3.6	−4.0	−4.1	−0.3
35	434*b*	−4.6	−7.4	−3.0	−0.5	−6.3	−0.3	−1.2
35	435*a*	−3.6	−5.0	−0.5	−2.1	+0.3	+0.3	+0.2
35	435*b*	....	....	−1.0	−1.6	....	....	+0.7
39	434*a*	−3.6	−11.6	+0.7	−2.3	−3.0	−0.2	−2.5
39	434*b*	−2.8	−3.6	−3.5	−5.0	−4.3	(−12.2)	−2.0
39	435*a*	−3.0	+0.3	−0.8	−0.3	−8.3	−2.6	−2.0
39	435*b*	....	....	−5.0	−4.5	....	....	−1.8
47	434*a*	+0.5	−8.6	−3.3	−4.3	−0.7	....	−2.0
47	434*b*	−5.0	−4.3	−0.5	−1.0	−6.4	−4.6	−0.2
47	435*a*	−0.2	−11.1	−2.6	−3.5	−3.1	−7.8	−0.3
51	434*a*	−2.0	−5.6	(+1.8)	−1.0	−2.1	−1.8	−0.7
51	434*b*	(−7.1)	−6.3	−2.6	−2.5	−9.2	−5.1	(−5.0)
51	435*a*	−3.1	−3.0	−1.0	+1.5	−3.8	−3.6	−1.2
S. 51	435*b*	....	....	....	....	....	....	−3.1

The coefficients A and B are defined by

$$\left.\begin{aligned} A &= 3\sin(2\phi - D) + \sin D \\ B &= 3\cos(2\phi - D) + \cos D \end{aligned}\right\} \tag{2}$$

in which

ϕ = heliographic latitude of the point observed,
D = heliographic latitude of sun's center.

Were D and i both zero, $k\Delta$ would equal $3\sin 2\phi$, which has zero values at $\phi = 90°$ N., 0°, and 90° S., a maximum at 45° N. and a minimum at 45° S. Since the observed displacement curves are approximately of this character, it follows that i must be a small angle (D is known to be small). This permits the calculation of the field-strength without further knowledge of i or λ, for applying equation (1) to equal northern and southern values of ϕ,

$$k(\Delta_n - \Delta_s) = 6\sin 2\phi\,(\cos D\cos i + \sin D\sin i\cos\lambda), \tag{3}$$

whence[1]

$$\frac{4}{CH_p}(\Delta_n - \Delta_s) = 6\sin 2\phi, \tag{4}$$

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[1] *Mt. Wilson Contr.*, No. 72, p. 11; *Astrophysical Journal*, **38**, 109, 1913.

in which

Δ is to be expressed in angstroms, and

H_p = field-strength in gaussian units at sun's magnetic pole,

C = separation of the n-components of the triplet in angstroms for a field of 1 gauss.

The omission of D, i, and λ has introduced an error of the order of i^2 (about 1 per cent), a precision that is ample, for the individual values of Δ are subject to uncertainties of 25 per cent or more.

For the determination of H_p it is convenient to apply equation (4) to the displacements observed at $\phi = 45°$, thus giving

$$H_p = \frac{4\Delta_{45}}{3C} \tag{5}$$

in which Δ_{45} is the mean of the absolute values for $\phi = 45°$ N. and S.

To determine Δ_{45}, equation (1) may be written

$$k\Delta = A \tag{6}$$

whence

$$\Delta_{45} = \Delta \frac{A_{45}}{A} \tag{7}$$

This neglects quantities of the order of i, but a combination of results for the observations in the northern and southern hemispheres, which with few exceptions are symmetrically distributed, reduces the error to one of the second order. In fact, the application of (6) to equal values of ϕ, N. and S., leads directly to (4), which is of this precision. The use of (7) is facilitated by the tabulation of A with the arguments ϕ and D.

To shorten the calculation, means were found for groups of displacements observed for each line in neighboring latitudes, the limits in general being 20°–29°, 30°–39°, 40°–49°, 50°–59°, 60°–69°, N. and S. The occasional values of Δ below 20° and above 69° were disregarded, since their contribution to the weight of Δ_{45} would have been insignificant.

The value of Δ_{45} having been found by a least-squares solution (the two hemispheres were treated separately), k was calculated by (6), which was then used for the detection of discordant

TABLE VIII

Displacements for Lines of Series VIII

Lat.	Plate	Δ
λ 4406.810		
N. 60°	470*a*	+4.0
60	470*b*	+3.0
52	470*a*	+3.5
52	470*b*	−0.3
52	480*a*	+0.2
48	470*b*	+1.0
47	480*a*	+2.5
41	470*a*	+3.5
40	480*a*	+5.6
36	470*a*	+3.3
36	470*b*	+3.5
35	480*a*	+2.8
31	480*a*	(+7.4)
N. 27	480*a*	+2.3
S. 33	463*a*	−4.8
33	463*b*	−4.5
33	464*a*	−1.3
33	464*b*	(+1.5)
38	463*a*	−4.8
38	463*b*	−4.0
38	464*a*	−3.0
38	464*b*	−1.8
43	463*a*	−3.1
43	463*b*	−2.5
43	464*a*	−5.0
43	464*b*	−5.9
51	463*a*	−2.6
51	463*b*	+0.2
51	464*a*	−2.3
51	464*b*	−2.0
57	463*a*	−1.7
57	463*b*	0.0
57	464*a*	−2.0
S. 57	464*b*	−5.1
λ 4418.499		
N. 60	470*a*	+2.6
60	470*b*	+4.6
52	470*a*	+3.3
52	470*b*	+1.0
52	480*a*	+1.3
48	470*a*	+3.3
48	470*b*	+2.3
47	480*a*	+2.5
41	470*a*	+4.8
41	470*b*	+1.2
40	480*a*	+5.8
36	470*a*	+2.6
36	470*b*	+3.1
35	480*a*	+3.3
31	480*a*	+2.0
N. 27	480*a*	+4.3
S. 33°	463*a*	−3.0
33	463*b*	−4.3
33	464*a*	−3.8
33	464*b*	−1.7
38	463*a*	−6.3
38	463*b*	−5.1
38	464*a*	(−6.6)
38	464*b*	−0.3
43	463*a*	−3.8
43	463*b*	−3.3
43	464*a*	−1.3
43	464*b*	−2.3
51	463*a*	−1.3
51	463*b*	−3.3
51	464*a*	−4.1
51	464*b*	−0.2
57	463*a*	−2.3
57	463*b*	−4.5
57	464*a*	−1.3
S. 57	464*b*	−2.6
λ 4421.733		
N. 60	470*a*	+2.0
60	470*b*	+1.3
52	470*a*	+1.7
52	470*b*	+3.0
52	480*b*	+5.0
48	470*a*	+3.3
48	470*b*	+2.3
46	480*b*	+5.6
41	470*b*	+0.3
40	480*b*	+4.3
36	470*a*	+4.0
36	470*b*	+1.5
35	480*b*	+5.3
31	480*b*	+1.3
N. 27	480*b*	+6.3
S. 32	478*a*	−8.9
32	478*b*	+1.3
32	479*a*	−5.3
32	479*b*	−8.9
33	463*a*	−6.8
33	463*b*	−7.9
33	464*a*	−2.1
36	478*a*	−7.6
36	478*b*	(+5.3)
37	479*a*	−5.3
37	479*b*	−5.9
38	463*a*	−6.3
38	463*b*	−9.9
38	464*a*	−7.9
S. 42	478*a*	(+5.0)
S. 42°	479*b*	−5.3
43	463*a*	−9.6
43	463*b*	−7.3
43	464*a*	−7.6
47	478*a*	−3.0
47	478*b*	−4.6
47	479*a*	−4.0
47	479*b*	−5.0
51	463*a*	−5.9
51	463*b*	−3.6
51	464*a*	(−11.4)
53	478*a*	−5.6
53	478*b*	−2.8
53	479*a*	−1.2
53	479*b*	−6.9
57	463*a*	+0.5
57	463*b*	−8.9
57	464*a*	−2.0
58	478*a*	−8.9
58	478*b*	−2.8
58	479*a*	−5.8
S. 58	479*b*	−2.6
λ 4430.785		
N. 52	480*a*	(−0.8)
47	480*a*	+2.3
40	480*a*	+3.3
35	480*a*	+2.0
31	480*a*	+3.0
N. 27	480*a*	+3.6
S. 33	463*a*	−2.5
33	463*b*	−3.0
33	464*a*	−1.0
33	464*b*	−2.8
38	463*a*	−5.1
38	463*b*	−5.3
38	464*a*	−4.0
38	464*b*	−2.6
43	463*a*	−5.1
43	463*b*	−3.3
43	464*a*	−3.6
43	464*b*	−1.8
51	463*a*	−0.7
51	463*b*	(+2.3)
51	464*a*	−3.1
51	464*b*	−2.5
57	463*a*	−1.5
57	463*b*	−3.6
57	464*a*	0.0
S. 57	464*b*	−4.0

TABLE VIII—*Continued*

Lat.	Plate	Δ	Lat.	Plate	Δ	Lat.	Plate	Δ
λ 4438.006			λ 4438.006—*Cont.*			λ 4438.006—*Cont.*		
S. 33°...	463*a*	+0.3	S. 43°...	463*a*	−0.5	S. 51°...	464*b*	−3.1
33....	463*b*	−3.6	43....	463*b*	−2.3	57....	463*a*	−3.3
33....	464*a*	−2.6	43....	464*a*	−0.7	57....	463*b*	−2.3
33....	464*b*	0.0	43....	464*b*	−2.0	57....	464*a*	+0.7
38....	463*a*	−3.5	51....	463*a*	−3.3	S. 57....	464*b*	−4.0
38....	463*b*	−3.3	51....	463*b*	+0.3			
S. 38....	464*b*	(+3.8)	S. 51....	464*a*	(−9.2)			

observations. Values giving residuals greater than three times the probable error were rejected, and the calculation was then revised for the determination of final values of Δ_{45} and k. Rejected observations appear in parentheses in Tables V–VIII.

The results are given in Table IX, which contains the values of Δ_{45} for each hemisphere without distinction as to sign, their differences and their means, the value of k, the original number of observations within the chosen limits for ϕ, the number rejected, and the probable error of a single observed value of Δ derived from a comparison with the theoretical displacement curve written in the form (6). The unit for Δ_{45} and the probable error is 0.001 mm. The table also indicates, in the last column, the value of one angstrom in millimeters, which is necessary for the transformation of Δ_{45} into angstroms.

In order to utilize all the available data, the results by van Maanen on λ 5929.898, Series I and III, and on λ 5812.139 and λ 5828.097, Series IV,[1] are also given in Table IX. To secure homogeneity these have been rediscussed by the foregoing method, which accounts for small differences between the values in Table IX and those previously published. The only results not included are the fragmentary measures of λ 5812 and λ 5828 in Series I and the results of Series II, which were from second-order spectra.

The consistency of the data is illustrated by the differences in the fifth column of Table IX, which by equation (1) are of the form

$$k(\Delta_{+45}+\Delta_{-45})=2\sin D\cos i+2\cos D\sin i\cos\lambda \tag{8}$$

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[1] *Mt. Wilson Contr.*, No. 71, p. 63; *Astrophysical Journal*, 38, 87, 1913.

and have the limiting values $2\sin(D+i)$ and $2\sin(D-i)$. The numerical results are such that i, as already assumed, cannot exceed a few degrees. Considering the difficulty of measurement and the smallness of the unit in which the differences are expressed (1 μ), the internal agreement is very good, especially for Series V and VII, which include later measures and plates of somewhat better quality.

TABLE IX

VALUES OF Δ_{45}

(Unit for Δ_{45} and P.E. is 0.001 mm)

SERIES	λ	Δ_{45}				k	NO. OBSER.		P.E. 1Δ	FACTOR
		N.	S.	Diff.	Mean		All	Rej.		
I....	5929.898	3.69	5.02	−1.33	4.36	0.70	71	7	±2.96	4.91
III...	5929.898	4.77	5.50	−0.73	5.14	0.59	125	9	3.38	4.91
IV...	5812.139	4.96	4.16	+0.80	4.56	0.66	96	10	1.61	4.87
	5828.097	4.44	3.74	+0.70	4.09	0.74	98	11	1.44	4.88
	5831.821	1.48	2.14	−0.66	1.81	1.69	86	6	1.66	4.88
	5856.312	5.36	3.55	+1.81	4.46	0.70	54	5	1.72	4.89
	5928.013	3.70	2.10	+1.60	2.90	1.12	74	5	2.15	4.91
V....	6007.540	2.44	2.31	+0.13	2.38	1.26	72	2	2.51	4.93
	6039.953	6.15	5.79	+0.36	5.97	0.50	85	6	2.17	4.94
	6079.227	4.81	5.07	−0.26	4.94	0.60	74	3	0.99	4.95
	6111.290	1.99	2.18	−0.19	2.08	1.44	66	4	1.49	4.96
	6119.740	4.90	4.42	+0.48	4.66	0.64	83	8	2.10	4.96
	6129.190	3.41	3.83	−0.42	3.62	0.83	80	5	2.08	4.96
	6149.458	3.64	3.73	−0.09	3.68	0.81	76	5	1.79	4.96
	6173.553	5.14	5.10	+0.04	5.12	0.58	90	5	1.58	4.97
VII..	5247.737	4.61	4.89	−0.28	4.75	0.62	184	12	1.50	4.72
	5250.817	2.60	2.50	+0.10	2.55	1.16	54	2	1.14	4.72
	5253.633	2.69	2.39	+0.30	2.54	1.18	54	3	1.14	4.72
	5263.486	2.00	2.06	−0.06	2.03	1.47	54	4	1.03	4.72
	5300.929	2.96	2.96	0.00	2.96	1.01	50	3	1.31	4.74
	5304.355	4.46	6.15	−1.69	5.30	0.58	47	3	2.94	4.74
	5328.515	2.12	2.02	+0.10	2.07	1.44	51	4	1.01	4.75
	5329.329	1.94	2.36	−0.42	2.15	1.40	52	3	1.17	4.75
	5329.975	2.75	3.98	−1.23	3.36	0.92	48	0	2.07	4.75
	5340.639	4.73	3.58	+1.15	4.16	0.73	45	3	2.60	4.75
	5348.511	1.79	1.29	+0.50	1.54	1.98	53	5	0.95	4.75
VIII.	4406.810	2.88	3.13	−0.25	3.00	1.00	34	2	1.28	4.50
	4418.499	3.14	3.04	+0.10	3.09	0.97	36	1	1.10	4.51
	4421.733	3.33	5.74	−2.41	4.54	0.72	52	3	2.12	4.51
	4430.785	3.09	3.04	+0.05	3.06	0.98	26	2	1.01	4.51
	4438.006	......	2.09	....	2.09	1.41	19	2	±1.44	4.51

5. CONFIRMATION OF THE EXISTENCE OF THE SUN'S GENERAL FIELD

It is appropriate at this point to remark upon the character of the evidence now presented as bearing upon the existence of the sun's general magnetic field. It is scarcely necessary to state that it confirms the results of the preliminary investigation. For example, a moment's inspection of Tables V–VIII shows that the displacements of the twenty-six additional lines agree with those found for the four lines discussed in *Mount Wilson Contribution*, No. 71. The algebraic signs are opposite in the northern and southern hemispheres and give the same magnetic polarity as was deduced from the earlier results. Such differences as occur relate mainly to the amplitude of the displacement-curves, and this is determined by the field-strength at the points in which the different lines originate and the magnitude of their respective Zeeman separations, modified to some extent perhaps by systematic influences depending upon line-intensity.

Comparing the variation of the displacements as a function of latitude with the theoretical behavior of a uniformly magnetized sphere, we find as before an agreement that is within the limits of the uncertainty affecting the measured displacements. Moreover, the small differences in the values of Δ_{45} for the northern and southern hemispheres, which have been collected in Table IX, leave no doubt as to the correctness of the original estimate that the inclination of the magnetic axis to the solar axis of rotation cannot exceed a few degrees.

Added weight is given to these conclusions by the precautions taken to avoid physiological error and by the independent confirmation of the displacements by measurers who were entirely ignorant of the data of observation. But by drawing upon the results of another investigation, which will be published in detail later,[1] we can make the case even stronger.

In the present paper we have been content to indicate that the inclination of the sun's magnetic axis does not exceed a few degrees and may be disregarded without appreciably affecting the results of the discussion. It is easy, however, to rearrange the fundamental equation (1) in such a way as to derive from suitably arranged observations made on a single day a value of the quantity

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[1] See, however, a preliminary account in *Mount Wilson Communication*, No. 50; *Proceedings National Academy of Sciences*, 4, 4, 1918.

$$Y = \tan i \cos \lambda \tag{9}$$

The longitude λ varies from day to day, and a comparison of the values of Y for different days extending over a sufficient interval will enable us to determine the inclination i and the period of revolution P. From an extended series of observations on three chromium lines, λ 5247, λ 5300, and λ 5329, it has been found that the calculated values of Y are actually in close agreement with a periodic function of the form of (9).[1]

It is of interest to consider the implications of this result. The curve of displacements defined by equation (1) is very nearly a sine curve. Disregarding D, which is always small, the curve will pass through the origin if i is zero. In general, for $\phi = 0$,

$$k\Delta = -2 \sin D \cos i + 4 \cos D \sin i \cos \lambda, \tag{10}$$

and since i as well as D is small, the curve always passes near the origin, Δ_0 having values that are sometimes positive and sometimes negative.

Thus the changing position of the magnetic axis caused by its revolution around the sun's axis of rotation produces a small shift of the displacement-curve in its own plane, together with some change of form; and from these second-order effects the inclination and period have been derived. Since the three lines selected for the investigation were observed on sixty-three days (the observations extend over an interval of one hundred and ten days), nearly two hundred separate curves enter into the calculation. Not only is the characteristic form revealed in every case, but, of far more importance as evidence of the reality of the field, the curves for the separate days are so related in form and in position with respect to the origin that the second-order quantities Y satisfy equation (9) throughout the entire series, including more than three complete revolutions of the magnetic axis about the axis of rotation.

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[1] It may be added that the resulting value of i is $6^\circ\!.2 \pm 0^\circ\!.4$, while the magnetic axis revolves about the sun's axis of rotation in a period of 31.79 ± 0.31 days.

Various details are illustrated by Figs. 3 and 4. The first gives a series of displacement curves for two dates, September 2 and 14, 1914—both the original observations and the theoretical curves—based upon the unknowns

$$x=\frac{\cos i}{k}, \qquad y=\frac{\sin i \cos \lambda}{k}$$

calculated from the data for the respective days. Aside from the close accordance of the plotted points, the shift in the position of the curves with respect to the origin is to be noted. Since D was sensibly constant and equal to $+7^\circ\!.2$, the change in the algebraic sign of Δ for $\phi=0$ indicates a similar change in the sign of $Y=\sin i \cos \lambda$; see equation (10). The northern end of the magnetic axis was accordingly directed toward the observer on September 2 (λ near 0°) and away from him on September 14 (λ near 180°). Fig. 4 shows the close agreement of the values of $Y=y/x$ with equation (9). These results, particularly those illustrated in Fig. 4, exhibit a degree of internal consistency which is a searching test of the validity of the conclusions as to the existence of the sun's general field.

On the other hand we must consider the significance of the negative results from the lines listed in Table IV, which are not affected to any measurable amount by the sun's general field, although all of them are susceptible to the action of the magnetic fields in sun-spots. Certain aspects of the question thus raised are discussed in a later section. At present it is sufficient to state that the probabilities established by the evidence favorable to the existence of the field are so great that the negative results are presumably to be attributed to some class distinction separating the two lists of lines.

Our experience indicates that a very considerable number of lines will ultimately be found to show the influence of the sun's general field. The list of elements represented by displaced lines now includes iron, chromium, nickel, vanadium, and titanium, but it will doubtless be possible to add other elements.

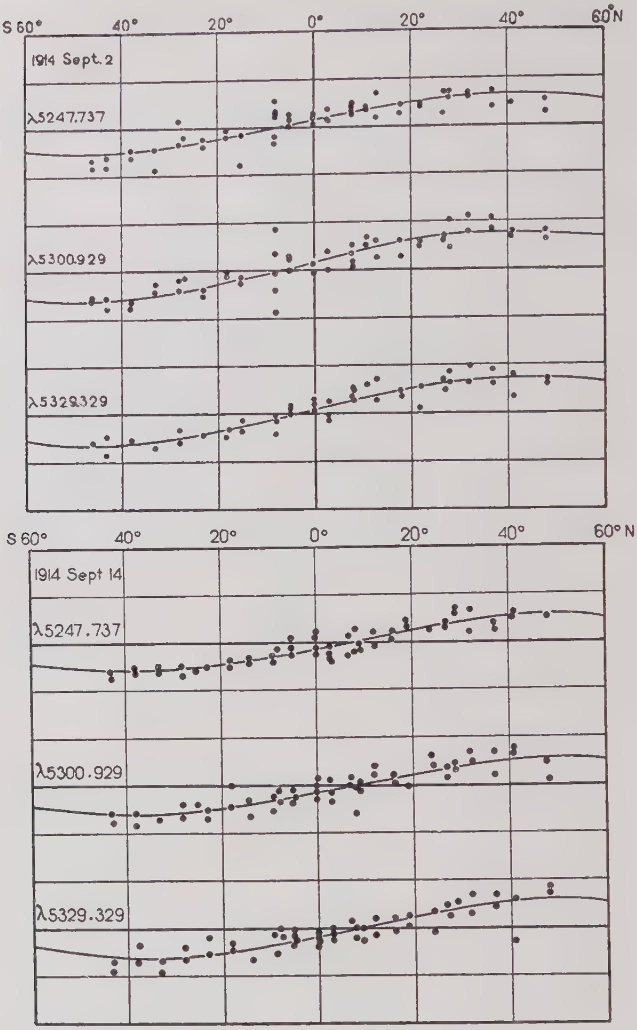

FIG. 3.—Displacement-curves for September 2 and 14, 1914. Abscissae are heliographic latitudes. Ordinates are displacements, the scale being 1 division of diagram = 0.005 mm. The curves, which correspond to equation (1), have been derived from the observed values of Δ. Their ordinates for $\phi = 0$ represent the combined influence of D, i, and λ. The data for the three lines give, as mean values of $Y = \tan i \cos \lambda$, $+0.213$ for September 2 and -0.159 for September 14. These are plotted as single points in Fig. 4, together with similar values of Y for each of the other dates.

6. CALCULATION OF THE FIELD-STRENGTH

The results in Table IX are now to be combined with the laboratory data in the fourth, fifth, and sixth columns of Table X, where C, the magnetic separation for a field of one gauss, is expressed in 0.0001 A as a unit. The values of Δ_{45} in the seventh column, expressed in the same unit, depend on the sixth and last columns of Table IX. The field-strength at the magnetic pole of the sun, which, it should be repeated, is here considered to be a uniformly magnetized sphere, is then found from C and Δ_{45} by equation (5).

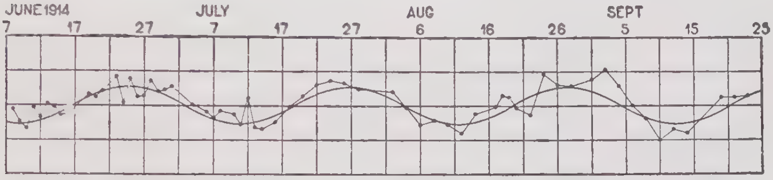

FIG. 4.—The curve $Y=\tan i \cos \lambda$. Each plotted point is derived from data of a single day similar to those illustrated in Fig. 3. Approximate values of i, P, and t_0 were read from a provisional curve. Differential corrections derived by a least-squares solution gave the final values, which correspond to the curve shown in the figure. The close agreement of the values of Y derived from the observed displacements with the theoretical curve is a most exacting test of the existence of the sun's general field.

Several of the laboratory data are from Mr. King's table[1] for iron, but most of the lines had to be specially observed. In spite of the great difficulties involved in the laboratory investigation of faint lines, Mr. Babcock has succeeded in determining C for all but four of the lines. One of these is unidentified, a second is a chromium line of intensity 0, and the others are the iron lines λ 5928.013 and λ 5929.898, both of solar intensity 2. In the previous paper doubt was expressed as to the reliability of the identification of λ 5929.898 with iron. The line has since been observed by Mr. Babcock in the spectrum of the core of the iron arc, but it does not appear in the spark and its separation by the magnet has not yet been determined. Hence we must still depend upon the approximate value of C derived from its separation in spots.

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[1] *Papers Mt. Wilson Observatory*, 2, Pt. 1; *Carnegie Institution Publication*, No. 153, 1912.

TABLE X

LABORATORY DATA AND FIELD-STRENGTH

λ	El.	Int.	No. Components	$\frac{\Delta\lambda}{a\lambda^2}$	C	Δ_{45}	H_p	Weight
4406.810	V	2	....	1.5	0.274	6.7	32.5	1
4418.499	Ti	1	6?	1.3	0.230	6.9	40.1	1
4421.733	V	0	12	2.4	0.45	10.1	29.9	1
4430.785	Fe	3	3	2.5	0.460	6.8	19.7	2
4438.006	V	0	4?	1.6	0.297	4.6	20.6	0.5
5247.737	Cr	2	3	2.54	0.653	10.1	20.6	3
5250.817	Fe	3	3?	1.5	0.381	5.4	18.9	2
5253.633	Fe	2	3?	1.5	0.377	5.4	19.1	2
5263.486	Fe	4	3	1.5	0.390	4.3	14.7	2
5300.929	Cr	2	3	1.9	0.498	6.2	16.6	2
5304.355	Cr	0	4	1.6	0.424	11.2	35.2	1
5328.515	Cr	2	9?	1.2	0.307	4.4	19.1	1
5329.329	Cr	3	....	1.7	0.440	4.5	13.6	1
5329.975	Cr	0	....	....	....	7.1	......	......
5340.639	Cr	0	9	1.6	0.438	8.8	26.8	2
5348.511	Cr	4	10?	1.6	0.429	3.2	9.9	2
5812.139	Fe	0	....	0.7	0.230	9.4	54.7	1
5828.097	......	0	....	....	....	8.4	......	......
5831.821	Ni	1	....	0.85	0.272	3.7	18.1	1
5856.312	Fe	2	....	1.0	0.306	9.1	39.6	1
5928.013	Fe	2	....	....	....	5.9	......	......
5929.898	Fe	2	....	....	0.72	9.9	18.3	1
6007.540	Ni	1	....	0.8	0.290	4.8	22.0	1
6039.953	V	0	4 or 6	1.5	0.502	12.1	32.2	2
6079.227	Fe	2	....	2.0	0.700	10.0	19.1	1
6111.290	Ni	2	3 or 4	0.9	0.315	4.2	17.8	1
6119.740	V	1	12	1.0	0.40	9.4	31.3	1
6129.190	Ni	1	....	1.1	0.332	7.3	29.3	1
6149.458	Fe	2	4?	1.3	0.446	7.4	22.2	1
6173.553	Fe	5	3	2.5	0.888	10.3	15.5	2

REMARKS RELATING TO LABORATORY DATA

λ 4406 Two groups each of *p*- and *n*-components, much widened.
4421 Weighted mean separation, based on measures of unresolved components.
4438 Components wide, but separation well determined; two sharp *p*-components.
5329.3 Only one *p*-component. Diffuse.
5329.9 Very weak and diffuse. Perhaps two *p*-components.
5340 Weak on laboratory plates.
5348 Two narrow groups of *n*-components; probably three or four in each, unresolved.
5831 *p*-components not observed; *n*-components not sharp.
5856 *p*-components not observed.
5928 Completely covered by an air line in spark. Inductance sufficient to cut out air line obliterates the iron line.
5929 Not observed in spark. *C* from observations of sun-spots.
6007 Difficult; *p*-components not observed.
6039 Two narrow groups of *n*-components, unresolved.
6079 Two *n*-components; violet *n*-component blended with adjacent line.
6111 Very difficult; *p*-components not observed; two *n*-components.
6119 Three groups of four components each. Measures of high weight, but *C* is approximate because of unequal intensities of *n*-components.
6129 Difficult; *p*-components not observed.

The results for the structure of the lines are far from complete. In several cases the number of components is uncertain or unknown. The number of normal triplets is relatively small; nevertheless the theory developed for lines of this class or, more generally, for those with three groups of components of definite intensity relations,[1] has been applied to all of the lines observed. The error thus involved in cases of unusual structure is probably small, for the field-strength is based upon the displacement at $\phi=45^\circ$, where the inclination of the line of sight to the lines of force is so small that the p-components and one group of the n-components are not transmitted to any appreciable extent by the analyzing apparatus of the spectrograph. Consequently the displacements must be nearly independent of the structure of the lines.[2] But Δ_{45} is not alone the result of observations at $\phi=45^\circ$, for it includes displacements observed throughout the interval $\phi=20^\circ$ to $\phi=69^\circ$. The substantial agreement of values of Δ_{45} found from different latitudes leads, however, to the conclusion that the influence of complex structure is unimportant.

The weights of H_p in the last column of Table X have been assigned after a consideration of various circumstances that affect the precision. Both faint and strong lines are difficult to measure. Thus far it has not been found possible to use lines fainter than 0, while the large scale of the photographs gives to lines of intensity 4 or 5 a width that greatly increases the difficulty of the settings. Lines of intensity 2 or 3 are perhaps most easily measured, but, independently of intensity, contrast and sharpness enter to such a degree that general statements must be cautiously made. Further, the number of observations naturally influences the precision of Δ_{45}, the reliability in this respect being indicated by the data in Table IX. On the laboratory side there is also a wide range in precision, according to the number, character, and the more or less perfect resolution of the components of the individual lines. The adopted weights represent an attempt to strike a balance between these different factors.

It will be noted that the value of H_p for λ 5929.898 (18.3 gausses) differs widely from that of 28 gausses previously published.

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[1] *Mt. Wilson Contr.*, No. 72, p. 11; *Astrophysical Journal*, 38, 109, 1913.

[2] *Mt. Wilson Contr.*, No. 72, p. 12; *Astrophysical Journal*, 38, 110, 1913.

The discrepancy arises from the large systematic difference between the measures of Miss Lasby and van Maanen. The original value was based upon the mean of the two series of measures, while, to secure homogeneity with the other results, that of Table X depends on the measures of van Maanen alone.

We desire at this point to express our obligations to Miss Richmond, Miss Wolfe, and Miss Felker of the Computing Division for their assistance with the least-squares reductions and the extensive calculations necessary for the determination of the field-strength from the individual lines.

7. VARIATION OF FIELD-STRENGTH WITH INTENSITY OF THE LINES

From an examination of Table X it is evident that the values of H_p vary with the intensity of the lines observed. To exhibit the nature of this relation, the data have been collected in Table XI, which also gives the weighted mean field-strength (the weights are in parentheses) corresponding to lines of each intensity for each of the elements thus far observed. It will be noted, that in general the values of H_p corresponding to a given intensity, at least for any single element, are approximately equal. In the case of iron, intensity 2, for example, the agreement is surprisingly good, with the exception of the result for λ 5856. The mean results are shown graphically in Fig. 5.

In forming the means, the iron lines λ 6079 and λ 6149, which are slightly enhanced, perhaps should have been omitted, since lines of this character probably correspond to a higher level than that indicated by unenhanced lines of the same intensity. The mean field-strength would not, however, have been appreciably modified. The single titanium line λ 4418 is also enhanced, though not to an important degree.

For iron and chromium we find a rapid decrease in field-strength with increasing line-intensity. For vanadium, nickel, and titanium the data are too slender to establish independently the nature of the relation; but, including lines of all intensities, the mean value of H_p for each of these elements is in general agreement with the results for iron and chromium. The field-strength for iron, line-

TABLE XI

Field-Strength, Intensity, and Level

Element	Intensity					
	0	1	2	3	4	5
Fe	5812 54.7(1)	...	5253 19.1(2)	4430 19.7(2)	5263 14.7(2)	6173 15.5(2)
	...	...	5856 39.6(1)	5250 18.9(2)	...	...
	...	...	5929 18.3(1)	...	...	...
	...	...	6079 19.1(1)	...	...	...
	...	...	6149 22.2(1)	...	...	...
Means	5812 54.7(1)	...	5753 22.9(6)	4840 19.3(4)	5263 14.7(2)	6173 15.5(2)
Level	270	...	335	370	410	450
Adopted	253	...	342	387	410	405
Cr	5304 35.2(1)	...	5247 20.6(3)	5329 13.6(1)	5348 9.9(2)	...
	5340 26.8(2)	...	5300 16.6(2)	...	...	...
	...	...	5328 19.1(1)	...	...	...
Means	5328 29.6(3)	...	5278 19.0(6)	5329 13.6(1)	5348 9.9(2)	...
Level	335	...	368	390	418	...
Adopted	335	...	368	390	418	...
Ni	...	5831 18.1(1)	6111 17.8(1)	...	...	...
	...	6007 22.0(1)	...	...	...	...
	...	6129 29.3(1)	...	...	...	...
Means	...	5989 23.1(3)	6111 17.8(1)	...	...	...
Level	...	361	375	...	...	...
Adopted	...	331	338	...	...	...
V	4421 29.9(1)	6119 31.3(1)	4406 32.5(1)	...	...	...
	4438 20.6(0.5)	...	...	...	...	...
	6039 32.2(2)	...	...	...	...	...
Means	5350 29.9(3.5)	6119 31.3(1)	4406 32.5(1)	...	...	...
Level	365	390	418	...	...	...
Adopted	315	317	387	...	...	...
Ti	...	4418 40.1(1)	...	...	...	...
Level	...	360	...	...	...	...
Adopted	...	361	...	...	...	...

intensity 0, is large in comparison with the corresponding value for chromium and may be considerably in error, for it depends upon a single line, difficult of measurement on the solar plates and troublesome to investigate in the laboratory because of its faintness.

In a series of important papers[1] Mr. St. John has shown in a convincing way that lines of increasing intensity represent successively higher levels in the solar atmosphere. In accordance with the suggestion on an earlier page, the data for field-strength and intensity in Table XI and Fig. 5 are therefore open to interpretation as the result of a variation of field-strength with level, and would indicate that the intensity of the sun's general field decreases rapidly in passing upward from the level represented by lines of intensity 0.

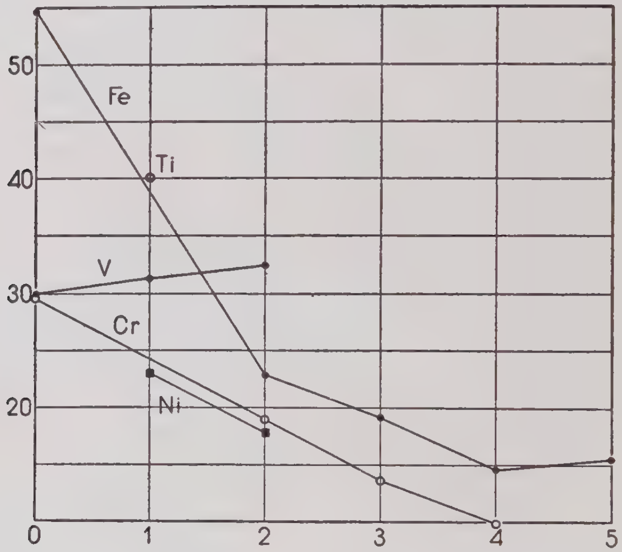

FIG. 5.—Variation of field-strength (ordinates, in gausses) with the Rowland intensity (abscissae) of lines in the solar spectrum.

In the main, the curves of Fig. 5 are in agreement with Mr. St. John's measures of radial velocities in sun-spot vortices. Thus we find from his chart[1] that lines of iron, chromium, vanadium, and nickel having the same intensity occur at approximately the same level, while titanium lines are relatively higher by one unit of intensity. In other words, we should expect the curves to lie close together, as they actually do. The systematic difference for iron and chromium is probably real. The lines observed may actually occur at slightly different elevations, as the two curves would indicate, for we are dealing with a limited number of lines observed over the general solar disk, while St. John's results are averages for large numbers observed at the outer edge of the penumbrae of sun-spots.

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[1] *Mt. Wilson Contr.*, Nos. 69, 74, 88; *Astrophysical Journal*, 37, 322, 38, 341, 1913; 40, 356, 1914.

But, as already stated, the correlation of the observed changes in field-strength with variations in level depends upon the assumption that the measured displacements are free from systematic errors which vary with the intensity of the lines observed. It is therefore important that the question of systematic errors be given careful attention.

8. SYSTEMATIC ERRORS

Comparing the results of different observers, the displacements determined by Miss Lasby[2] appear to be 50 per cent larger than those by van Maanen, and seem also to have been in excess of those found by Mr. Adams in his trial measures.[3] On the other hand the mean values for Miss Richmond and Miss Felker in Table I are appreciably smaller than those by van Maanen. Further, disregarding the inconclusive discordance for λ 5928, the measures by van Maanen are in satisfactory agreement with those from the Koch registering photometer, although the data are insufficient for final conclusions.

Personal errors, however, we may suppose to be approximately constant, and should not therefore affect the relative values of the field-strength derived from lines of different intensities. Depending upon the measures of a single observer, the data discussed in this paper should be homogeneous, and it is highly improbable that the important relation between field-strength and line-intensity should be a consequence of personal error.

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[1] *Mt. Wilson Contr.*, No. 74, p. 5; *Astrophysical Journal*, 38, 345, 1913.

[2] *Mt. Wilson Contr.*, No. 71, p. 63; *Astrophysical Journal*, 38, 87, 1913.

[3] *Mt. Wilson Contr.*, No. 71, p. 32; *Astrophysical Journal*, 38, 56, 1913.

Approaching the question of systematic errors from a different direction, we are confronted by the following apparently contradictory results:

1. The three lines λ 5329.329 (Cr, 3), λ 5812.139 (Fe, 0), and λ 5828.097 (—, 0) show displacements in the second order which are sensibly equal, when reduced to the same scale, to those observed in the third order (van Maanen estimates the intensity of λ 5329 on Mount Wilson plates to be 2).

2. In third-order spectra the lines λ 5247.737 (Cr, 2) and λ 5929.898 (Fe, 2) both show displacements attributable to the sun's general field, although neither is appreciably displaced in second-order spectra (van Maanen's estimate of the intensity of λ 5247 on Mount Wilson plates is 3).

3. The lines λ 6173.553 (Fe, 5) and λ 6302.709 (Fe, 5) have wide separations of nearly equal magnitudes in the spectra of sunspots. The former shows one of the largest displacements thus far observed in the case of the sun's general field (the measures are of third-order spectra); but the latter, λ 6302, has no measurable displacement in the first, second, or third orders.[1] (On Mount Wilson photographs λ 6302 is decidedly stronger than λ 6173 and much more difficult of measurement.)

It is difficult to find in these results any influence certainly due to intensity. The fact that the intensities of the lines under (1) are less than those under (2) might be suspected of having something to do with the failure of λ 5247 and λ 5929 to show displacements in the second order. But we should then be at a loss to know the bearing of this inference upon the third-order results for lines of high intensity, such as λ 6173 and λ 6302, referred to under (3); and the difficulty would only be increased when we attempted to account for the behavior of the faint line λ 6012.450 (Ni, 1), which has a very large displacement in the spectra of sunspots and none at all in the general-field photographs. Clearly something besides line-intensity is involved in these phenomena.

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[1] This statement refers to the mean result from a number of photographs extending over a wide range of heliographic latitude. Small displacements are shown by individual groups of spectrum strips which are probably real in part, although perhaps not the result of a magnetic field. Small irregularities of photographic density may produce appreciable shifts, but these will be accidentally distributed, and have nothing to do with the systematic displacements produced by the general field. Note in this connection the results for the registering photometer given on p. 6.

There are reasons for believing that the amount and distribution of the photographic density across a line are factors of importance in the measurement of minute shifts.[1] Both the lines which are undisplaced in the second order have sharp edges and show strong contrast on the photographs taken in this order. The third-order images of these lines, and both second- and third-order images of the lines mentioned under (1), are much flatter. Their density-curves have neither the steepness nor the height that characterize the second-order photographs of λ 5247 and λ 5929; and, in general, lines that show displacements do not possess intensity-curves of this type. The minimum density within a line (the reference is to the negative) and the contrast vary with exposure and development. Very faint lines naturally give difficulty in measurement, but low density and excessive contrast also produce unsatisfactory results. The ease of setting is of course greater in the case of sharp narrow lines, but for such lines displacements have not been detected.

These facts may perhaps be explained by supposing that it is the point of minimum density—the highest point of the intensity-curve—rather than the edges of the line that determines the value of a setting. In cases of excessive contrast the middle of the line is likely to be under-exposed and it will then be impossible to locate the minimum with precision, for throughout the central section there will be no appreciable deposit of silver.

The explanation also applies to the peculiar behavior of λ 6302 described under (3). On the Mount Wilson photographs its intensity is at least two units greater than that of λ 6173. In the third order it is so broad and diffuse that satisfactory measurement is impossible, while in the first and second orders the contrast and density are such that the point of minimum density cannot be located; λ 6173, on the other hand, is well suited to measurement.

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[1] In addition to the accompanying discussion, see also Hale, *Mt. Wilson Contr.*, No. 71, p. 50; *Astrophysical Journal*, **38**, 74, 1913.

It seems likely, therefore, that the form of the intensity-curve rather than intensity in the usual sense is a controlling factor in the detection and measurement of the displacements. This conclusion is not directly an answer to the inquiry as to the existence of systematic errors depending upon intensity, but it seems to account for the irregularities that might have been attributed to such an influence, and we are thus left without any evidence which would indicate that such errors have entered into the results.[1] Tentatively, we are therefore disposed to accept the relation between field-strength and intensity illustrated by the curves of Fig. 5 as an indication of a decrease in the field with increasing level in the solar atmosphere.

9. DISCUSSION OF THE RESULTS IN RELATION TO LEVEL IN THE SOLAR ATMOSPHERE

To examine further the results in relation to elevation in the solar atmosphere, they have been compared with the flash-spectrum measures of Mitchell.[2] A number of the lines employed were observed by him in the chromosphere; but in the nature of the case the precision of the calculated levels is less than that required to make the comparison effective, and we are therefore obliged to use mean levels. To this end the results calculated from the length of the chromospheric arcs were taken from Mitchell's tables for individual lines and grouped into means for each intensity unit of Rowland's scale. All blends and bracketed wave-lengths were omitted, and to avoid photographic influences only those lines between λ 4000 and λ 5000 were included.

This limitation is necessary in order that the calculated levels for the different elements may be comparable. The distribution of the lines throughout the spectrum is different for different elements, and, since the sensitiveness of the photographic equipment changes with the wave-length, systematic differences between the different intensity-and-level curves are likely to enter unless the data relate to the same limited region of the spectrum. An exception, however, was made in the case of nickel, for which the region λ5000–λ6000 was also used in order to compensate for irregularities in the interval λ4000–λ5000.

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[1] A method of testing this question by measurements in the second and third orders of lines slightly displaced by the weak fields surrounding sun-spots will be tried in the future.

[2] *Publications McCormick Observatory*, 2, Pt. 2; *Astrophysical Journal*, 38, 407, 1913.

The mean levels thus derived (in kilometers) have been entered in Table XI. Those for iron, chromium, and nickel show a regular increase in elevation with decreasing field-strength. For vanadium and titanium the results are discordant, but the evidence is inconclusive. Although the levels for these are systematically higher than for the corresponding intensities of other elements, the extreme range, including all elements, is small, and apparently that portion of the sun's general field effective in producing the observed displacements is limited to a very thin shell in the solar atmosphere. The results are illustrated in Fig. 6.

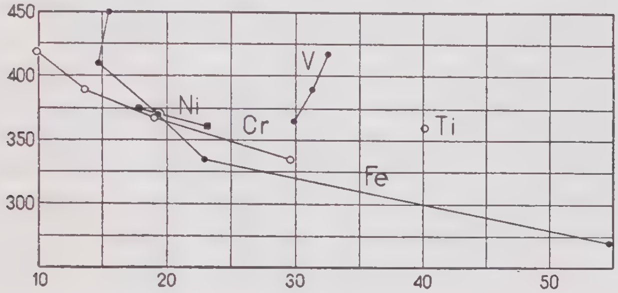

Fig. 6.—Variation of field-strength (abscissae, in gausses) with elevation in the solar atmosphere (ordinates, in kilometers). The levels require correction as described in sec. 9.

Various reservations and exceptions to these conclusions immediately suggest themselves and have now to be considered.

First, we have tacitly assumed that lines of a given element and intensity correspond to the same level in the sun's atmosphere independently of wave-length. But we have reason to believe that the levels determined from line-intensity are affected by scattering,[1] which certainly varies with the wave-length; in addition there may be influences depending on wave-length which are peculiar to the different elements. We can remove the effect of scattering by the application of an appropriate correction; but possible disturbances arising from other causes will be so involved with various uncertainties inherent in the observation of flash spectra that their consequences are necessarily disregarded.

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[1] St. John, *Mt. Wilson Contr.*, No. 69, p. 17; *Astrophysical Journal*, **37**, 338, 1913.

Secondly, elevations derived from flash spectra must be accepted with caution, owing to the fact that for different elements lines having the same solar intensity may behave quite differently in the chromosphere, some showing much greater intensities in the flash spectrum than others. Since lines of high intensity are represented in such spectra by relatively long arcs, the calculated levels may exhibit differences that are wholly fictitious. Such, for example, is the case with vanadium as compared with the other elements observed.

Thirdly, since the evidence for relatively greater elevation in the case of the enhanced lines seems unquestioned, it is not permissible to disregard this fact in the combination of the data now under discussion.

For a change in wave-length of 1000 A the influence of scattering is equivalent to about one unit of intensity,[1] lines to the red corresponding to a relatively lower level than their observed solar intensities would indicate. Levels assigned on the basis of the observed intensity will therefore be too high in the red and too low in the blue. The corrections apparently should vary directly as the fourth power of the wave-length, and may be derived with the aid of the levels in Table XI, care being taken to use for each element the rate of change in elevation per unit of intensity corresponding to the element and intensity in question.

As a matter of convenience we reduce the elevations to λ 5300, the wave-length of the chromium lines. The corrections in kilometers are given in Table XII.

To study the consequences of abnormal behavior of any element in the chromosphere, we may compare the intensities of its lines in the flash spectrum with those of the flash-spectrum lines of other elements. Flash intensities do not ordinarily correspond to those of the general solar spectrum. Lines of faint solar intensity commonly have flash intensities which are one or more units higher than the Rowland values, while the stronger lines of the general solar spectrum have relatively low intensities in the chromosphere.

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[1] St. John, *loc. cit.*

TABLE XII

Corrections to Level Depending on Wave-Length

Element	Intensity					
	0	1	2	3	4	5
Fe....	5812, −17		5753, −18	4840, +17	5263, 0	6173, −45
Ni....		5989, −17	6111, −21			
V.....	5350, 0	6119, −28	4406, +17			
Ti....		4418, +21				

For the intensities involved in this investigation the behavior of iron and chromium in the chromosphere is substantially the same, and the levels calculated for their respective lines should be comparable, in so far as the point now under discussion is concerned. But with nickel, vanadium, and titanium there are deviations to be considered, which are indicated by Table XIII. For example, the relatively high flash intensities of vanadium as compared with iron and chromium lines of the same solar intensity will lead to calculated levels that are too high, unless suitable corrections are applied. To derive such corrections, the flash-spectrum intensities may be plotted against the levels given in Table XI under the corresponding solar intensities. The changes in level corresponding to the differences of flash intensity in the last three columns of

TABLE XIII

Flash Intensities

Solar Intensity	Flash Intensity				Deviations from Fe and Cr		
	Fe, Cr	Ni	V	Ti	Ni	V	Ti
0..........	0.4	0.6	0.9	0.5	−0.2	−0.5	−0.1
1..........	0.7	1.1	1.4	1.0	−0.4	−0.7	−0.3
2..........	1.0	1.5	2.1	1.5	−0.5	−1.1	−0.5
3..........	1.4	1.4	3.0	1.8	0.0	−1.6	−0.4
4..........	2.0	1.3	4.0	2.2	+0.7	−2.0	−0.2

Table XIII can at once be read from the curves thus defined. The results thus found are given in Table XIV.

TABLE XIV

CORRECTIONS DEPENDING ON ABNORMAL FLASH INTENSITY

Element	Solar Intensity		
	0	1	2
Ni	....	−13	−16
V	−50	−45	−48
Ti	....	−20	....

Finally, to arrive at a rough correction of the levels of the enhanced lines λ 6079 and λ 6149 which will reduce them to the system of the other lines, we may make use of the results of Mr. Adams on the displacement of lines at the sun's limb.[1] These show that the enhanced lines are shifted by larger amounts than unenhanced lines of the same intensity. For λ 6079 and λ 6149 he finds shifts of +0.009 and +0.010 mm, respectively. These are equal to the displacements of unenhanced lines whose intensities are at least two units greater than those of the lines in question. We accordingly find corrections of +75 km. The result is not very reliable, but the scanty data available suggest that the corrections should be larger rather than smaller. The corresponding effect upon the level for iron lines of intensity 2 given in Table XI is +25 km.

The application of this and the corrections in Tables XII and XIV to the levels in Table XI gives the values which are provisionally adopted as corresponding to the mean field-strengths. These results also appear in Table XI opposite the word "Adopted."

With this revision of the levels, the curves of Fig. 6 assume the form shown in Fig. 7. Notwithstanding the tentative character of several of the corrections, an improved agreement is noticeable in various directions. The iron and chromium curves are now nearly coincident, and four of the five vanadium lines, which before were seriously discordant, are now in excellent agreement with iron and chromium. The fifth vanadium line, λ 4406 (2), and the single titanium line, λ 4418 (1), remain discordant, and the agreement for nickel is less satisfactory than before. The titanium line is slightly enhanced and presumably something should be added to the adopted level, but the data necessary for the calculation of the correction are lacking. It is not surprising, however, that individual lines should have levels which differ from the mean level of all lines of the same intensity; and the uncertainty of the corrections which reduce the results for nickel to λ 5300 is more than sufficient to account for the systematic deviation shown by that element in Fig. 7.

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[1] *Mt. Wilson Contr.*, No. 43; *Astrophysical Journal*, **31**, 30, 1910.

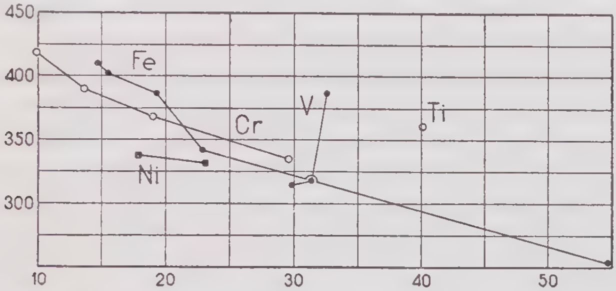

FIG. 7.—Adopted variation of field-strength with level in the sun's atmosphere. The elevations (ordinates) are in kilometers and the field-strengths (abscissae) in gausses.

The main point of this detailed discussion of corrections is that none of the disturbing influences mentioned is capable of modifying essentially what seems to be a fundamental relation between field-strength and level in the sun's atmosphere. Qualitatively the results of Fig. 7 are the same as those of Fig. 6, and quantitatively, even, the differences are confined within narrow limits.

10. INTERNAL EVIDENCE

It is now possible to point out certain internal agreements in the results that add much to the weight of the conclusions.

First, there is a wide range in the Zeeman separation of the components of the various lines under the influence of a constant field. Nevertheless, the measured displacements lead to closely accordant values for the field. As an example, the iron lines λ 5253 and λ 6149, of intensity 2, coefficient of separation about 0.4, give for H_p the same value as that derived from λ 5929 and λ 6079, also of intensity 2, whose coefficient is 0.7. In fact the ten lines of intensity 2 may be divided into two groups according to large and small values of C, respectively, with mean results as follows:

C	H_p	Weight	No. of Lines
0.676	19.8	5	3
0.378	22.5	9	7

The two groups of lines give nearly the same value for H_p, although for a given field-strength the separations of one group as observed in the laboratory are nearly twice those of the other. Their solar displacements must therefore bear a similar relation to each other, and the precision of the measures has accordingly been such as to reflect these differences in the behavior of individual lines.

Secondly, reference has been made to the systematic difference in field-strength for iron and chromium lines of the same intensity illustrated in Fig. 5. The difference does not appear in Fig. 7, and upon examination we find that this is due to the circumstance that the calculated levels for chromium are higher than those for iron (see Table XI), the differences in field and intensity compensating each other in such a way that for a given level the calculated strength is the same whether we use the lines of iron or chromium. In other words the hypothesis of a change of field with level reconciles the differences of Fig. 5, and the observed displacements for these two elements differ by just the amount required for them to yield the same field-strength for a given elevation.

Thirdly, the application of the corrections derived in the preceding section led in general to an improved agreement. Since all of the corrections are based upon the hypothesis of levels, the accordance of the final results is in favor of the applicability of this hypothesis.

11. FAILURE OF CERTAIN LINES TO REVEAL THE GENERAL FIELD

The influence of the density and contrast of a spectral line upon the detection of displacements due to the sun's general field has already been discussed. The apparently great importance of these factors raises a question as to the part they may have played in producing the negative results summarized in Table IV. It is certain, however, that the character of these lines is not wholly responsible for the absence of measurable displacements, and it is unlikely that it has had any considerable influence, for many of the lines are of a satisfactory quality.

It is evident, therefore, that in the case of Table IV we are dealing with an anomaly of another kind. All of the lines there listed show displacements in the spectra of sun-spots. For some of these lines the separation of the components in spots is less than the average spot-separation of the lines in Table I, but the behavior of others (for example, λ 6005, λ 6012, and λ 6081) in spots is entirely comparable with that of the lines in Table III. The line λ 6012 (Ni, 1) is especially noteworthy. Its components are more widely separated by the magnetic fields in sun-spots than those of any of the lines in Table III excepting λ 6173 (Fe, 5); but on the general-field plates it shows no appreciable displacement, although other lines of similar intensity on the same photographs, such as λ 6007 and λ 6039, are certainly displaced.

There is obviously a lack of parallelism between the results for spot fields and for the sun's general field. At present no complete explanation of these differences can be given; we can only offer various suggestions that later experience may prove to be of more or less significance.

In general it is to be noted that sun-spot fields extend over a much greater range of level than can at present be explored in the case of the sun's general field. Moreover, the conditions of pressure, temperature, and ionization are more or less different in the two cases; and it is not surprising that complete parallelism in results should not be found for fields existing under such diverse conditions. It seems not improbable that, in the sun's general atmosphere, some of the lines listed in Table IV originate outside the thin shell of a hundred or more kilometers' thickness which seems to include that part of the field now accessible to observation.

A careful examination of the displacements in sun-spot vortices by Mr. St. John and Miss Ware reveals no essential difference between the two classes of lines. On the other hand, Mr. King, from a consideration of temperature conditions, finds for chromium and vanadium, at least, some indication of a class distinction. His conclusions are as follows:

The iron and nickel lines of Table III are faint in the arc and for the most part have not been observed in the furnace. The chromium and vanadium lines are usually strong at all furnace temperatures. The vanadium lines λ 4438 and λ 6120 are relatively strong at the lowest temperature at which the spectrum appears.

Table IV contains no lines which are in any respect low-temperature lines. The iron lines are of the type seeming to require not only high temperature but high-vapor density. They are shown best by the core of the arc and by the tube-arc, and with great difficulty by the furnace and the spark. Two titanium lines and one calcium line are produced by the furnace, but are faint or absent at low temperature.

12. THE LOCAL-WHIRL HYPOTHESIS

In *Mount Wilson Contribution*, No. 71, it was suggested that the general magnetic field of the sun might result from the combined effect of a great number of local whirls. Evidence opposing this view was offered, but the material available for discussion was insufficient, and we may now return to a consideration of the suggestion, which is regarded favorably by both Birkeland[1] and Brunt.[2]

The hypothesis assumes the existence of a large number of minute spots or pores, too small to be distinguished as such, but having magnetic and other properties similar to those of visible spots. Recent investigations at Mount Wilson show these properties to include:

1. An almost universal tendency of spots to occur in pairs, the chief members of which are of opposite magnetic polarity. This tendency to form bipolar groups was shared by the few extremely small spots which appeared near the last sun-spot minimum, when the general magnetic field was under observation.

2. The preceding members of bipolar spot-groups, in the great majority of cases, are of opposite polarity in the northern and southern hemispheres.

3. Since the last sun-spot minimum the preceding members of bipolar groups have been opposite in polarity to those observed in the same hemisphere before the minimum.

[1] *Comptes Rendus*, **157**, 394, 1913.

[2] *Astronomische Nachrichten*, **196**, 169, 1913.

4. The magnetic axes of sun-spots are in approximate coincidence with solar radii.

5. The strength of the magnetic field of a spot is roughly proportional to its area.

Minute bipolar spots, even if distributed uniformly over the sun, could not account for the general field, for the following reasons:

1. If the members of each bipolar group were of equal field-strength, they would exactly neutralize each other in their combined effect.

2. If, as is usually the case, the preceding members of bipolar groups were larger, on the average, than the following members, the polarity of a general field due to their predominating magnetic influence should have reversed at the sun-spot minimum. Our observations show, on the contrary, that the polarity of the general field did not reverse at the minimum.

Suppose we assume, however, that the minute dark spaces between the brighter granules of the solar surface represent small single spots, opposite in polarity in the northern and southern hemispheres. It is not clear why they should fail to show the bipolar characteristics of larger spots, but, as single spots with almost no indications of bipolar structure are not very uncommon, we may make this assumption. It is more difficult to admit the possibility that their polarity did not reverse at the sun-spot minimum, because single spots, as well as bipolar groups, shared in this remarkable change. In any case, however, the hypothesis must face the fact that the magnetic axes of visible spots are very nearly radial, and that the granulated structure is best seen at the center of the sun, as though we were looking down through the bright granules into the darker regions between them. Under such conditions it is hard to see how we could fail to observe the Zeeman effect to the best advantage (along the lines of force) at the center of the sun, where the general magnetic field shows no displacements. If this objection were waived, it would still be necessary to assume a distribution of the granules such as to account for the observed inclination of the magnetic axis to the axis of rotation (6°2). Finally, there would remain the serious difficulty, pointed out by Brunt, of supposing that the total charge of electricity in a pore, or in a minute vortex between the granules, is almost as great as that required to account for the intense magnetic field in a large sun-spot.

While for the foregoing reasons we are not inclined to give favorable consideration to the hypothesis of local whirls, further attempts will be made to overcome the observational difficulties of photographing the spectra of the dark spaces between the granules on a sufficient scale to detect possible local Zeeman effects.

In considering the bearing of our results on theories of the solar magnetism it should be remembered, quite apart from the question of systematic errors, that our highest values do not necessarily represent the full intensity of the sun's field. On the contrary, the apparent variation of the field-strength with level in the solar atmosphere renders it probable that more intense fields may ultimately be detected at lower levels. This change of field-strength, if actually as rapid as the results seem to indicate, will also afford a useful criterion of a satisfactory theory, which must furthermore account for the observed inclination of the magnetic axis to the sun's axis of rotation.

13. SUMMARY

The present investigation is a continuation of that in *Contribution*, No. 71. Measures of displacements are given for twenty-six additional lines in the solar spectrum belonging to the elements iron, chromium, nickel, vanadium, and titanium (Tables V–VIII). Eighteen other lines (Table IV), all of them susceptible to the influence of magnetic fields in sun-spots, show no measurable shift. The twenty-six lines, which through changes in position indicate the presence of a magnetic field (Table IX), confirm the results detailed in *Contribution*, No. 71, and seem to place beyond reasonable doubt the conclusion that the sun behaves approximately as a uniformly magnetized sphere (secs. 1, 2, and 5), with the magnetic axis only slightly inclined to the solar axis of rotation and a polarity corresponding to that of the earth.

Laboratory data are available for twenty-seven of the thirty lines (indirectly, for one line) known to be influenced by the sun's general field. Combined with displacements observed in the solar pectrum, these yield for each line a value of the field-strength at the magnetic pole (sec. 6 and Table X). An intercomparison of results (Table XI) shows that the field decreases with increasing values of the Rowland intensity of the respective lines (sec. 7). Since line-intensity increases with the level at which the various lines originate, it would appear that the strength of the sun's general field falls off rapidly with increasing elevation in the solar atmosphere.

Using Mitchell's observations of the flash spectrum, we find that the part of the field now accessible to observation lies within the bounding surfaces of a thin shell in the solar atmosphere, whose thickness seems to be of the order of 150 km (Table XI, Fig. 6). The application of certain necessary corrections (Tables XII and XIV) improves the internal agreement and leaves outstanding as discordant only two of the twenty-seven lines (Fig. 7). With proper allowance for peculiarities in the behavior of different elements and in the Zeeman effect for individual lines, the results indicate that definite values of the calculated field-strength always correspond to definite levels in the solar atmosphere.

Systematic errors in the measured displacements varying in an appropriate manner with the intensity of the spectral lines would explain the observed dependence of field-strength upon line intensity; but there is no indication of the existence of such errors (sec. 8), and the internal evidence, on the other hand, is strongly in favor of the hypothesis of changing field-strength with changing level (sec. 10).

The anomalous behavior of the lines which show no displacement is not satisfactorily explained. Because of possible peculiarities of pressure, temperature, and electrical conditions necessary for the emission of these lines, they may originate outside the very limited region within which it has been possible thus far to observe the sun's general field (sec. 11).

The underlying causes of the sun's general magnetic field remain obscure. The evidence bearing on the local-whirl hypothesis is unfavorable to its acceptance as an adequate explanation of the existence of the field (sec. 12).

Mount Wilson Solar Observatory
February 1918

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THE ORBIT OF THE SPECTROSCOPIC BINARY BOSS 46

By WALTER S. ADAMS and GUSTAF STRÖMBERG

The star Boss 46 = B.D. 50°46 ($\alpha = 0^{h}12^{m}.4$; $\delta = +50° 53'$, 1900.0) belongs to the class of spectroscopic binaries in which the calcium lines give values of the radial velocity differing widely from those furnished by the other lines in the spectrum. The variable velocity of the star was first found from spectrograms obtained in 1914 at Mount Wilson, and the announcement of the variation was made in the *Annual Report of the Observatory* for the year 1916. The visual magnitude of the star is 6.0 on the Harvard scale; its total proper motion is 0".011 annually; and its spectral type as determined from our photographs is B3p. The lines in its spectrum, with the exception of those due to calcium, are faint and diffuse, and a spectrograph of low dispersion has been employed for nearly all of the observations. The precision obtained for the calcium lines is considerably higher than for the remainder of the lines in the spectrum.

The list of photographs is given in Table I, together with the Greenwich Mean Time of the middle of each exposure and the phase based upon a period of 3.5225 days and referred to the epoch 1917, January 1.0. In view of the long interval covered by the observations the period is known with a considerable degree of accuracy. The results for the velocity, as derived from the hydrogen and helium lines on the one hand and the calcium lines on the other, are combined into mean values for plates taken near the same phase. These normal values, together with the mean phases, are given in Table I, the number of plates being indicated by the figures in parentheses. It was not possible to measure the calcium lines upon all of the negatives, since they occur near the end of the spectral region photographed, and a dense negative is required for the purpose. For this reason the mean phase and the number of plates differ in several cases for the hydrogen and the calcium results.

TABLE I

PLATE NO.	DATE	G.M.T.	PHASE	MEAN PHASE		MEAN VELOCITY		O−C		REMARKS
				H and He	Ca	H and He	Ca	H and He	Ca	
						km	km	km	km	
4849	1916, May 20	$23^{h}42^{m}$	$0^{d}.424$	$0^{d}.408$ (2)	$0^{d}.392$ (1)	−241	−34	+ 0.8	−1.6	
5514	1917, Feb. 5	14 48	0.392							
5384	1917, Jan. 1	15 51	0.660	0.678 (4)	0.672 (3)	−251	−32	−10.6	+2.8	
6278	1917, Oct. 6	21 42	0.627							
6279	1917, Oct. 6	22 23	0.656							
6329	1917, Oct. 31	15 05	0.70							
5287	1916, Dec. 4	17 15	0.899	1.010 (4)	1.010 (4)	−173	−37	+ 9.8	−3.0	
6376	1917, Nov. 25	14 51	1.03							
6377	1917, Nov. 25	15 19	1.05							
6378	1917, Nov. 25	15 48	1.06							
4854	1916, May 21	23 27	1.417	1.434 (5)	1.426 (3)	− 25	−24	+ 0.1	+5.0	
5472	1917, Jan. 30	15 04	1.448							
6229	1917, Sept. 30	16 41	1.463							
6230	1917, Sept. 30	17 00	1.476							
6281	1917, Oct. 7	15 28	1.367							
4305	1915, Aug. 18	0 00	1.718	1.699 (3)	1.699 (3)	+ 81	−24	− 7.7	−3.1	
6237	1917, Sept. 30	21 51	1.678							
6238	1917, Sept. 30	22 23	1.700							
6291	1917, Oct. 25	18 22	1.88	1.885 (2)	1.885 (2)	+140	−26	−10.9	−8.1	
6292	1917, Oct. 25	18 42	1.89							
4916	1916, July 10	23 13	2.092	....	....	....	....	....	....	Process Plate
5461	1917, Jan. 13	14 47	2.050	2.057 (5)	2.072 (3)	+194	−18	+10.8	−1.7	
5462	1917, Jan. 13	15 52	2.094							
6296	1917, Oct. 25	21 45	2.02							
6297	1917, Oct. 25	22 02	2.103							
5016	1916, Aug. 18	20 51	2.247	2.246 (2)	....	+189	....	+ 3.1	....	
5029	1916, Sept. 9	0 03	2.245							40″ Camera
3950	1914, Dec. 25	16 10	2.399	2.382 (2)	2.399 (1)	+153	−13	−15.3	+2.5	
5018	1916, Aug. 18	23 40	2.364							40″ Camera
4922	1916, July 11	23 16	3.094	3.164 (2)	3.094 (1)	− 80	−14	− 2.4	+2.0	Seed 23 Plate
5034	1916, Sept. 9	23 48	3.233							40″ Camera
5338	1916, Dec. 10	18 03	3.410	3.486 (3)	3.486 (3)	−164	−27	+10.1	−3.2	
5452	1917, Jan. 11	15 09	3.586							
5493	1917, Feb. 1	15 25	3.462							

Except as noted in the table a dispersion of one prism and a camera of 18 inches (45.7 cm) were used in the spectrograph. The plates with but two exceptions were Seed Gilt Edge 27. With this combination the exposure times averaged from 15 to 25 minutes. This relatively short exposure is advantageous in view of the very rapid change of velocity in some portions of the velocity-curve.

The lines measured upon the photographs and the wave-lengths employed are as follows:

Ca	3933.825	He	4388.100
Ca	3968.625	He	4471.646
He	4026.342	Mg	4481.400
H	4101.900	Fe	4549.642
He	4143.919	$H\beta$	4861.527
$H\gamma$	4340.634	He	4922.096

The spectrum of the iron arc was used for comparison purposes on all of the spectrograms.

THE ORBIT AS DERIVED FROM THE HYDROGEN AND HELIUM LINES

The well-known graphical method of Lehmann-Filhés was used for the derivation of the approximate elements of the system. These elements were then corrected by means of differential formulae, a least-squares solution being used and the eleven normal values of Table I being assigned weights according to the number of plates involved. A convenient summary of the formulae to be employed is given by Plummer in his article on the determination of the orbits of spectroscopic binaries.[1] The resulting definitive elements are as follows, the letters having their usual significance. The errors given are probable errors.

$$U=3.5225 \text{ days (assumed)}$$

$$K=217.4\pm3.4 \text{ km}$$

$$\omega=323^\circ\!.0\pm10^\circ\!.2$$

$$e=0.094\pm0.020$$

$$T=1917, \text{ January } 2.865\pm0.098 \text{ G.M.T.}$$

$$\gamma=-44.9\pm5.5 \text{ km}$$

$$a \sin i=10{,}480{,}000 \text{ km}$$

$$\frac{m_1^3}{(m+m_1)^2}\sin^3 i=3.71 \text{ sun's mass}$$

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[1] *Astrophysical Journal*, **28**, 212, 1908.

The differences between the observed values of the radial velocity and those computed from these elements are given under O−C in Table I. The average deviation is 7.4 km. A graphical representation of the velocity-curve and of the observed velocities is given in Fig. 1.

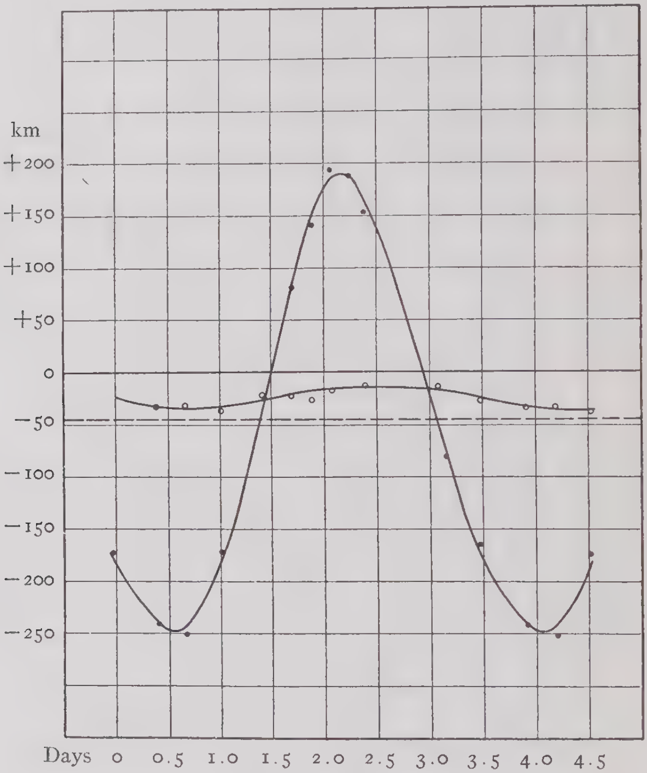

FIG. 1

The most convenient form for the calculation of the radial velocity at any phase of the period, and hence for the comparison of future observations with these results, is by means of a Fourier's series connecting velocity with phase. If we put $\tau=\frac{2\pi}{U}(t-t_0)$ we may represent the velocity V by the equation

$$\begin{aligned} V = a_0 + a_1 \cos \tau + a_2 \cos 2\tau + \ldots . \\ + b_1 \sin \tau + b_2 \sin 2\tau + \ldots . \end{aligned}$$

The epoch t_0 has been taken as 1917, January 1.0. The coefficients in this series have been determined in the usual way with the following result:

$$V=-49.3-145.3\cos\tau+18.2\cos 2\tau+10.0\cos 3\tau-5.2\cos 4\tau$$
$$-147.0\sin\tau+18.2\sin 2\tau-7.3\sin 3\tau+6.1\sin 4\tau$$

The agreement of the radial velocities computed from this formula with those observed is considerably better than that of the velocities calculated from the elements, the average deviation being 3.0 km instead of 7.4. This result is due to the use of nine constants as compared with the five constants of the orbit.

THE CALCIUM LINES

An examination of the radial velocities obtained from the calcium lines seems to give distinct evidence of a small variation with the same period as that for the hydrogen lines within the limits of error of the observations. If we apply to these results the method of representation by a Fourier series, we obtain the following equation:

$$V=-23.5-1.88\cos\tau+1.29\cos 2\tau-0.58\cos 3\tau$$
$$-10.41\sin\tau-0.94\sin 2\tau-0.42\sin 3\tau$$

The velocities calculated from this formula when compared with the observed values give the residuals shown in Table I. The average deviation is 3.3 km. The velocity-curve for the calcium lines is shown in Fig. 1, the observations being represented by small circles. The motion of the system is −23.5 km.

THE SPECTRUM OF THE SECOND COMPONENT

Several of the photographs show evidences of the presence of the second component of the system, and on a very few plates we have been able to secure measurements of velocity. They are, however, too few in number to be used in the calculation of the orbit. The secondary spectrum appears to be of nearly the same type as the primary, and the lines are seen most clearly when the principal star shows its maximum positive velocity.

DISCUSSION OF THE RESULTS

The principal features of interest in connection with these results are as follows:

1. The variation of velocity shown by the hydrogen and helium lines is remarkably large, amounting to nearly 450 km.

2. The calcium lines indicate a variation in velocity of about 20 km with the same period as that shown by the hydrogen and helium lines.

3. The motion of the system as derived from the two sets of lines differs by about 20 km.

It is evident that the variation of velocity indicated by the calcium lines favors the view that the calcium gas producing these lines is connected with the binary system and is not a detached cloud in space. This is in accordance with the observations of Jordan,[1] Lee,[2] and others on spectroscopic binaries of similar character. On the other hand, the difference in the motion of the system as derived from the hydrogen and the calcium lines would seem to indicate an independent source of origin for the latter in accordance with the original suggestion of Hartmann.[3] It seems to us probable that the calcium gas is actually connected with the stellar system, and that the difference in the apparent motion of the system is not to be interpreted wholly on the basis of velocity. A systematic displacement of the calcium lines relative to the other lines in the spectrum might readily come about from a marked difference in the distribution of the gases around the stars. Whether it could amount to as much as 20 km is, however, doubtful. A very probable source of difference may be the presence of the spectrum of the secondary component. As we have already stated, this is seen most clearly at the time of maximum positive velocity of the principal component. At other times it would blend with the stronger spectrum and thus introduce a possible source of systematic error into the measured displacements.

An important result which may have a direct bearing upon this question was obtained by Beal at the Allegheny Observatory in

[1] *Publications of the Allegheny Observatory*, **2**, 63, 1910.

[2] *Astrophysical Journal*, **37**, 1, 1913.

[3] *Astrophysical Journal*, **19**, 268, 1904.

1915.[1] A series of observations upon the spectroscopic binary η Orionis, which belongs to the class with abnormal calcium lines, showed that the motion of the system had undergone a change of 25 km in the interval between 1902 and 1915. Whether this result is interpreted as the effect of the presence of a third body, or as a change due to variations in the physical and mechanical conditions in the system, it is clear that a factor is introduced which may go far toward explaining the results found in the case of Boss 46.

MOUNT WILSON SOLAR OBSERVATORY
March 1918

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[1] *Report of the Eighteenth Meeting of the American Astronomical Society.*

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THE VARIATION WITH TEMPERATURE OF THE ELECTRIC FURNACE SPECTRA OF CALCIUM, STRONTIUM, BARIUM, AND MAGNESIUM

By ARTHUR S. KING

APPARATUS AND METHODS

The methods used in this investigation for observing the temperatures at which spectral lines appear and the rate at which they increase with rising temperature have been the same, in the main, as those employed for the examination of other spectra.[1] The electric furnace was operated *in vacuo*, except for occasional tests under other conditions, and the spectra were photographed in the first and second orders of a 15-ft. concave grating mounted in the vertical spectrograph in the Pasadena laboratory.[2] Spectrograms made with a 1-meter concave grating were used for a few lines in the ultra-violet and extreme red to supplement those of higher dispersion.

To obtain the various spectra the furnace was charged with calcium metal in small pieces, strontium chloride or bromide, barium chloride, and magnesium metal, both as powder and as fragments cut from rods.

A temperature of 1650° C., produced by a potential of 15 volts applied to the graphite tubes regularly used, yielded a considerable number of lines for each element. This accordingly was taken as the low-temperature stage. As intervals of 350° showed distinctive changes in the spectra, temperatures of approximately 2000° and 2350° were chosen as the medium and high temperatures, respectively. The spectrum of each of the substances was photographed at still higher temperatures, but the change beyond 2350° consisted chiefly in a broadening of lines already present, with more frequent reversals. At these temperatures calcium, strontium, and barium gave strong continuous spectra, which tended, through the equalization of emission and absorption, to conceal most of the sharp lines, while those usually showing reversal appeared as wide absorption lines, often with a width of several angstroms. This rendered impossible any comparison with the relative intensities at lower temperatures, so that the useful plates were limited to a temperature range within which the spectra always consisted of emission lines.

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[1] *Mt. Wilson Contr.*, Nos. 66, 76, 94, 108; *Astrophysical Journal*, **37**, 239, 1913; **39**, 139, 1914; **41**, 86, 1915; **42**, 344, 1915.

[2] *Mt. Wilson Contr.*, No. 84; *Astrophysical Journal*, **40**, 205, 1914.

For the identification of the lines and a comparison of their intensities with those of the vacuum furnace, the arc in air was used, with the result that certain lines, especially in the spectra of calcium and barium, showed notable differences in structure in the two sources. Lines which in the air-arc are extremely diffuse, often without a distinct maximum and sometimes showing only as wide hazy patches, appear in the vacuum furnace as sharp lines which permit of close measurement, two or three such lines sometimes taking the place of the diffuse arc line. The reduced pressure in the furnace is in a measure responsible for this condition, but other conditions in the arc discharge contribute to the effect. A somewhat detailed study has been made of lines of this type in the spectrum of barium, including their changes under various conditions of temperature and pressure and the measurement of their furnace wave-lengths. This has yielded a considerable addition to the list of barium lines in the ultra-violet. Their peculiarities will be discussed later in the paper.

The line-classification adopted in previous papers has been adhered to as closely as possible, but in the extension into the ultra-violet a limit is reached beyond which no lines are emitted by the furnace at the given temperature, even with prolonged exposure. This limit proceeds steadily toward shorter wave-lengths as the temperature rises and furnishes the chief point of resemblance between the emission of furnace lines and that of the continuous spectrum of an incandescent solid. The effect on the classification is that lines beyond the point where the lowest temperature gives a spectrum go automatically into classes not lower than Class III, while lines too far in the ultra-violet to be given by the medium temperature chosen must be placed in Class IV or V. It may happen as a result of this method that lines such as λ 2852 of magnesium or λ 3072 of barium are placed in Class III because they are too short in wave-length to be produced at the low furnace temperature, though they are very strong when the temperature is sufficient to show a spectrum in this region. In the visible region lines of this character would probably go into Class I. Lines belonging to a series which extends into the ultra-violet cannot be expected to go as a whole into the same temperature class, since the ultra-violet members may not be produced by a temperature which, for members of the same series in the visible region, results in a high intensity. While this condition must be considered as entering into the method of classification, the character of a line is indicated clearly enough for most purposes by noting the intensities for those temperatures at which the line in question is radiated.

EXPLANATION OF THE TABLES

The first column of each table contains the wave-lengths of lines as measured by Exner and Haschek[1] in the spectrum of the arc in air. In the second column are wave-lengths on the international system. For calcium the measurements of Crew and McCauley[2] with the vacuum arc are used, as the structure of lines in this source is similar to that of furnace lines and their list is more complete than that of Exner and Haschek, frequently giving values for groups of two or three lines which blend into a hazy line in the arc in air. For strontium the measurements of Hampe[3] on the international system are entered in the second column, and for magnesium those of Nacken,[4] the source in each case being the arc in air. For barium the international wave-lengths, beginning with λ 3640, are those for the arc in air given by Schmitz;[5] but to the violet of this point measures of the barium lines in the vacuum furnace made by the writer, with the assistance of Miss Brayton of the Computing Division, have been used, since the vacuum furnace lines are more numerous and include components whose wave-lengths differ appreciably from those of the corresponding diffuse blends of the arc in air.

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[1] *Spektren der Elemente bei normalem Druck*, Leipzig, 1911.

[2] *Astrophysical Journal*, **39**, 29, 1914.

[3] *Zeitschrift für wissenschaftliche Photographie*, **13**, 348, 1914.

[4] *Ibid.*, **12**, 54, 1913.

[5] *Ibid.*, **11**, 209, 1912.

The sign † adjacent to a wave-length refers to a remark at the end of the table, which often includes important data regarding the character of the line.

Following the plan of previous papers the remaining columns in the tables give the intensity estimates for lines in the arc and for three furnace temperatures. The method of assigning lines to classes is also the same. Nebulous lines are indicated by "n" after the value of the intensity, very pronounced haziness being denoted by "N." The letters "r" and "R" indicate partial and complete self-reversal. Lines of Class I and Class II are strong at low temperature, those of Class II strengthening more rapidly as the tube becomes hotter, while Class I includes the lines for which the low-temperature furnace is especially favorable. Lines of Class III are absent or faint at low temperature, but appear at medium temperature, and are usually considerably stronger at high temperature. Class IV appears at the highest furnace temperatures, while Class V is usually absent in the furnace or, if present, the lines are faint compared with their arc intensities. "A" after the class number indicates that the line in question is relatively weak in the arc—usually not more than half as strong as in the high-temperature furnace.

For calcium and strontium a column headed "Series" is introduced, in which a line is placed in a proper pair, triplet, or "single-line" series according to the notation of Saunders.[1]

CALCIUM

Table I indicates the relative condition of the calcium lines for various furnace temperatures and for the arc in air. The number of lines in the furnace spectrum is practically the same as that observed by Crew and McCauley in the vacuum arc, the low pressure bringing out certain lines which in the arc in air are either quenched or blended with neighboring widened lines. Many calcium lines retain a high intensity at low temperature. In the red, certain lines are especially strong at medium temperature. Lines of Class V are found in the strong arc pairs λλ 3159, 3179 and λλ 3706, 3737, which have not been obtained in the furnace. The flame line λ 4227 dominates the furnace spectrum at all the temperatures

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[1] *Astrophysical Journal*, 32, 153, 1910.

TABLE I

TEMPERATURE CLASSIFICATION OF CALCIUM LINES

λ Exner and Haschek	λ Crew and McCauley (I. A.)	Arc	Furnace			Class	Series
			High Temperature	Medium Temperature	Low Temperature		
2995.06....	2994.953	5	5	1	....	III	T
2997.41....	2997.309	5	5	1	....	III	p
2999.74....	2999.651	4	4	1	....	III	T
3000.96....	3000.865	5	5	1	....	III	
3006.97....	3006.864	6	6	2	....	III	p
3009.30....	3009.212	5	5	1	....	III	T
....	3107.388	1N	1	....	....	IV	T_2
....	3117.656	1N	2	....	....	IV A	T_2
....	3136.003	1N	4	1	....	IV A	T_1
....	3140.782	3N	6	1	....	IV A	T_1
....	3141.164		3	....	....	IV A	T_1
....	3150.747	4N	8	2	....	III A	T_1
....	3151.280		3	....	....	IV A	T_1
3159.01....	3158.877	10	....	....	....	V	P_1
....	3164.618	1N	1	....	....	IV	T_2
....	3169.854	1N	2	....	....	IV A	T_2
3179.50....	3179.340	15	....	....	....	V	P_1
....	3180.521	1N	4	1	....	III A	T_2
3181.43....	3181.283	4	....	....	....	V	P_1
....	3209.930	2n	6	4	....	III A	T_1
3215.0	3215.145	5n	8	5	....	III A	T_1
....	3215.334		4	2	....	III A	T_1
3225.6	3225.883	8n	10	6	....	III A	T_1
....	3226.129		5	3	....	III A	T_1
3269.37....	3269.090	1n	2	1	....	III A	T_2
3274.95....	3274.661	2	4	2	....	III A	T_2
3286.35....	3286.060	4	6	4	....	III	T_2
3344.52....	3344.508	8n	10	5	1	III	T_1
3350.25....	3350.198	25n	15	8	2	III	T_1
....	3350.361		10	5	1	III	T_1
3361.95....	3361.918	35n	20	12	3	III	T_1
....	3362.131		10	5	1	III	T_1
3468.70....	3468.484	4	5	2	1	II	T_2
3475.01....	3474.774	8	8	4	2	II	T_2
3487.82....	3487.611	12	10	6	3	II	T_2
3624.19....	3624.107	20	20	10	1	III	T_1
3630.87....	3630.749	30	30r	15	3	III	T_1
3631.07....	3630.973	15	15	5	1	III	T_1
3644.53....	3644.400	40	40R	15	5	III	T_1
3644.90....	3644.760	15	20	5	1	III	T_1
....	3644.990	2	3	1	....	III	T_1
....	3673.448	1	4	1	....	III A	t
....	3675.307	2	8	2	....	III A	t
....	3678.240	3	12	3	....	III A	t
3706.18....	3706.022	10	....	....	....	V	P_2
3737.06....	3736.903	12	....	....	....	V	P_2
....	3748.374†	....	6	....	....	IV A	t
....	3750.349†	....	12	1	....	IV A	t
....	3753.367†	....	15	1	....	III A	t

TABLE I—*Continued*

λ Exner and Haschek	λ Crew and McCauley (I. A.)	Arc	Furnace			Class	Series
			High Temperature	Medium Temperature	Low Temperature		
....	3872.552†	?n	3	1	....	III A	t
....	3875.807†	?n	4	2	....	III A	t
....	3889.141†	....	2	1	....	III A	SL_3
3933.81....	3933.664	400R	60R	30	20	II	PH
3949.10....	3948.899	6	6	3	1	III	T_2
3957.22....	3957.054	10	10	6	2	III	T_2
3968.63....	3968.465	350R	50R	25	18	II	PH
....	3972.578†	....	3	1	....	III	SL_3
3973.91....	3973.716	12	12	8	3	III	T_2
....	4058.912	1n	1	....	....	IV	SL_2
4093.00....	4092.649	8	4	3	1	III	t
4095.30....	4094.944	12	8	6	2	III	t
4098.9.....	4098.552	15	10	8	3	III	t
....	4108.554†	10N	8	3	....	III	SL_3
4226.90....	4226.731	500R	1000R	500R	300R	I	SL
4240.61....	4240.455	6	6	2	tr	III	SL_2
4283.20....	4283.008	40	30R	30	30	I	p
4289.50....	4289.363	40	30R	30	30	I	T
4299.18....	4298.989	30	25R	25	25	I	T
4302.70....	4302.525	60r	50R	40	40	I	p
4307.90....	4307.738	45	35R	35	30	I	
4318.80....	4318.648	45	35R	35	30	I	T
4355.50....	4355.099	25	15	8	2	III	SL_3
4425.60....	4425.428	50	30R	40	30	I	T_1
4435.17....	4434.948	60r	50R	50	40	I	T_1
4435.88....	4435.673	40	25r	35	25	I	T_1
4455.00....	4454.765	80	70R	60	50	I	T_1
4456.10....	4455.875	40	25r	35	30	I	T_1
4456.84....	4456.612	10	10	10	5	II	T_1
....	4507.854	1	1	....	....	IV	
4509.8.....	4509.446	3	3	1	....	III	
4512.7.....	4512.281	5	5	2	....	III	
4527.35....	4526.944	30	15	8	3	III	SL_2
4578.88....	4578.570	30	10	10	6	II	t
4581.77....	4581.414	40	15	15	8	II	t
4586.22....	4585.868	50	20	20	12	II	t
4685.35....	4685.264	12	6	3	....	III	
4847.38....	4847.292	2	5	1	....	III A	
4878.38....	4878.132	50	25	20	4	III	SL_3
....	5019.981†	....	2	1	....	III A	vp
5041.83....	5041.613	40	15	5	2	III	SL_2
5189.00....	5188.846	50	25	5	2	III	
5260.58....	5260.375	2	2	1	....	III	
5261.87....	5261.701	20	20	10	2	III	
5262.40....	5262.238	25	25	12	2	III	
5264.41....	5264.237	20	20	10	2	III	
5265.73....	5265.559	40	40	30	4	III	
5270.44....	5270.272	60	60	40	6	III	
5349.69....	5349.470	25	20	8	2	III	
5513.16....	5512.978	20n	5	2	....	III	
5582.20....	5581.973	25	20	10	2	III	

TABLE I—*Continued*

λ Exner and Haschek	λ Crew and McCauley (I.A.)	Arc	Furnace			Class	Series
			High Temperature	Medium Temperature	Low Temperature		
5588.94....	5588.746	80	50	50	8	III	
5590.30....	5590.109	20	15	10	2	III	
5594.70....	5594.464	60	40	40	6	III	
5598.69....	5598.484	50	50	30	4	III	
5601.50....	5601.283	30	30	10	2	III	
5603.09....	5602.829	25	20	10	2	III	
5857.69....	5857.476	100	80	15	2	III	
5867.89....	5867.578	1	1	....	....	IV	
6102.92....	6102.716	80	80	100	40	II	T_2
6122.49....	6122.216	100	80	150	60	II	T_2
6161.57....	6161.309	10	2	1	....	III	
6162.41....	6162.177	150	60	150	70	II	T_2
6164.05....	6163.749	10	5	2	....	III	
6166.70....	6166.443	15	10	5	....	III	
6169.30....	6169.034	25	15	8	1	III	
6169.82....	6169.576	40	25	15	2	III	
6439.35....	6439.086	150	100	125	50	II	
6450.01....	6449.811	50	40	20	8	II	p
6455.83....	6455.606	10	10	4	2	II	p
6462.80....	6462.576	125	80	100	40	II	
6471.90....	6471.659	40	30	15	6	II	
6494.02....	6493.789	80	60	60	25	II	p
6499.84....	6499.648	30	25	15	6	II	p
6573.00....	6572.783	8	30	150	100	I A	
6717.90....	6717.688	30	25	4	1	III	
....	7148.123	10	12	12	5	II	p
....	7202.161	3	4	4	2	II	p
....	7326.099	2	2	1	....	III	

REMARKS

λ	
3748–3753.	Appear faintly in vacuum arc.
3872–3875.	In band at λ 3883.
3889.	Appears in vacuum arc.
3972.	Appears in vacuum arc.
4108.	Very hazy in arc; about 2 A wide.
5019.	Appears in vacuum arc.

used. Being very sensitive to widening influences, its intensity at a given temperature depends largely on the vapor density, as was noted in a previous investigation.[1] If a large quantity of vapor is present, as in the experiments which gave the intensities noted in Table I, the line changes from a narrowly reversed condition at low temperature to a great width at high temperature. A width of reversal of 6.6 A has thus been obtained, the wings of the line blending with the continuous spectrum. Its progressive strengthening with temperature increase would place it in Class II; but it is essentially a low-temperature line and is not notably strong at high temperature unless much vapor is present. It may properly be classified, therefore, as a special type belonging in Class I.

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[1] *Mt. Wilson Contr.*, No. 32; *Astrophysical Journal*, **28**, 389, 1908.

The H and K lines, λ 3934 and λ 3969, appear at temperatures as low as 1650°; they strengthen rapidly with temperature increase but remain narrow, though it is possible to obtain reversals at high temperature. Their intensities in the arc are enormously greater than in the furnace and offer in this respect a strong contrast to λ 4227.

A large proportion of the calcium lines having been placed in series, it is of interest to note their behavior at different temperatures. As would be expected, the component lines of a series pair or triplet are affected alike; but lines belonging to series groups in different parts of the spectrum may appear in different classes on account of the rapid falling off of the low-temperature spectrum as the wave-length decreases. Thus the intensities of T_1 series lines in the group which has its strongest line at λ 4455.00 are nearly equal at the three temperatures and the lines are assigned to Class I (with the exception of the component λ 4456.84), while the ultra-violet members of the same series fall off rapidly with decrease of temperature and are placed in Classes III and IV. To determine whether the ultra-violet members of a series weaken more rapidly when the temperature is reduced than those of greater wave-length requires careful attention to photographic differences, but this effect has occasionally appeared when furnace spectra at different temperatures were recorded on the same plate. Of the three groups with strongest lines at λλ 4455.00, 3644.53, and 3361.95 the more rapid falling off of the violet groups at reduced temperature was very distinct. This corresponds to the result previously observed for the principal series of caesium.[1]

The alternating single-line series SL_2 and SL_3 show a varied behavior but must be regularly classed as high-temperature lines. In the blue they show a decided contrast with the T_1 series lines

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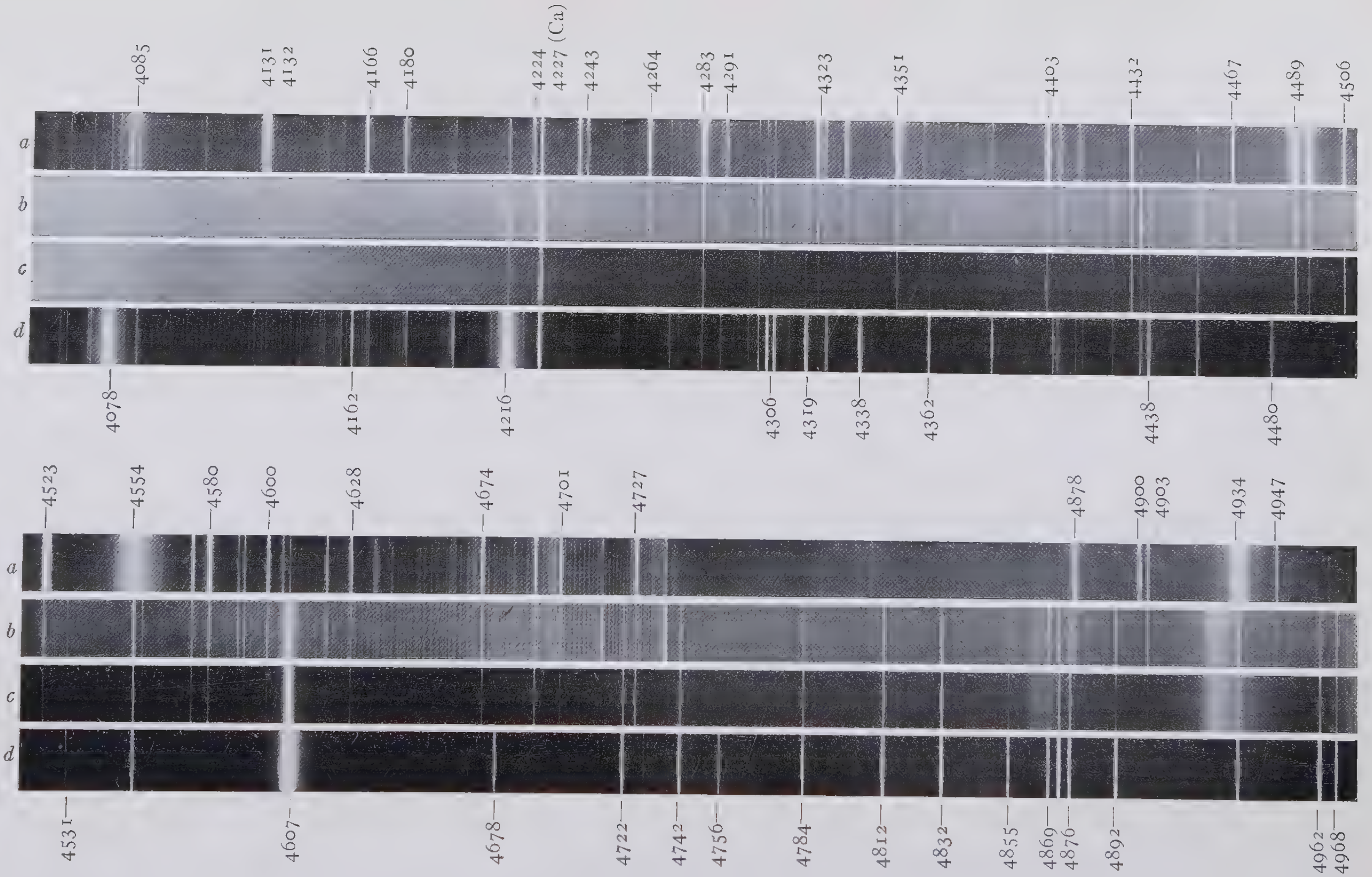
[1] *Astrophysical Journal*, **21**, 236, 1905.



Furnace and Arc Spectra of Barium and Strontium

a. Arc spectrum of barium
b. Furnace spectrum of barium and strontium at 2050°
c. Furnace spectrum of barium and strontium at 1750°
d. Arc spectrum of strontium

and with the group of six near λ 4300. The several lines of this latter group, like those of the similar group near λ 3000, are affected alike at different temperatures.

In the region of greater wave-length the groups λ 5261–λ 5270 and λ 5582–λ 5603 behave in each case as a unit and their lines go into Class III. The lines farther to the red vary greatly in character, a notable feature being the high relative strength at the medium temperature in the case of several lines. λ 6573, whose high intensity in sun-spot spectra has been taken as evidence of the low temperature of these regions, is unique among the calcium lines, being faint in the arc and much weaker at high temperature than at the lower temperatures. Apparently it can be used with confidence as a low-temperature indicator.

STRONTIUM

The strontium lines are widely varied, but their notable features are for the most part sufficiently indicated by the data in Table II. The behavior of the strong lines λλ 4078, 4216, and 4607 in furnace and arc is very similar to that of their counterparts, H, K, and λ 4227 of the calcium spectrum. In the ultra-violet the arc gives a number of hazy, very unsymmetrical lines which reduce to sharp lines in the vacuum furnace, but without the complex structure sometimes observed for lines of this type in the case of barium. As in the case of calcium, the members of "single line" series are notably high-temperature lines. In the red there is a lack of the lines of Class I which appear in this region for both calcium and barium; excepting the strong lines λ 6408 and λ 6504 of Class II the dominant low-temperature lines are in the region of shorter wave-length.

Plate I shows the furnace spectra of a mixture of strontium and barium at temperatures of about 2050° and 1750° C., respectively, with the arc spectra of these substances above and below. The exposure times for the two temperatures were in the ratio of 11 to 1 and show lines of Class I, such as λ 4742 and λ 4812 of nearly the same intensity, while the larger gradations of lines in Classes II and III for these two temperatures are evident. The high intensity of λ 4607 in the furnace is shown, together with the relative faintness of λ 4078 and λ 4215.

TABLE II

Temperature Classification of Strontium Lines

λ Exner and Haschek	λ Hampe (I.A.)	Arc	Furnace			Class	Series
			High Temperature	Medium Temperature	Low Temperature		
2569.55....	2569.502	20	10	....	....	IV	SL_1
2932.00....	2931.856	30	15	10	....	III	SL_1
3301.86....	3301.739	50	15	10	....	III	T
3307.6.....	3307.548	50n	15	10	....	III	p
3322.40....	3322.237	30	15	8	....	III	T
3330.20....	3330.011	30	8	8	....	III	
3351.45....	3351.258	150	50	20	....	III	p
3366.51....	3366.339	50	30	12	....	III	T
3371.10....	....	2	....	....	....	V	
3380.98....	3380.721	50	....	....	....	V	P_1
3390.09†...	....	4N	3	2	....	III	T_1
3400.39†...	....	8N	4	3	....	III	T_1
3411.7†....	....	12N	6	4	....	III	T_1
3456.8.....	3456.52	8n	2	1	....	III	T_2
3457.7.....	3457.54	15n	10	6	....	III	T_1
3464.68....	3464.470	50	....	....	....	V	P_1
3475.09....	3474.901	10	....	....	....	V	P_1
3477.3.....	3477.2	8n	8	5	....	III	T_1
3499.4†....	3499.61	20n	10	8	....	III	T_1
3504.9.....	3504.27	2n	1	....	....	IV	T_2
3547.8†....	3548.1	15n	10	6	1	III	T_1
3577.5†....	....	2n	?	?	....	III ?	T_2
3628.7.....	3628.37	3n	2	1	....	III	T_2
3629.1.....	3629.12	10n	6	3	....	III	T_1
3653.30....	3653.26	12n	6	4	....	III	T_1
3654.0.....	3653.91	3n	3	1	....	III	T_1
3706.0.....	3705.90	15n	8	5	1	III	T_1
3940.90....	3940.806	20	8	4	....	III	T_1
3969.40....	3969.270	30	8	5	1	III	T_1
3970.12....	3970.049	20	7	3	....	III	T_1
4030.55....	4030.386	40	15	8	1	III	T_1
4032.50....	4032.387	20	12	3	....	III	T_1
4033.25†...	4033.19	6	?	?	....	III ?	T_1
4061.3.....	4061.1	8n	4	2	....	III	t
4071.22†...	....	10n	6	3	....	III	t
4077.89....	4077.714	400r	40r	25	12	II	PH
4087.88....	4087.46	12n	....	....	....	V	t
4161.99†...	4161.812	30	?	?	....	IV ?	P_2
4215.70....	4215.515	300r	30	15	6	II	PH
4305.68....	4305.459	40	15	5	1	III	P_2
4308.5.....	4308.13	20n	10	8	2	III	t
4313.7.....	4313.23	3n	4	2	....	III	SL_2
4319.4.....	4319.090	25n	15	10	2	III	t
4326.64....	4326.444	8	10	6	2	II	T_2
4338.2.....	4337.704	30n	20	12	3	III	t
4361.88....	4361.710	20	20	15	4	III	T_2
4407.0.....	4406.11	10n	10	5	1	III	SL_3
4412.80....	4412.620	4	5	4	1	III	

TABLE II—*Continued*

λ Exner and Haschek	λ Hampe (I.A.)	Arc	Furnace			Class	Series
			High Temperature	Medium Temperature	Low Temperature		
4438.21....	4438.044	25	20	15	6	II	T_2
4451.99....	4451.803	2	3	2	....	III	
4480.9.....	4480.540	10n	10	5	....	III	SL_2
....	4531.340						
4531.52....	....	10	12	8	3	II	
....	4531.359						
4607.51....	4607.340	600R	600R	400R	300R	I	SL ?
4678.4.....	4678.304	20n	10	8	2	III	SL_3
4701.2†....	4700.7	2	?	....	....	IV ?	
4704.35†...	4704.0	2	?	1	....	III	
4707.5†....	4707.1	2	?	1	....	III	
4714.35†...	4714.0	3	?	2	....	III	
4722.48....	4722.272	30	25	25	20	I	p
4729.8.....	4729.48	4n	4	2	....	III	
4742.08....	4741.917	30	30	25	20	I	T
4755.60....	4755.467	12n	10	4	1	III	SL_2
4784.51....	4784.323	30	30	25	18	I	T
4812.03....	4811.867	40	40r	30	25	I	p
4832.27....	4832.075	50	50r	40	30	I	T_1
4855.20....	4855.078	20	20	12	3	III	t
4868.92....	4868.739	20	20	15	3	III	t
4869.45....	4869.194	4	5	2	....	III	t
4872.70....	4872.485	40	30	25	20	I	T_1
....	4876.062	15	20	15	8	II	T_1
4876.38....	4876.323	20	20	20	10	II	T
4892.20....	4892.009	25	25	20	6	II	t
4892.90....	4892.666	5	5	3	....	III	t
4962.43....	4962.244	40	40	30	20	I	T_1
4968.03....	4967.928	20	20	15	6	II	T_1
4971.79....	4971.649	2	5	3	2	II A	T_1
5156.38....	5156.068	8	8	6	1	III	SL_3
5213.22†...	5212.968	3	3	2 ?	....	III	
5222.50....	5222.200	20	10	8	3	II	
5225.35....	5225.110	20	10	8	3	II	
5229.51....	5229.266	20	10	7	3	II	
5238.82....	5238.548	30	15	12	6	II	
5257.10....	5256.897	50	20	20	10	II	
5330.11....	5329.816	8	8	5	1	III	SL_2
5451.20....	5450.818 5450.834	15	15	10	2	III	
5481.19....	5480.840	40	30	20	15	II	
5486.40....	5486.121	15	15	10	2	III	
5504.50....	5504.166	30	20	15	8	II	
5522.01....	5521.752	25	20	12	5	II	
5535.10....	5534.796	15	15	8	2	III	
5540.29....	5540.033	15	20	10	2	III	
5543.44....	5543.327	15n	12	3	....	III	
5970.6.....	5970.103	12n	10	3	....	III	
6159.15....	6158.967	1	1	....	....	IV	
6272.32†...	6272.052	2	3	?	....	IV ?	

TABLE II—*Continued*

λ Exner and Haschek	λ Hampe (I.A.)	Arc	Furnace			Class	Series
			High Temperature	Medium Temperature	Low Temperature		
6346.05†...	6345.760	10	?	?	?	IV?	
6364.23....	6363.932	8	8	5	1	III	
6370.23....	6369.959	15	12	8	2	III	
6381.00....	6380.740	30	20	15	3	III	
6386.84....	6386.507	40	20	15	4	III	p
6388.50....	6388.245	25	15	10	3	III	
6408.76....	6408.465	100	60	30	20	II	
6446.89†...	6446.676	12	6	?	?	III	
6466.01†...	6465.788	10	4	?	?	III?	
6504.21....	6503.990	80	50	30	20	II	
6547.00†...	6546.785	20	15	5	?	III?	
6550.59†...	6550.253	60	25	8	?	III?	p
6617.50†...	6617.268	50	20	10	?	III?	
6643.70†...	6643.545	20	15	5	?	III?	

REMARKS

λ	
3390–3400.	Wide and hazy in arc. λ from Kayser and Runge.
3411.	Wide and hazy in arc.
3499.	Shaded to violet in arc.
3547.	Shaded to violet in arc.
3577.	Disturbed by band.
4033.	Disturbed by band. λ from Kayser and Runge.
4071.	Measured by writer.
4161.	Very faint if present in furnace.
4701–4714.	Disturbed by carbon band.
5213.	Disturbed by band.
6272.	Disturbed by band.
6346.	Concealed by band.
6446–6466.	Disturbed by band.
6547–6643.	Concealed by band at low temperature.

BARIUM

Table III gives the characteristics of the furnace spectrum of barium. Since the grouping of lines according to series is not so marked as with either calcium or strontium, the chief feature to be noted in this respect is that the behavior of the strong lines λ 4554 and λ 4934 is similar to that of H and K of calcium and λ 4078 and λ 4216 of strontium, while the low-temperature line λ 5536 corresponds to λ 4227 of calcium and λ 4607 of strontium. The furnace results thus bear out the resemblance observed between these lines in other light-sources.

The differences of structure shown by furnace spectra for many lines in the ultra-violet as compared with the arc in air warranted

TABLE III

TEMPERATURE CLASSIFICATION OF BARIUM LINES

λ Exner and Haschek	λ King (I.A.)	Arc	Furnace			Class
			High Temperature	Medium Temperature	Low Temperature	
2596.87	...	8n	5	...	...	IV
2647.42	...	2	2	...	...	IV
2702.70	...	8	3	...	...	IV
2739.38	...	3	1	...	...	IV
2785.36	...	15	8	...	...	IV
3071.72	3071.592	100R	100R	50R	...	III
3108.3	3108.21	10n	5	1	...	III
...	3117.34	*	3n	1	...	III
...	3117.638	*	3n	1	...	III
...	3119.202	3N	2	1	...	III
...	3132.602	*	8	5	...	III
...	3135.72	*	8n	5n	...	III
...	3137.700	*	10	5	...	III
...	3155.336	*	10	5	...	III
...	3155.673	*	10	2	...	III
...	3158.046	*	12	4	...	III
...	3158.54†	*	12N	6	...	III
...	3165.598†	*	25	15	...	III
...	3173.69	*	15n	10	...	III
...	3183.156	*	30	15	...	III
3184.5	3183.96†	?N	15	8	...	III
...	3193.912†	*	25n	10	...	III
...	3193.967†			5	...	III
3204.5	3203.700†	25N	25	15	...	III
...	3221.630†	40N	30	20	...	III
...	3222.188†		40	25	...	III
3223.0	3222.441†		8	3	...	III
3253.2	3253.067	5n	5	2	...	III
...	3261.961†	80N	50r	25	...	III
3263.0	3262.336†		60R	25	...	III
3270.30	3270.115	4n	4	2	...	III
...	3272.405	2n	2	1	...	III
...	3281.503†	70N	40r	20	...	III
3282.1	3281.772†		15	6	...	III
3315.92	3315.753	8n	8	5	...	III
...	3322.797†	80N	50r	25	...	III
3323.3	3323.058†		10	4	...	III
3357.15	3356.804†	80N	40r	25	...	III
3377.33	3376.975†	80N	60R	35	...	III
...	3377.391†		20	10	...	III
...	3413.835	3n	3	1	...	III
3420.70	3420.322†	100N	70R	40	...	III
...	3421.008†		25	15	...	III
...	3421.476†		30n	15	...	III
...	3426.453†	4n	3	?	...	III ?
...	3427.85	3n	3	1	...	III
3464.7†	3463.741	?N	40	25	...	III
3501.31	3501.115	200R	150R	75r	20	II

* Not visible in barium arc in air, probably on account of extreme diffuseness.

TABLE III—*Continued*

λ Exner and Haschek	λ King Schmitz (I.A.)	Arc	Furnace			Class
			High Temperature	Medium Temperature	Low Temperature	
3525.30†	3524.975	80n	50r	25	2	III
3529.64	3529.480	15	10	6	....	III
3531.8†	3531.345	30N	40	20	1	III
3545.00†	3544.663	80n	60R	30	3	III
3547.99	3547.696	20n	15	10	1	III
3562.19	3561.942	10	10	7	....	III
3566.83	3566.660	10	8	6	....	III
3576.24	3576.036	10	10	6	1	III
3577.81	3577.615	30	10	8	1	III
3579.91†	3579.670	80n	50R	30	5	III
3586.69	3586.505	10	10	6	1	III
3588.32	3588.099	10	10	7	1	III
3590.15	3589.950	3	8	4	....	IIIA
3593.50	3593.204	15n	15	8	1	III
3599.62	3599.396	15	10	8	1	III
3611.20	3610.957	15n	15	10	1	III
3630.85	3630.641	40	30	20	3	III
3637.2†	3636.832	40N	40r	20	2	III
....	3639.715 Schmitz	2	2	1	....	III
3640.56	3640.391	10	8	5	....	III
3662.71	3662.523	15	8	6	....	III
3664.82	3664.598	3	3	1	....	III
3675.45	3675.268	2	1	1	....	III
3688.8	3688.473	20N	15	8	....	III
3701.90	3701.716	3	3	2	....	III
3794.88	3794.771	15	15	10	2	III
3862.07†	3861.905	15	8?	8?	2	III
3889.43	3889.314	20	10	8	6	II
3891.90	3891.788	50	....	....	....	V
3892.90	3892.653	20	10	7	2	III
3900.56	....	2	2	2	....	III
3906.15	....	2	2	2	....	III
3910.03	3909.922	40	20	15	8	II
3917.38	....	4	4	3	....	III
3935.91	3935.715	50	15	10	6	II
3938.03	3937.876	20	10	8	2	III
3945.3†	3945.60	10N	15	7	....	III
3947.5†	3947.51	5N	8	4	....	III
3975.6	3975.362	4n	5	2	....	III
3983.06	....	2	1	1	....	III
3993.55	3993.395	80	20	12	6	II
3995.85	3995.663	30	15	10	2	III
3998.05	....	3	2	2	....	III
4026.55†	....	2n	1?	....	....	IV?
4081.1	4081.347	6n	8	4	....	III
4085.0	4085.322	30N	30	12	2	III
4087.6	4087.371	8n	20	8	....	III A
4110.35	4109.884	2	1	1	....	III
4130.83	4130.683	80	....	....	....	V
4132.58	4132.444	20	20	20	15	I

TABLE III—*Continued*

λ Exner and Haschek	λ Schmitz (I.A.)	Arc	Furnace			Class
			High Temperature	Medium Temperature	Low Temperature	
4106.17	4166.017	20	....	....	....	V
4179.58	4179.372	8n	20	10	....	III A
4224.20	4223.957	12	20	12	2	III
4239.85	4239.576	10n	15	10	1	III
4242.80	4242.619	10	15	10	1	III
4264.45	4264.386	15n	30	15	2	III A
4283.31	4283.111	100	40	40	20	II
4291.33	4291.165	12	15	10	2	III
4323.05	4323.004	20n	35	20	3	III
4325.34	4325.152	10	15	10	1	III
4332.97†	4332.919	10n	10?	10	2	III
4350.60	4350.375	80	35	30	12	II
4359.76	4359.554	5	8	5	....	III
4402.80	4402.550	60	30	30	15	II
4407.07	4406.846	15	15	12	2	III
4413.89	4413.679	8	10	6	1	III
4432.10	4431.914	40	25	25	15	II
4467.30	4467.129	12	15	12	2	III
4489.10	4488.973	60n	50	35	6	III
4493.79	4493.641	50n	40	30	5	III
4506.13	4505.936	40	30	25	15	II
4523.48	4523.237	60n	40	25	12	II
4525.19	4524.946	35	....	....	....	V
4554.21	4554.038	1000R	100r	80	70	II
4574.02	4573.881	40	30	25	15	II
4579.82	4579.667	80	40	30	20	II
4589.83	4589.762	8n	15	12	2	III A
4591.92	4591.825	10n	15	12	2	III
4599.97	4599.751	30	20	15	3	III
4605.10	4605.012	8n	15	8	2	III A
4620.19	4619.978	20n	15	8	2	III
4628.45	4628.330	25n	25	20	3	III
4636.7†	4636.333	15N	?	....	....	IV?
4642.5†	4642.038	4N	?	....	....	IV?
4652.0†	....	5N	?	....	....	IV?
4663.0†	....	10N	?	....	....	IV?
4673.75	4673.621	30	25	20	12	II
4691.85	4691.630	35	30	25	15	II
....	4699.108†	15N	10	6	1	III
4700.67	4700.446	20	12	10	2	III
4724.96	4724.742	3	4	2	....	III
4726.68	4726.455	40	30	25	15	II
4732.0†	....	5N	?	?	....	III?
4739.5	....	5N	?	1	....	III
4807.8	....	2N	1	1	....	III
4877.8	4877.650	30N	25	12	2	III
4900.11†	4899.971	35	3?	....	....	IV?
4903.05	4902.898	15	15	10	2	III
4934.26	4934.099	700R	70R	60	50	II
4947.50	4947.350	8	10	4	1	III
5055.14†	5054.975	5	?	?	....	IV?

TABLE III—*Continued*

λ Exner and Haschek	λ Schmitz (I.A.)	Arc	Furnace			Class
			High Temperature	Medium Temperature	Low Temperature	
5160.10†	5159.919	20n	?	?	....	IV ?
5175.72†	5175.619	5	?	?	....	IV ?
5177.52†	5177.448	2	?	?	....	IV ?
5267.18†	5267.033	20	?	?	....	IV ?
5277.75†	5277.625	3	?	?	....	IV ?
5303.00†	5302.808	6	?	?	....	IV ?
5305.92†	5305.758	4	?	?	....	IV ?
5309.10†	5308.952	4	?	?	....	IV ?
5405.20	5404.920	5n	10	5	....	III A
5424.85	5424.616	100	100R	50	10	II
5437.56	5437.393	4	10	5	....	III A
5473.90	5473.689	3	10	4	....	III A
5519.28	5519.115	200	150	70	20	II
5535.70	5535.534	1000R	1000R	500R	400	I
5593.50	5593.297	3	5	3	....	III
5620.2	....	4n	?	....	....	IV ?
5620.6	....	20n	?	....	....	IV ?
5680.45†	5680.173	2	5	3	....	III A
		10	20	10	....	III A
5709.89	5709.546	2n	3	1	....	III
5713.80	5713.554	4n	6	2	....	III
5778.00	5777.695	400r	200R	70	20	II
5784.29	5784.105	4	6	2	....	III
5800.60	5800.299	100	80	50	4	III
5805.92†	5805.712	20	30	40	8	II
5819.19	5818.906	5	20	12	....	III A
5826.56	5826.294	150	125r	60	30	II
5853.91	5853.699	200	40	20	....	III
5907.81	5907.656	15	30	20	10	II A
5962.7	5962.445	2	7	2	....	III A
5965.00	5964.787	5	5	2	....	III
5972.00	5971.715	100	80	60	50	II
5978.69	5978.496	4	10	6	....	III A
5997.39	5997.102	100	80	60	40	II
6019.70	6019.505	100	90	60	40	II
6063.44	6063.149	200	125r	80	60	II
6083.67	6083.441	5	15	5	....	III A
6111.10	6110.808	300r	200R	100	80	II
6129.50	6129.335	3	5	2	....	III
6141.95	6141.760	600r	50	30	5	III
6235.5†	....	30n	20	6 ?	....	III
6323.6†	....	3	5	?	....	III ?
6341.93	6341.697	150	100	100	80	I
6411.95	....	4	20	10	....	III A
6451.11	6450.842	125	100	100	80	I
6483.21	6482.936	200	125	100	60	II
6497.21	6496.902	600r	75	75	8	III
6499.10	6498.776	300r	300r	200	125	II
6527.60	6527.324	250	150	150	100	I
....	6564.38†	2	5	2 ?	....	III A
6581.00	6580.77†	6	10	4	....	III

TABLE III—*Continued*

λ Exner and Haschek	λ Schmitz (I.A.)	Arc	Furnace			Class
			High Temperature	Medium Temperature	Low Temperature	
6595.65.....	6595.351	200	125	100	80	I
....	6642.40†	1n	3	?	....	III A
6654.27.....	6654.120	10	15	4	....	III
6675.50.....	6675.280	80	70	80	60	I
6694.08.....	6693.875	70	60	80	60	I

λ	Remarks
3158.	Very diffuse at high temperature.
3165.	Shaded to red at high temperature.
3184.	Appears as hazy patch in arc. Blend V in furnace.
3193.	Not resolved at high temperature.
3204.	Very hazy in arc.
3221.630.	Reduced in atmospheric furnace to one-third strength of λ 3222.188.
3222.	Very hazy in arc.
3222.441.	Shows in atmospheric furnace as shading to λ 3222.188.
3261.961.	Reduced in atmospheric furnace to two-thirds strength of λ 3262.336.
3262.336.	Unsymmetrically reversed at high temperature.
3281.772.	Fades to trace in atmospheric furnace.
3322.797.	Unsymmetrically reversed at high temperature.
3323.058.	Not visible as separate line in atmospheric furnace.
3357.	Unsymmetrical to red at high temperature.
3377.391.	Fades to one-tenth strength of λ 3376.975 in atmospheric furnace.
3421.008.	Fades to one-tenth strength of λ 3420.322 in atmospheric furnace.
3421.476.	Very faint and diffuse in atmospheric furnace.
3426.	Disturbed by band.
3464.	Hazy patch in arc. Faint and diffuse in atmospheric furnace.
3525.	Unsymmetrically reversed at high temperature.
3531.	Very hazy in arc. Resembles λ 3464.
3545, 3579, 3637.	Unsymmetrically reversed at high temperature.
3862.	Disturbed by band.
3945, 3947.	Very hazy in arc. Measured in vacuum furnace.
4026.	Disturbed by band.
4332.	Blend with V at high temperature. Probable intensity of V line subtracted.
4636–4663.	Disturbed by carbon.
4699.	Measured in vacuum furnace.
4732.	Disturbed by carbon.
4900.	High-temperature line may belong to carbon.
5055–5309.	Disturbed by band. Faint, if present in furnace.
5680.	Double, not fully resolved.
5805.	Very strong at medium temperature.
6235–6323.	Disturbed by band.
6564–6580.	Measured in vacuum furnace.
6642.	Shaded to red in arc. Measured in vacuum furnace.

a special measurement of these lines as produced by the furnace. These differences consist not merely in a sharpening of the lines, but, in several cases, of a resolution of diffuse arc lines into two or three components in the furnace. For lines remaining single in both furnace and arc the dissymmetry almost disappears in the furnace spectrum, so that wave-lengths very nearly free from widening influence are obtained.

Photographs for the measurement of furnace lines were made in the second order of the 15-ft. concave grating spectrograph, the dispersion being 1 mm = 1.86 A. The furnace was operated at about 2000°, which gave very sharp lines, without the reversals combined with incipient dissymmetry which appeared at higher temperatures. Standard lines were obtained by mixing a small amount of iron with the barium in charging the furnace. As far as λ 3640, the wave-lengths on the international system in the second column of Table III were deduced from measurements of these plates, the mean of three complete sets being taken. For the iron lines the values of Burns[1] were used. Since the data on pressure-shift are scanty for this region, his wave-lengths, which are for the iron arc at atmospheric pressure, were used for the vacuum source without correction.

The different appearance of the furnace lines as compared with those of the arc in air is shown in Plates II and III. The arc spectra above and below, in each plate, were photographed alongside the furnace spectra at 2350° and 2000° respectively (spectra *b* and *c*), and the close coincidence of symmetrically reversed lines such as λ 3071 and λ 3501 in arc and furnace showed a freedom from instrumental displacement. The arc lines become more and more diffuse as the wave-length decreases until the lines to the violet of λ 3204, while they cannot be said to be absent, are so hazy as to be indistinguishable even on strong photographs. In the furnace they are well defined and fairly sharp and have been measured on the furnace plates. The other barium lines shown are so wide and unsymmetrical in the arc in air that only rough measurements are possible. Measurements of the furnace lines give not only a close value for the position of the unwidened line, but, in some cases, the wave-lengths of component lines whose presence is not indicated in the arc. This results from the fact that lines are frequently resolved into components which in the arc in air are

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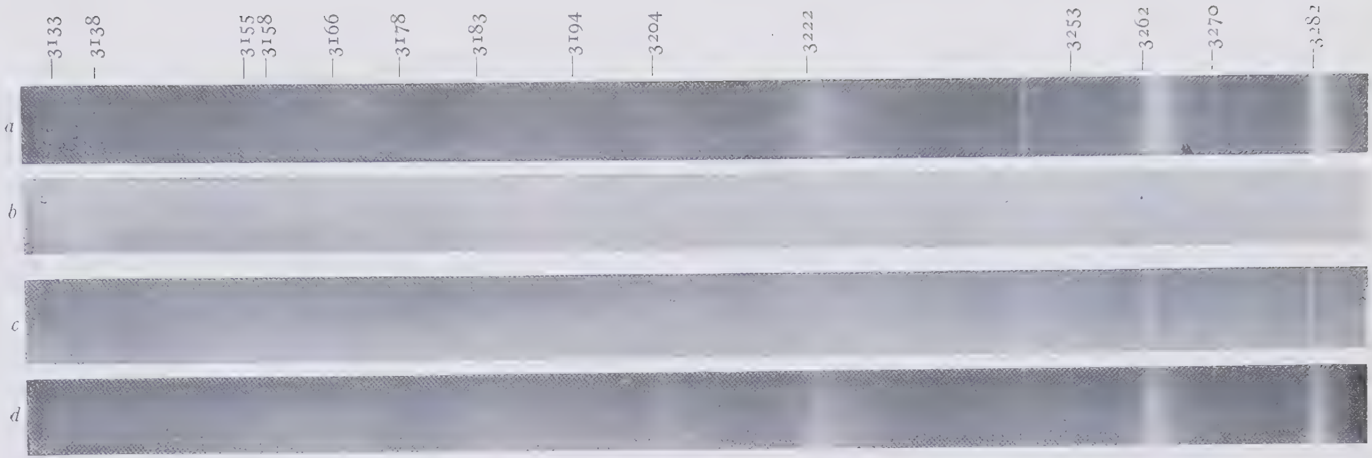
[1] *Lick Observatory Bulletin*, **8**, 27, 1913.

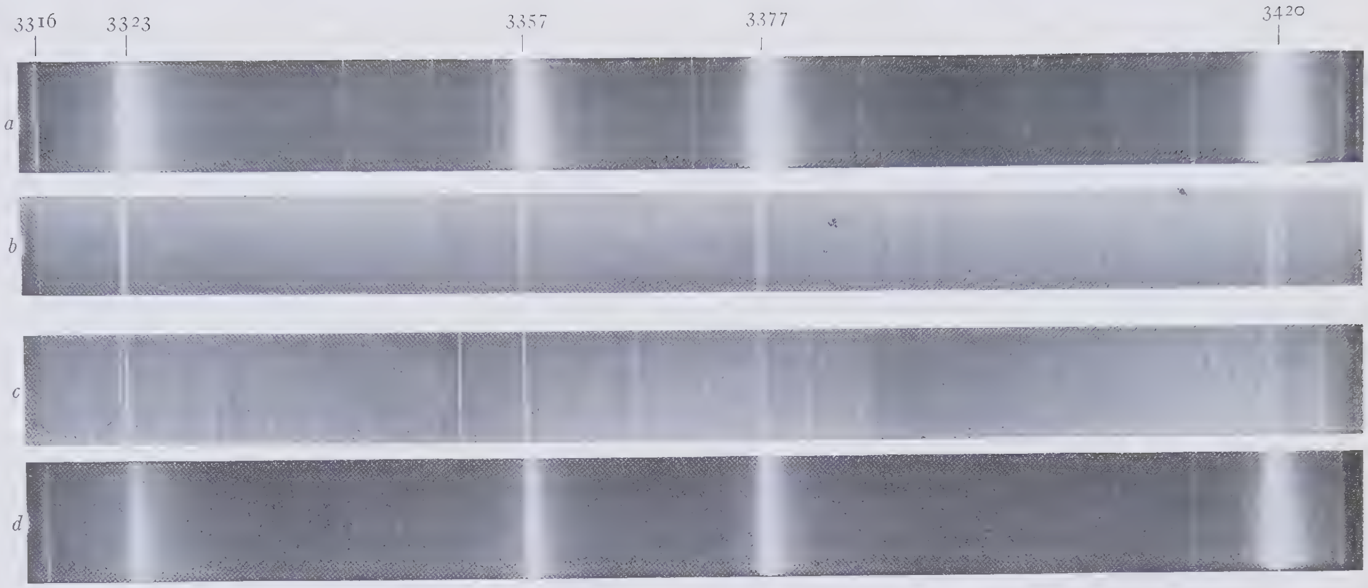
PLATE II



Furnace and Arc Spectra of Barium

a, *d*. Arc spectrum at atmospheric pressure
b. Furnace spectrum at 2350°
c. Furnace spectrum at 2000°

PLATE III



Furnace and Arc Spectra of Barium

a, *d*. Arc spectrum at atmospheric pressure
b. Furnace spectrum at 2350°
c. Furnace spectrum at 2000°

merely wide, unsymmetrical lines with no indication of structure. These components sometimes occur in pairs, as for λλ 3262, 3282, 3377, and sometimes with a third component, as in the case of λ 3222 and λ 3420. The position of the chief component is at the extreme violet edge of the arc line; this is also the case when there is but one component, as, for example, with λ 3357. The extra components, when they occur toward the red, might be considered as blending into a shaded line in the arc, but both λ 3222 and λ 3262 show a component to the violet, where nothing appears in the arc. A violet component of λ 3323 is shown in Plate III*c*, but this is due to a strontium impurity.

The question arises as to whether this resolution into components is due solely to the low pressure in the furnace or in part to the absence of the discharge conditions of the arc. To test this, photographs were made at approximately the same temperature (2300°) with the furnace in vacuum and at atmospheric pressure, the results being compared with the spectrum of the arc in air. The results showed that the furnace at atmospheric pressure gives an intermediate spectrum to the extent that the extra lines characteristic of the vacuum furnace are weakened and an unsymmetrical widening of the principal components has commenced; but the use of atmospheric pressure has by no means changed the furnace structure into that of the arc. The distinctive features of the furnace lines, such as the violet components of λ 3222 and λ 3262, are still present.

Another method of making the furnace lines approach the structure given by the arc is to increase the temperature of the vacuum furnace with plenty of barium present. This, as may be seen from Plate III*b*, leaves the relative intensity of the furnace components almost unchanged from the condition at lower temperature (Plate III*c*), but produces an unsymmetrical reversal of the stronger lines. This dissymmetry evidently is a beginning of the strong one-sidedness shown by the arc lines.

The type of excitation occurring in the arc is thus chiefly responsible for the fading out of certain components and the strong dissymmetry shown by the arc lines in this region of the barium spectrum. Increase of pressure reinforces the widening action but is only slightly effective in changing the intensity of the components of furnace lines.

Occurrence of barium lines in the solar spectrum.—That barium occurs in the solar atmosphere is rendered fairly certain by the strong arc pair λ 4554 and λ 4934, which correspond in each case with a close doublet in the solar spectrum. λ 5853.91 and λ 6141.95 also occur as strong solar lines. All of these lines are much weaker in the furnace than in the arc. Lines strong in both arc and furnace are faint in the sun, and often unidentified with barium by Rowland. Thus λ 5535.70, which dominates the furnace spectrum, if present in the sun, has an intensity of only 0. Other strong furnace lines are still fainter in the solar spectrum, the tendency being toward a weakening of those lines for whose appearance the furnace is especially favorable. Most of the ultra-violet lines of barium, strongly widened in the arc and reduced to sharp lines in the furnace, are of a type which should be present in the solar spectrum, though probably faint. It is of interest to examine some of the cases in which a wide arc line is replaced by two or three narrow lines in the furnace. Seven groups were taken, from λ 3183 to λ 3421, and their wave-lengths on the Rowland system were deduced from neighboring iron lines. In some cases the position of the barium line coincides with that of a stronger line of another substance, but when this is not the case a solar line of intensity from 00 to 0000 invariably appears close enough to the position of the furnace line to be within the error of measurement. There is thus considerable probability that the solar conditions are such as to produce a resolution of these lines into the components which occur in furnace spectra. Such conditions, according to present evidence, appear to be a moderately high temperature combined with low pressure in the region occupied by the barium vapor.

MAGNESIUM

The magnesium lines listed in Table IV show a great variety of types. The low-temperature spectrum consists of the triplet near λ 3830, the single line λ 4571, and the b group λλ 5167–84. λ 2852, while one of the strongest lines observed during the furnace investigations, is beyond the ultra-violet limit reached by photographs of the low-temperature spectrum. λ 4571 is a low-temperature line of extreme type, in that it is decidedly weakened both in the furnace

TABLE IV

TEMPERATURE CLASSIFICATION OF MAGNESIUM LINES

λ Exner and Haschek	λ Nacken (I.A.)	Arc	Furnace			Class
			High Temperature	Medium Temperature	Low Temperature	
2733.7	2733.55	5n	1	....	....	IV
2736.8	2736.60	10n	2	....	....	IV
2776.82	2776.704	10	4	....	....	IV
2778.40	2778.289	10	4	....	....	IV
2779.95	2779.853	12	5	....	....	IV
2781.51	2781.431	10	4	....	....	IV
2783.08	2782.989	10	4	....	....	IV
2795.64	2795.545	60	10	....	....	IV
2802.82	2802.718	40	8	....	....	IV
2852.25†	2852.128	300R	1000R	400R	....	III
2937.00	2936.754	2	2	....	....	IV
2938.70	2938.487	4	4	1	....	IV
2942.22	2942.016	8	8	2	....	III
3091.20	3091.093	20	20	5	....	III
3093.14	3093.011	40	40	12	....	III
3097.08	3096.914	50	50	15	....	III
3330.09	3329.934	10	10	5	....	III
3332.31	3332.163	15	20	15	....	III
3336.83	3336.688	20	30	20	....	III
3829.51†	3829.364	40	?	20	15	II
3832.49†	3832.306	80r	?	40	30	II
3838.45†	3838.283	100r	?	50	40	II
4059.15	4057.81	5n	3	1	....	III
4167.8†	4167.65	10n	5?	?	....	III?
4352.35	4351.940	30	?	1	....	IV
4571.31†	4571.114	5	?	80	80	IA
4703.40†	4703.069	40	?	....	....	V
5167.50	5167.303	40	20	10	5	II
5172.87†	5172.673	80	40	25	15	II
5183.78†	5183.600	125	60	40	20	II
5528.70	5528.466	10	....	....	....	V
5711.38	5711.127	1	....	....	....	V

REMARKS

λ	
2852.	Line may attain enormous intensity in furnace. Reversals up to 30 A wide have been observed.
3829–3838.	Difficult at high temperature on account of λ 3883 band. Apparently not stronger than at medium temperature.
4167.	Disturbed by band.
4352.	Concealed by band at high temperature.
4571.	Very faint, if present at high temperature. Weakens as emission line; appears at 2500° as narrow absorption line.
4703.	Disturbed by band; very faint if present.
5172–5183.	Reversed at 2500°.

at high temperature and in the arc. It has not been possible to obtain this line strongly widened, as the conditions which give widening diminish its strength. At high temperature it is likely to appear as a narrow absorption line if the continuous background is strong enough. Its characteristics are very similar to those of λ 6573 of calcium, and, like that line, it is a good indicator of low temperature.

The furnace shows only faintly λ 4352 and λ 4703, both of which are strong in the arc and not enhanced in the spark. The enhanced line λ 4481 is not listed in Table IV, as it does not appear in the ordinary carbon arc containing magnesium. As would be expected, no trace of it is found in the furnace spectrum.

In the blue and green a series of spectrograms made in a previous investigation,[1] when hydrogen at pressure up to 20 cm was used in the furnace, were compared with spectra given by the vacuum furnace, but the hydrogen appeared to have no effect on the line spectrum.

SUMMARY

The foregoing pages summarize an investigation of the furnace spectra of calcium, strontium, barium, and magnesium from the standpoint of the lines appearing at each of three stages of temperature. The lines are classified according to the temperature at which they appear and their rate of change with increase of temperature. Special note is made of connection with series relations, of change of characteristic features of the furnace spectrum with the wave-length, and of differences of structure in furnace lines as compared with those of the arc. The differences of structure, in the case of barium, have led to the measurement of a number of ultra-violet lines not given in tables of arc wave-lengths.

Mount Wilson Solar Observatory
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[1] *Mt. Wilson Contr.*, No. 114; *Astrophysical Journal*, **43**, 341, 1916.

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STUDIES BASED ON THE COLORS AND MAGNITUDES IN STELLAR CLUSTERS

SIXTH PAPER: ON THE DETERMINATION OF THE DISTANCES OF GLOBULAR CLUSTERS

By HARLOW SHAPLEY

I. INTRODUCTION AND SYNOPSIS

The derivation of parallaxes for globular clusters will contribute to several problems of general astrophysical interest. Through the introduction of a linear scale we may expect to learn much of the actual dimensions of these stellar groups, and of the relation to distance from their centers, of the luminosity, mass, spectrum, and star density, and, eventually, of the internal motions. Since we may also be able to estimate at least a lower limit for the total mass involved in each group, the opportunity is promising for a contribution to the rather meager supply of observed facts for studies in stellar dynamics. Further, a knowledge of the distances of these widely distributed systems, and of the highly luminous variable stars for which parallaxes will be obtained in the course of the same research, will add to our conception of the dimensions of the visible stellar universe. These problems are of enough importance to justify a considerable effort in the study of the distances of globular clusters and their distribution in space. As direct measurement of cluster parallaxes is out of the question, the procedure must be by other methods, such as those involving proper motions and luminosity correlations.

The range in the absolute brightness of stars in many globular clusters is now known to exceed ten magnitudes, and we have as yet no reason to believe that the dispersion of luminosity in any of the typical globular clusters is not strictly comparable with the known dispersion in the general galactic system. Our ordinary investigations of cluster magnitudes, therefore, involve many stars of relatively great luminosity. Among these very brightest stars reddish Cepheid variables of long period frequently appear, while among the stars two or three magnitudes fainter is found the typical cluster-type variable—a Cepheid with a first-type spectrum and a period less than a day.

For some years we have known that the longer-period Cepheids in the galactic system are also giant stars, and a casual examination of their motions indicates a fairly small dispersion in actual luminosity. The possibility is at once suggested, therefore, of utilizing the motions of galactic Cepheids to obtain a mean value of the absolute magnitude of such stars, which, when compared with observed magnitudes of analogous objects in globular clusters, permits an estimate of the distances of the clusters themselves. One obvious advantage of operating with a group of giant stars is that in many of the most distant clusters only the stars of highest luminosity are within reach of our greatest telescopic power. It is also clearly of prime importance in problems of this nature, which involve faint stars and great distances, to have reliable systems of magnitudes established and to be able to ignore with safety the general scattering of light in space.

The methods and results of the investigation of cluster parallax will be discussed in Papers VI to XII, inclusive, of the series of studies of magnitudes in clusters. The present discussion begins with the derivation of a mean absolute magnitude for the isolated Cepheids whose motions are well determined, and then considers the remarkable relation between the period of variation and the total light-emission of such stars. Examples of this interesting correlation, which may perhaps be considered a fundamental law in Cepheid variation, are found among the variables in no less than six different stellar organizations as well as among the galactic Cepheids. Combined with the very homogeneous and accordant data derived from motions, it permits the deduction of absolute magnitudes, and hence of absolute distances, with a surprisingly small computed probable error. In fact, unless some intrinsic weakness in the procedure or some overlooked alternative is found, we may believe that the distances of the Cepheid variables, and of the extremely remote globular clusters in which Cepheid variables have been studied, can be determined with a percentage accuracy rarely excelled and usually unequaled by direct measures of the nearest stars.

Now it happens that in many globular clusters no variable stars are as yet known; in others the Cepheids similar in period to those for which motions are known in the galactic system are of rare occurrence compared with the typical cluster-type Cepheid with period less than a day. We find, however, that the median magnitude of the typical cluster-type variable shows such an extremely small dispersion in a given cluster that a constant value can be assumed; and when we adopt the reasonable and important hypothesis that the equality of the total light-emission is universal for such variables, the corresponding absolute median magnitude can be accurately derived from the analysis of the motions, magnitudes, and periods of the longer-period Cepheids. The determination of the parallaxes of certain clusters then becomes a by-product of the study of their typical variable stars.

A further step correlates this median magnitude with the magnitude of the brightest stars in a cluster. A means is thus afforded of computing distances of all the clusters (whether or not they contain known variables) for which apparent photographic magnitudes of the brightest stars are measured. Finally, we find from the results derived by the processes outlined above that the parallax of a cluster is very definitely related to its angular diameter, and this yields a method of obtaining from already available photographs the distances of all globular clusters so far discovered.

In the second article (the seventh paper of the general series) the individual distances of globular clusters are derived; their highly significant distribution in space is also discussed, and some computations made of the linear dimensions, of the concentration of stars, and, provisionally, of the total masses in cluster systems.

The parallaxes and co-ordinates in space of all Cepheid variables, for which magnitudes and periods are known, appear in the eighth paper. The chief uncertainty in the results is due to the lack of accuracy and homogeneity in published values of magnitudes. For the isolated cluster-type variables accurate values of the period are not essential in deriving distances, as the dispersion in absolute magnitude is small for periods less than a day.

The next paper contains three notes on certain theoretical and observational aspects of Cepheid variation. An interpretation is offered for some of the results obtained in studying the distances of clusters and variables, the relation of color to period is examined observationally for cluster-type variables, and a composite color-curve is derived for more than a hundred variable stars.

An inquiry into some problems of the frequency of stellar luminosities appears in the tenth paper. The magnitudes of several thousand cluster stars have been measured for this discussion. Some light is thrown on the meaning of the median magnitude of variables, and the propriety of its use for parallax work, rather than the use of maximum or minimum, is established.

The eleventh paper treats briefly of the distances and distribution in space of various objects and classes of objects. The data derived from clusters and variables is supplemented by the results of other investigators for different types and groups of stars.

The last article, summarizing the facts and indications of the preceding papers, contains a preliminary attempt to discern the general plan of the sidereal system. The arrangement proposed has its principal justification in that it appears to be a simple interpretation of the new data and at the same time does not seem to be inconsistent with previous observational results bearing on the structure of the universe.

In the accumulation of the observational material for these studies much credit is due Miss Davis for the measurement of large numbers of stars in many difficult cluster fields. She has also assisted throughout in the reduction and discussion of the observations. Mrs. Shapley has collaborated in the treatment of most of the problems and, in particular, is responsible for large parts of the tenth and twelfth papers. Valuable assistance in the preparation of the papers for the press has been given by Miss Connor. Mr. Pease has freely permitted the use of his long-exposure cluster photographs. Data relative to parallaxes have been kindly furnished by Mr. van Maanen and relative to spectroscopic results by Mr. Adams. Special acknowledgment is due Mr. Seares for many valuable suggestions and criticisms and for painstaking editorial supervision of the whole series of cluster papers.

II. THE MEAN ABSOLUTE MAGNITUDE OF CEPHEID VARIABLES

Hertzsprung[1] and Russell[2] have computed the mean absolute magnitude of Cepheid variables from the proper motions given in the *Preliminary General Catalogue* of Boss, and the former has published his work in some detail. With a few corrections to the observational data the computations are now repeated and somewhat extended. The material is not extensive, and its sufficiency for this problem might well be questioned if it were not for the fortunate circumstances that the data are complete for each star and in most respects homogeneous; that no evidence of preferential motion is found; and that the peculiar motions of such stars, without exception, are small compared with their parallactic drifts.[3] Hertzsprung tabulates 13 stars, but two of them are not typical Cepheid variables and have been excluded;[4] no others have sufficiently accurate proper motions to be added to the list.[5]

Table I contains observational data relative to the group of 11 Cepheids. With the exception of Polaris, all are near the

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[1] *Astronomische Nachrichten*, **196**, 201, 1913; see also *Zeitschrift für wissenschaftliche Photographie*, **5**, 94, 107, 1907.

[2] *Science*, N.S., **37**, 651, 1913.

[3] See n. 1, p. 15.

[4] Both the period and light-curve of κ Pavonis are subject to considerable perturbation according to Gould (*Uranometria Argentina*, p. 244, 1879), and Roberts also notes the star as an exception to ordinary Cepheid variation. The light-variation of l Carinae is peculiar; "an irregular and ill-defined secondary maximum has frequently been observed" (Roberts, *Astronomical Journal*, **21**, 89, 1901). Albrecht has just reported a variation in the spectrum of l Carinae from F8 to G5 (*Popular Astronomy*, **25**, 519, 1917), and it may be the star is not so abnormal as was believed when the computations for this paper were made.

The typical Cepheid characteristics of the eleven stars used are attested not only by their light-curves, but also in every case by spectroscopic study. We have no velocity-curves for l Carinae and κ Pavonis. The inclusion of both in the discussion would change the final result by less than its probable error. Including κ Pavonis would greatly decrease the certainty of the computed absolute magnitudes, though not altering them seriously, while the inclusion of l Carinae alone would make no appreciable difference in either the result or its probable error.

[5] The values given by Perrine in *Astrophysical Journal*, **41**, 308, 1915, for Y Ophiuchi and SZ Tauri are too uncertain for the present work. The proper motion of RR Lyrae, period 13.5 hours, is rightly excluded; see the eighth paper of this series, "The Luminosities and Distances of 139 Cepheid Variables," *Mt. Wilson Contr.*, No. 153, 1917.

TABLE I

OBSERVATIONAL MATERIAL FOR ELEVEN CEPHEIDS

Boss No.	Name	R.A. 1900	Decl. 1900	Galactic		Median Magnitude	Spectrum	Period	τ	v	λ	V
				Longitude	Latitude							
												km/sec.
325..	α Ursae Minoris	$1^h 22^m.6$	$+88° 46'$	90°	+27°	2.12	F8	$3^d.9681$	$+0''.016$	$+0''.041$	60°	−15
637..	SU Cassiopeiae	2 43.0	+68 28	101	+ 9	5.9	A8–F5	1.9495	+ 8	+ 14	75	− 7
1629..	RT Aurigae	6 22.1	+30 34	151	+11	5.3	A7–G1	3.72806	− 4	+ 24	118	+22
1815..	ζ Geminorum	6 58.2	+20 43	164	+13	4.0	G	10.15382	− 2	+ 9	126	+ 7
4493..	X Sagittarii	17 41.3	−27 48	329	− 1	4.7	F1–G5	7.01188	− 3	+ 23	59	−14
4564..	W Sagittarii	17 58.6	−29 35	330	− 5	4.7	A8–G2	7.5946	+ 9	+ 12	61	−29
4632..	Y Sagittarii	18 15.5	−18 54	340	− 4	5.8	F4–G4	5.7734	+ 6	+ 12	50	+ 4
5071..	η Aquilae	19 47.4	+ 0 45	8	−14	4.05	A8–G5	7.176382	+ 1	+ 12	39	−14
5098..	S Sagittae	19 51.5	+16 22	23	− 7	5.8	F4–G3	8.381613	− 1	+ 4	30	−12
5370..	T Vulpeculae	20 47.2	+27 52	40	−11	5.8	A9–G1	4.435521	− 11	+ 10	36	− 1
5807..	δ Cephei.......	22 25.4	+57 54	73	+ 1	4.14	F0–G2	5.366386	+0.003	+0.012	52	−17

galactic plane; their distribution in galactic longitude is satisfactory. The median visual magnitudes differ in some instances from those used by Hertzsprung; the spectral types, taken mainly from *Contribution* No. 124,[1] are known to be variable for all but two of the stars. The proper motions have been reduced to the parallactic system of co-ordinates, v being counted positive in the direction of the antapex. Without further reduction the parallactic motion is conspicuously evident, the unweighted algebraic means being

$$\left.\begin{aligned}\tau&=+0''.002\\ v&=+0''.016\end{aligned}\right\} \qquad (1)$$

All values of v are positive. The distances from the apex of the solar motion, λ, are very favorable for the weight of the solution.

Curves of velocity-variation have been determined for all of these stars, mainly at the Lick Observatory. Seven of the values of V, the observed radial velocity of the center of mass, are taken from Duncan's compilation in *Lick Observatory Bulletin*, No. 151. The value for α Ursae Minoris, computed by Miss Hobe, has been given by Campbell,[2] and those for X Sagittarii[3] and δ Cephei[4] were computed by Moore. The value for SU Cassiopeiae is obtained from an unpublished discussion by Adams and Shapley of the variations in light, velocity, and spectral type.[5]

Some of the steps in the reduction appear in Table II. To eliminate possible effects of the dispersion in distance the proper motions were reduced to the common apparent magnitude +5, which is very near the mean apparent median magnitude of the variables, +4.8. The weights of the individual values of the reduced parallactic motion, $v_5/\sin\lambda$, were determined in the manner suggested and used by Hertzsprung.[6] They depend on the distribution of velocities for this type of star, on the probable errors given by Boss for the individual proper motions, and on the distances from the solar apex. The weighted mean value and its probable error are

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[1] *Astrophysical Journal*, **44**, 273, 1916.

[2] *Lick Observatory Bulletin*, **6**, 19, 1910.

[3] *Ibid.*, **5**, 111, 1909.

[4] *Ibid.*, **7**, 153, 1913.

[5] *Mt. Wilson Contr.*, No. 145, 1917. *Note added to proof, April, 1918:* The source of the visual magnitude 6.23 is *Harvard Annals*, **50**, and not the *Preliminary General Catalogue*, as erroneously printed (*Astrophysical Journal*, **47**, 50, 1918).

[6] *Astronomische Nachrichten*, **196**, 203, 1913.

$$\frac{v_5}{\sin\lambda}=+0''.0161\pm 0''.0016. \qquad (2)$$

It is of interest to compare this value with (1). The corresponding value derived by Hertzsprung, if we reduce his result to the same apparent magnitude, is

$$\frac{v_5}{\sin\lambda}=+0''.0146\pm 0''.0022$$

from which he derived for the mean parallax of a Cepheid of the fifth apparent magnitude

$$p_5=0''.0035\pm 0''.00054,$$

and for the mean absolute magnitude, corresponding to the mean period of 6.6 days,

$$M=-2.3\pm 0.35.$$

The larger probable error of his value of $v_5/\sin\lambda$ is attributable mostly to the inclusion of the highly discordant κ Pavonis.

TABLE II

SOLUTION FOR MEAN PARALLAX

BOSS No.	OBSERVED		$\frac{v_5}{\text{SIN }\lambda}$	RELATIVE WEIGHT	PARALLACTIC v_5	RESIDUAL v_5	V_0	π (M = −2.35)
	τ_5	v_5						
325..	+0″.004	+0″.011	+0″.013	99	+0″.014	−0.003	− 4km	0″.0128
637..	+0.012	+0.021	+0.022	48	+0.016	+0.005	− 2	22
1629..	−0.005	+0.028	+0.032	61	+0.014	+0.014	+12	30
1815..	−0.001	+0.006	+0.007	83	+0.013	−0.007	− 6	54
4493..	−0.003	+0.020	+0.023	80	+0.014	+0.006	− 4	39
4564..	+0.008	+0.011	+0.013	69	+0.014	−0.003	−18	39
4632..	+0.009	+0.017	+0.022	12	+0.012	+0.005	+18	24
5071..	+0.001	+0.008	+0.013	48	+0.010	−0.002	+ 2	52
5098..	−0.001	+0.006	+0.012	16	+0.008	−0.002	+ 6	24
5370..	−0.016	+0.014	+0.024	11	+0.009	+0.005	+16	24
5807..	+0.002	+0.008	+0.010	79	+0.013	−0.005	− 4	0.0050

With the value of the sun's velocity adopted by Hertzsprung, 20 km/sec., equation (2) gives $p_5=0''.00386$ and $M=-2.06$; but later determinations from radial motions point to a higher velocity of the sun. We have the following recent values, based on a greater proportion of giant stars than were included in the early determinations and therefore probably more reliable because of smaller peculiar velocities:

		km/sec.	Mean Error
Charlier,[1]	156 B-type stars (mag.<5.0), $V_\odot$	$=21.26$	
Gyllenberg,[2]	284 B-type stars	$=22.06$	± 0.91
Gyllenberg,	291 A-type stars	$=19.77$	± 1.45
Strömberg,[3]	1400 F, G, K, M, giant stars	$=21.48$	± 1.02

Adopting $V_\odot = 21.5$ km/sec.,

$$\frac{v}{p \sin \lambda} = 4.535$$

$$p_5 = 0''.00354 \pm 0''.00035 \qquad (3)$$

and the absolute magnitude, corresponding to the mean period of 5.96 days, is

$$M = -2.26 \pm 0.22.$$

Computing for each star that part of the v-component due to parallactic motion, $0''.0161 \sin \lambda$, and subtracting these values in the sixth column of Table II from the observed values of v_5 in the third column, we obtain in the seventh column values for the peculiar velocity parallel to the direction of the sun's motion. The values of τ_5 and of "residual v_5" are in satisfactory agreement.[4] Their mean values, without regard to sign, are

$$\tau_5 = 0''.0056 \pm 0''.0010.$$

$$\text{Residual } v_5 = 0.0052 \pm 0.0007.$$

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[1] *Meddelande från Lunds Astronomiska Observatorium*, Serie II, No. 14, 1916.

[2] *Ibid.*, No. 13, 1915.

[3] *Mt. Wilson Contr.*, No. 144, 1917.

[4] A considerable part of the agreement is a necessary consequence of the magnitude of the observational errors relative to those of the proper motions. The average probable error for the annual motion in one direction is about $\pm 0''.004$. As the peculiar motions are not greatly in excess of the errors of observation, possibly less weight should have been given to (6) in obtaining the final value (7). The difference between (3) and (7), however, is less than half the probable error of either.

Combining all twenty-two values, with half weight for the residual $v_5=$components because one constant has been derived from them, we obtain the mean value for the peculiar proper motion in one direction

$$\mu_5'=0''.0055\pm0''.00064, \tag{4}$$

a value about one-third that of the derived parallactic motion.

The observed radial velocities corrected for the sun's motion[1] are given in the eighth column of Table II. Their arithmetical mean value is

$$V_0=8.35\pm1.29 \text{ km/sec.} \tag{5}$$

and we derive from (4) and (5) an independent value of the mean parallax

$$p_5=\frac{4.74\,\mu_5'}{V_0}=0''.0031\pm0''.0005. \tag{6}$$

Probably the very close agreement of (3) and (6) is partly chance. If in the place of (5) we use a value based upon all known radial velocities of Cepheids, we add to the eleven values in Table II nine others, most of which are only rough estimates,[2] and obtain $V_0=9.58$ km/sec., $p_5=0''.0027$.

Giving double weight to (3), or, what amounts to the same, combining (3) and (6) with regard to their probable errors, we obtain the final values:

$$\left.\begin{array}{l} p_5=0''.0034\pm0''.00030 \\ M=-2.35\pm0.19 \end{array}\right\} \tag{7}$$

The probable errors of the foregoing mean values include the errors in the observed proper motions and radial velocities and some uncertainties of reduction; but they do not include the errors

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[1] The value 21 km/sec. was used for the sun's velocity in this computation, but no difference in the mean value would result from using 21.5 km/sec.

[2] The additional stars are SU Cygni, Y Ophiuchi, SZ Tauri, T Monocerotis, and five southern Cepheids for which very approximate velocities have been estimated recently by Paddock from a few plates of each (*Lick Observatory Bulletins*, **9**, 68, 1917). The value for T Monocerotis is a little more than a guess (*Astrophysical Journal*, **23**, 266, 1906). The velocity curve of SZ Tauri is by Haynes (*Lick Observatory Bulletins*, **8**, 85, 1914), and for the others Duncan's table furnishes information.

(relatively much less important in the derivation of M) in the apparent magnitudes and the velocity of the sun, nor the uncertainty arising from any systematic motion or drift of the Cepheids as a group.

From the value (7) and the apparent median magnitudes in the seventh column of Table I, the individual parallaxes have been derived and entered in the last column of Table II. The probable error of each value depends only on that of M and the uncertainty of the corresponding apparent magnitude; its average is about ±0″.0007, or 15 per cent of the tabulated parallaxes. Better values of these parallaxes are derived later.[1]

TABLE III

Boss Number	Observer	Rel. π and P.E.	Absolute π	π from Parallactic Motion and Period
325	Flint *et al.*	±0″.018	+0″.028	+0″.016
637	van Maanen	+0.008±0.003	+0.010	+0.004
1815	Abetti, Miller	+0.019±0.009	+0.025	+0.004
5071	Mitchell	+0.001±0.009	+0.006	+0.005
____(X Cygni)	Miller	0.000±0.008	+0.006	+0.001
5532	Lee and Joy	−0.016±0.014	−0.011	+0.018

Direct measures of the parallaxes of four of the 11 Cepheids here considered are available for comparison with the results of the present investigation. The data are shown in the first four lines of Table III. Flint's parallax (absolute) of Polaris, +0″.008±0″.016, is combined with six of the more recent determinations listed by Kapteyn in *Groningen Publications*, No. 24. The values of the last column are taken from a later paper of this series. The absolute values are systematically larger than those based upon the parallactic motion, the mean difference being +0″.010 for the first four stars, which corresponds to a difference of approximately two magnitudes in the absolute brightness. This discrepancy is large and raises a question as to possible sources of error.

We note, however, that the inclusion of X Cygni and the short-period variable β Cephei (Boss 5532)[2] reduces the difference in the mean values to $+0\overset{''}{.}003$; and, further, according to an unpublished investigation by van Maanen, there is some evidence that the directly measured parallaxes require systematic corrections sufficient to bring the mean value below that derived from the parallactic motion by $0\overset{''}{.}001$. These circumstances illustrate the uncertainty of any conclusion based upon a small number of directly measured parallaxes whose values are as minute as those of the Cepheid variables appear to be.

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[1] *Mt. Wilson Contr.*, No. 153, the eighth paper of this series.

[2] Their parallaxes are determined later by the method used to calculate the final values for the 11 Cepheids, although they could not be included in the original group.

On the other hand, the smallness of the proper motions makes the calculated parallactic motion sensitive to any systematic errors in the observed motions, provided the errors are so large or so distributed that the mean value of p_5 is affected. An attempt to determine directly from the observations a systematic correction to the proper motions is not expedient, however, as the number of stars is small and they are widely scattered over the sky. The reduced individual motions should reveal any conspicuous error.

If we assume a systematic correction to the proper motions of the amount used by Kapteyn (*Contributions*, Nos. 82 and 147) for some of the B-type stars in the southern hemisphere, and suppose that its effect is not obliterated in the mean result, the deduced absolute brightness might be changed by about one-tenth of a magnitude, an amount clearly insufficient to question the general accuracy of the present result.

In the introduction to the *Preliminary General Catalogue* Boss has suggested as provisional systematic corrections to his proper motions:

$$d\mu_\alpha = +0\overset{s}{.}00021 - 0\overset{s}{.}00015 \sin\alpha \tan\delta$$

$$d\mu_\delta = -0\overset{''}{.}0023 \cos\alpha$$

These corrections have been applied to the proper motions of the Cepheids and revised values computed for the τ- and υ-components. In Table IV the probable errors of the proper motions, as given by Boss, should be compared with the proposed systematic corrections. (For the first star, Polaris, a systematic correction of this general nature is of course not appropriate.)

In nearly every case the probable error exceeds the systematic correction, and in the average it is about three times as large. The revised values of τ and υ should be compared with the analogous values in Table I. The average correction to υ is just one-half the probable error of the proper motion in one direction, and the systematic correction to the parallactic motion is also about one-half of its probable error. The corresponding correction to the adopted absolute magnitude would be less than $+0.05$, an amount safely negligible. We conclude, therefore, that unless the systematic errors are assumed to be of a wholly different order of magnitude from those derived by Kapteyn and Boss the mean parallax of these Cepheids is essentially correct.

TABLE IV

Boss No.	P.E. of μ_α	P.E. of μ_δ	$d\mu_\alpha$	$d\mu_\delta$	Revised		$\Delta\upsilon$
					τ	υ	
325....	$\pm 0^{s}.00343$	$\pm 0''.0008$	$(-0^{s}.00225)$	$(-0''.0022)$	$+0''.015$	$+0''.041$	$0''.000$
637....	88	40	$-$ 4	$-$ 17	$+$ 7	$+$ 16	$+$ 2
1629....	37	35	$+$ 12	$+$ 2	$-$ 2	$+$ 23	$-$ 1
1815....	11	17	$+$ 16	$+$ 6	0	$+$ 7	$-$ 2
4493....	23	30	$+$ 13	$+$ 1	$-$ 2	$+$ 22	$-$ 1
4564....	35	44	$+$ 12	$+$ 0	$+$ 11	$+$ 12	0
4632....	74	80	$+$ 16	$-$ 2	$+$ 8	$+$ 12	0
5071....	20	29	$+$ 21	$-$ 10	$+$ 3	$+$ 15	$+$ 3
5098....	31	40	$+$ 25	$-$ 11	0	$+$ 8	$+$ 4
5370....	59	63	$+$ 27	$-$ 15	$-$ 11	$+$ 13	$+$ 3
5807....	± 0.00020	± 0.0017	$+0.00031$	-0.0021	0.000	$+0.014$	$+0.002$

There remains, however, the possibility of systematic error in the mean distance due to a preferential drift of the Cepheids as a class—a drift which, in order to escape ready detection by means of the peculiar motions, must be either very small or nearly in the direction of the solar motion. Suppose, for example, that the mean absolute magnitude of the eleven Cepheids were zero rather than -2.35. This corresponds approximately to the extreme systematic difference indicated by the directly measured parallaxes. Then the mean parallactic motion would be $0''.05$, that is, three times the observed value (which is determined with a computed error of only 10 per cent). To harmonize this assumption with the observed parallactic motion we must assume an annual preferential drift of $0''.03$ in a direction deviating but a few degrees from the solar apex. Such a drift should reveal itself in the radial velocities, which show no effect of the required order of magnitude. As a matter of fact, the mean component of these velocities (corrected for solar motion) in the direction of the sun's apex is +3.3 km/sec. with a probable error of ±3.5. The corresponding effect on the mean parallactic motion would be of the order of 0″.003, or one-tenth that required; and the uncertainty, arising from this source, in the mean parallax given in (7) is perhaps of the order of 0″.0006.

TABLE V

Boss No.	p'_5	Probable Error	M	Period	π
325........	0″.0029	±0″.0009	−2.7	$3^{d}.97$	0″.0110
637........	49	13	−1.6	1.95	32
1629........	71	11	−0.7	3.73	63
1815........	15	10	−4.1	10.15	24
4493........	51	10	−1.5	7.01	58
4564........	29	11	−2.7	7.59	33
4632........	49	26	−1.6	5.77	33
5071........	29	13	−2.7	7.18	45
5098........	26	22	−2.9	8.38	18
5370........	53	27	−1.4	4.44	36
5807........	0.0022	±0.0010	−3.3	5.37	0.0033

A general solution for the preferential motion, based on all the Cepheids for which radial velocities are available,[1] gives the following values for the velocity of the hypothetical drift, its rectangular components, and the position of its apex:

$$V=6.4\pm3.0\text{ km/sec.}$$

$$X=+0.2\pm6.2\qquad Y=-1.8\pm2.6\qquad Z=+6.1\pm3.0$$

$$A=275^\circ\ \pm195^\circ\qquad D=+74^\circ\ \pm131^\circ$$

The relative magnitudes of the probable errors show that as far as this material goes there is no appreciable motion of the Cepheids as a class.

Referring again to the data of Table II, we assume, for the purpose of computation, that the v-component for every star in the foregoing tables is wholly parallactic. Then the parallax of each variable, reduced to apparent magnitude +5, is $p'_5=0.22\ v_5/\sin\lambda$, and the corresponding absolute magnitudes are as given in Table V. Proceeding as above, where a constant value of M was used, new values of π are obtained. Their fairly close agreement with the results of the last column of Table II further emphasizes the relative smallness of the neglected peculiar motion.[1]

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[1] See n. 2 on p. 10.

The chief interest of Table V, however, lies in the comparison of the absolute magnitude with the period of variation. A marked correlation is at once evident,[2] and if we combine the stars in order of period into small groups so as to eliminate some of the effect of the neglected peculiar motion in v and some of the errors of observation, we find

Number of Stars	Absolute Magnitude	Period
1	−4.1	10.15 days
3	−2.8	7.72
3	−2.1	6.05
3	−1.6	4.05
1	−1.6	1.95

The probable error of the mean absolute magnitude would be further reduced by allowing for this correlation between luminosity and period.[3]

III. THE RELATION OF PERIOD TO LUMINOSITY

From the plot[4] of the numbers in the fourth and fifth columns of Table V a new absolute magnitude is obtained for each star, and these smoothed values are plotted against the logarithm of the period as large circles in Fig. 1. From a final curve, based on such material as this, we shall presently derive the absolute magnitudes and distances of all Cepheids for which the periods are known.[1]

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[1] This agreement further suggests that the number of stars is sufficient to give a dependable value of the mean parallax. We may, in fact, group the stars in threes, either at random, or in the order of any characteristic (except period), and the separate means will almost invariably give a value differing by less than half a magnitude from the mean value for the eleven stars.

[2] This might be interpreted as a peculiar distribution of the v-components depending on the length of the period, but further work on the relation of period to luminosity completely disposes of this unlikely alternative.

[3] The correction, however, is very small because the principal source of the probable error remains in the dispersion of the residual v_5-components. Since $M=f(P)$, we observe that the reduced parallactic motion, $v_5/\sin\lambda$, is also a function of the period, and that for the extremes of period the residual v_5 values really contain some portion (positive or negative) of the parallactic motion. A new solution, which involved the reduction of the observed proper motions to apparent magnitudes depending on the period rather than to the common apparent magnitude $+5$, naturally gave no appreciable difference in the mean parallax and absolute magnitude, and reduced the probable error of the latter by less than 0.02.

[4] The deviations from the smooth curve correspond in general to the residual v_5-components, providing we grant that M is uniquely defined by P.

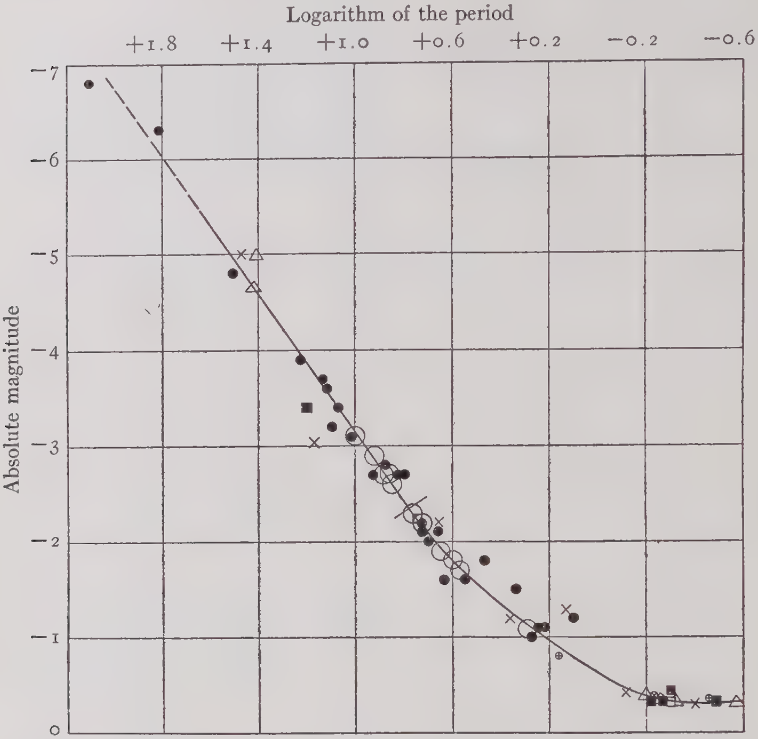

FIG. 1.—Luminosity-period curve of Cepheid variation. The various symbols designate variables from seven different systems. The short bisecting line at absolute magnitude −2.35, log period 0.775, indicates the mean values for Cepheids of known proper motion. Most of the symbols for periods less than a day represent averages of about ten variables. Of the six largest deviations, four refer to values of particularly low weight. Table XI contains co-ordinates of the adopted curve.

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[1] The eighth paper of this series contains the results for the individual stars (*Mt. Wilson Contr.*, No. 153). Without correcting for the progression of color with period, and assuming a linear relation between period and luminosity, absolute magnitudes have already been computed for two-thirds of the long-period Cepheids by Hertzsprung (*Astronomische Nachrichten*, **196**, 205, 1913) and by Russell for a paper by Russell and Shapley (*Astrophysical Journal*, **40**, 417, 1914). The individual results were not published.

Some years ago Miss Leavitt found a similar relation between the apparent photographic brightness and the length of period for the Cepheid variable stars in the Small Magellanic Cloud.[1] In order to support the result derived above for isolated Cepheids and to obtain a definitive luminosity-period curve, we may reduce her results to the absolute visual system of the present work. From certain globular clusters information that is still more valuable may be obtained through the correlation of the luminosities of the long- and short-period Cepheids. The various sources will be taken up separately.

1. *Small Magellanic Cloud.*—The magnitudes given by Miss Leavitt for stars in the Magellanic clouds are based upon estimates on ordinary photographic plates and are referred to a provisional scale and zero-point. The uncertainty of the zero-point is of no importance for our immediate purpose. As the magnitude scale is probably nearly correct, we shall adopt it as it stands, giving diminished weight to the brightest and faintest stars, and transforming the median brightness of the variables from photographic to visual apparent magnitudes.

The reduction to the visual system will be very small for the short-period variables, but probably as much as two magnitudes for some of the red stars with periods longer than ten days. In the absence of direct color or spectral determinations the change to visual magnitudes must be made on the basis of length of period, using the data already collected for an earlier paper.[2] With a few modifications based on recent spectral classifications,[3] the material is given again in Table VI, the last two columns of which are plotted in Fig. 2. The curve as drawn in the figure has been used for all color corrections; but for periods greater than one day a linear formula

$$\text{Color-index} = -0.55 + 1.5 \log P$$

represents satisfactorily the change of color with period. The probable interpretation of the curve is discussed in a later article.[4]

[1] *Harvard Circular*, No. 173, 1912; *ibid.*, No. 79, 1904; *Harvard Annals*, **60**, 106, 1908.

[2] *Mt. Wilson Contr.*, No. 92; *Astrophysical Journal*, **40**, 448, 1914.

[3] *Mt. Wilson Contr.*, No. 124; *Astrophysical Journal*, **44**, 273, 1916.

[4] "Three Notes on Cepheid Variation," the ninth paper of this series.

Of the 969 known variables[1] in the Small Magellanic Cloud the periods of 25 have been determined. The designation, logarithm of the period, median[2] photographic magnitude, reduction for color-index by means of the curve in Fig. 2, and the adopted visual magnitude are given for each star in successive columns of Table VII.

TABLE VI

PERIOD AND SPECTRAL TYPE

Limits of Spectrum	Mean Spectrum	Number of Stars	Color-Index	Log. of Mean Period
B0 to A9...	A4	15	+0.15	−0.26
F0 to F9...	F5	27	+0.6	+0.78
G0 to G9...	G5	31	+1.0	+1.04
K0 to K9...	K5	9	+1.4	+1.26
M+....	Ma	3	+1.6	+1.52

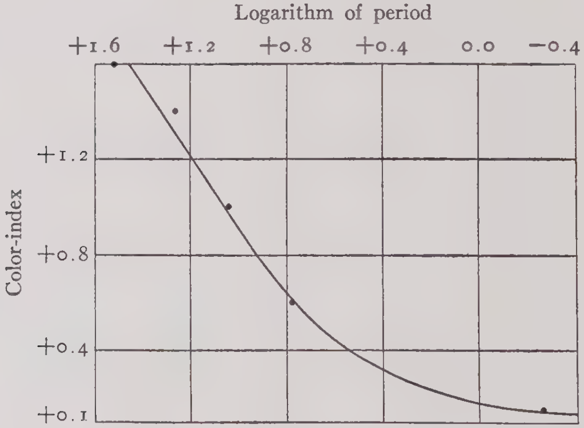

FIG. 2.—Change of median color with period for Cepheid variables

The visual magnitudes of the 25 variables were reduced as follows to the absolute system represented in Fig. 1. The provisional luminosity-period curve, based on the isolated variables as described in an earlier paragraph, covers only a part of the interval for which periods have been determined in the Small

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[1] *Harvard Annals*, **60**, No. IV, 1908.

[2] The mean of the values for maximum and minimum.

Magellanic Cloud. To reduce to the absolute scale, values of the absolute magnitude corresponding to the observed periods of the second column have been read from the provisional curve for the 13 variables with $\log P$ between 0.27 and 1.00. The results are in the sixth column of Table VII; in the seventh are the

TABLE VII

CEPHEID VARIABLES IN SMALL MAGELLANIC CLOUD

Harvard Variable	Log. Per.	Median Pg. Mag. (Leavitt)	Color-Index from Curve	Median Vis. Mag.	Absolute Mag. from Curve	Difference	Provisionally Adopted Abs. Mag.
1505....	0.098	15.4	+0.2	15.2	....	....	−1.2
1436....	0.221	15.6	0.3	15.3	....	....	1.1
1446....	0.246	15.6	0.3	15.3	....	....	1.1
1506....	0.273	15.7	0.3	15.4	−1.1	−16.5	1.0
1413....	0.337	15.2	0.3	14.9	1.3	16.2	1.5
1460....	0.464	15.0	0.4	14.6	1.5	16.1	1.8
1422....	0.545	15.3	0.4	14.8	1.7	16.5	1.6
842....	0.632	15.3	0.5	14.8	1.9	16.7	1.6
1425....	0.658	14.8	0.5	14.3	2.0	16.3	2.1
1742....	0.698	14.9	0.5	14.4	2.1	16.5	2.0
1646....	0.726	14.9	0.6	14.3	2.2	16.5	2.1
1649....	0.727	14.8	0.6	14.2	2.2	16.4	2.2
1492....	0.799	14.3	0.6	13.7	2.3	16.0	2.7
1400....	0.823	14.4	0.7	13.7	2.4	16.1	2.7
1355....	0.874	14.4	0.8	13.6	2.7	16.3	2.8
1374....	0.924	14.5	0.8	13.7	−2.8	−16.5	2.7
818....	1.015	14.2	0.9	13.3			3.1
1610....	1.066	14.0	1.0	13.0	Mean diff.	−16.4	3.4
1365....	1.094	14.3	1.1	13.2			3.2
1351....	1.116	13.9	1.1	12.8			3.6
827....	1.129	13.8	1.1	12.7			3.7
822....	1.224	13.8	1.3	12.5			3.9
823....	1.504	13.2	1.6	11.6			4.8
824....	1.818	12.1	2.0	10.1			6.3
821....	2.104	11.6	+2.0	9.6			−6.8

differences between the magnitudes from the curve and the apparent magnitudes. The mean difference, −16.4, is the reduction constant, which, applied to all apparent median magnitudes, gives in the last column the corresponding absolute magnitudes. In making this transformation, the purpose of which is to relate *change* of period with *change* in absolute visual magnitude, we make no assumption regarding the actual luminosity of Cepheid variables in the Magellanic clouds; but as soon as we use the reduction constant as a measure of distance we assume, of course, that variables of a given period are of comparable luminosity, whether they are in the general galactic system or in separate stellar organizations such as globular clusters and the Magellanic clouds.

The absolute magnitudes of the last column of Table VII, plotted in Fig. 1 as dots, show that the same relation holds for the Cepheids in the Small Magellanic Cloud and in the galactic system, and that an improved and extended luminosity-period curve may be based upon the combined data. The relation appears so definite that the prediction of the length of period on the basis of magnitude estimates should be possible for most of the 944 other variables in the Small Magellanic Cloud.[1] Two or three hundred of them are fainter than any for which the period has been determined. It is very probable that they are cluster-type variables with periods of the order of twelve hours. With no correction either for color or for divergence of magnitude scale their median magnitudes on the absolute system are about −0.3, agreeing almost exactly with the mean value determined below for cluster-type variables in globular clusters.

It is important to note further that magnitude 16.0 in Miss Leavitt's provisional system marks an abrupt and definite fainter limit to the median brightness of the variables in the Small Magellanic Cloud. The plates, which were made with the 24-inch Bruce telescope with exposures varying from two to five hours, are sufficient to test this matter, for minima are observed as faint as 17.0, but, with one possible exception, no maximum is recorded fainter than 16.0. A similarly definite fainter limit to the interval of magnitude throughout which Cepheids occur has been observed in globular clusters, particularly in ω Centauri, and Messier 3, 5, and 13. The concurrent progression toward a definite limit of luminosity, spectrum, and period suggests that, in the evolutionary sequence of stars, Cepheid variation is abruptly limited at or near the blue end of the giant series because of the changing physical conditions in the interior of the gaseous masses.

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[1] Without doubt nearly all variables in both clouds belong to the Cepheid class. In the Small Cloud, however, four variables have an observed range of at least three magnitudes, and are probably long-period variables rather than Cepheids. Possibly a few others have larger ranges of variation than shown by the plates examined by Miss Leavitt and are also long-period variables. It seems to be a definite observational fact that no star that otherwise has typical Cepheid characteristics is known to have a range in excess of two magnitudes. The value, photographically, falls usually between 0.8 and 1.5, regardless of the length of period. Hence the appropriateness of Eddington's search for the physical reason of an upper limit to the amplitude of a pulsation in a gaseous star ("The Pulsation Theory of Cepheid Variables," *Observatory*, **40**, 290, 1917).

2. *ω Centauri.*—Of 132 variables observed by Professor Bailey in the southern cluster ω Centauri,[1] periods were determined for 93, of which 3 are long-period Cepheids. From the observations and notes it has been possible to derive approximate results for two others with periods in excess of a day. In Table VIII the correction to visual magnitude and the reduction to the absolute system follow the same plan as that for the Small Magellanic Cloud. The magnitudes of the fifth column were read from the improved luminosity-period curve, but since at this point in the discussion the curve has not yet been extended to periods less than a day, the three groups of cluster-type variables were not used in the reduction. The light-curve of No. 29 is hardly typical of Cepheids, and its magnitude also appears somewhat discordant. In obtaining the reduction constant, −13.80, half weight was given to the approximate magnitudes for Nos. 48 and 61. The numbers in parentheses in the first column refer to the total number of stars in each subgroup. Plotting as crosses in Fig. 1 the values of the second and last columns, the luminosity-period curve is further improved for the ordinary Cepheids and is extended to the cluster-type variables.

TABLE VIII

CEPHEID VARIABLES IN ω CENTAURI

Designation	Log. Per.	Median Pg. Mag. (Bailey)	Visual Mag.	Mag. from Curve	Difference	Provisional Abs. Mag.
1..........	1.47	10.45	8.80	−4.70	−13.50	−5.00
29..........	1.17	11.94	10.76	−3.60	−14.43	−3.04
48..........	0.66	12.08	11.60	−2.00	−13.60	−2.20
60..........	0.13	12.74	12.52	−1.01	−13.51	−1.28
61..........	0.36	12.92	12.60	−1.35	−13.95	−1.20
a(37).......	−0.23	13.55	13.40			−0.40
b(19).......	−0.12	13.54	13.37	Mean diff.	−13.80	−0.43
c(34).......	−0.40	13.61	13.49			−0.31

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[1] *Harvard Annals*, **38**, 1902.

3. *Messier 5.*—Professor Bailey has recently determined the periods and light-curves of about 70 variables in Messier 5.[1] Of the three with periods longer than a day, two are certainly Cepheids. The third, No. 50 of Bailey's list, with a period of 106 days, is one component of a close bright double. . Its observed magnitude is certainly diminished by the Eberhard effect, possibly to a great extent. Professor Barnard has observed visually all three of these stars.[2] In a recent letter he states that the measures of No. 50 do not give the rapid rise to maximum light that is typical of Cepheid variation. If the star were included without correction for the Eberhard effect, it would deviate more than three magnitudes from the curve.

TABLE IX

Cepheid Variables in Messier 5

Designation	Log. Per.	Median Pg. Mag. (Bailey)	Visual Mag.	Mag. from Curve*	Difference	Provisional Abs. Mag.
42	1.41	11.72	10.20	−4.55	−14.75	−4.98
50	2.03	13.60	11.6	....	....	−3.6
84	1.42	12.08	10.53	−4.57	−15.10	−4.65
Double Max. (8)	−0.57	14.98	14.84	....	....	−0.34
Group 1	−0.32	14.98	14.85	−0.33	−15.18	−0.33
Group 2	−0.26	14.98	14.83	−0.39	−15.22	−0.35
Group 3	−0.20	14.96	14.80	−0.45	−15.25	−0.38
				Mean diff.	−15.18	

* It should be noted that the luminosity-period curve is slightly corrected after each accretion of data so that the magnitudes in the fifth column are not those derivable from the final curve.

A group of 8 variables with double maxima or peculiarly short periods is found among the cluster-type variables of Messier 5.[3] That they are single stars with an average period of six and one-half hours seems the most probable hypothesis; they are accordingly used to extend the luminosity-period curve and to show that no appreciable decrease in luminosity occurs as the periods become less than twelve hours.

Bailey has collected into three equal groups, in order of length of period, the thirty typical cluster-type variables with most definite light-curves. Each group is given weight 5 in determining the reduction constant. The material is discussed in Table IX and, omitting No. 50, the results are plotted in Fig. 1 as triangles.

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[1] *Harvard Annals*, **78**, Part 2, 1917.

[2] *Astronomische Nachrichten*, **147**, 243, 1898.

[3] *Harvard Circular*, No. 193, 1916.

4. *Messier 3.*—Bailey's monograph on Messier 3 contains the light-curves of 110 stars, none of which has a period exceeding a day.[1] In his catalogue the only bright star that is certainly variable is one that appears to be irregular. A bright Cepheid, however, was found near the center of the cluster by Barnard, who has determined the period and published a light-curve.[2] The median photo-visual magnitude of this star has been determined from plates made at Mount Wilson and measured by Miss Davis and the writer. Although the light-curve derived by Barnard is more nearly symmetrical than is usual for Cepheid variables, the color and the change of color between maximum and minimum, which is typical of this kind of variable, is clearly indicated by the Mount Wilson measures. A small correction to the final magnitude for Eberhard effect would be appropriate, because of the star's situation in the densest part of the cluster, and would probably eliminate its deviation from the curve in Fig. 1.

One variable of Bailey's list, No. 37, has been specially studied at Harvard[3] and on a series of Mount Wilson plates.[4] Its period is about one-half that of the typical variable of Messier 3, resembling in this respect, as well as in the shape of the light-curve, the eight

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[1] *Harvard Annals*, **78**, Part 1, 1913.

[2] *Astronomische Nachrichten*, **172**, 345, 1906. Barnard suspected another bright star of variation, No. 19, in his nomenclature. Referred to his comparison star No. 8 no variation is apparent on the Mount Wilson plates so far examined. The data for photographic magnitudes are as follows:

Plate	Date	19−8	Plate	Date	19−8
2372	1915, April 16	$0^{M}00$	3679.......	1917, March 28	$+0^{M}06$
2463	June 7	+0.10	3680.......	28	+0.04
2506	July 6	+0.04			

These stars are among the very brightest in the cluster, and, as might have been inferred from previous investigations, they are red. The color-index of No. 19 is approximately +1.8 magnitudes, and of No. 8, more than two magnitudes, according to measures on five photographic and three photo-visual plates. There is some possibility of Eberhard effect as both stars are near the center.

[3] *Harvard Circular*, No. 193, 1917; *Harvard Annals*, **78**, Part 2, 1913.

[4] *Publications of the Astronomical Society of the Pacific*, **29**, 110, 1917.

stars of Messier 5 that are designated "Double Max." in Table IX. It is entered singly in Table X. The 54 typical cluster-type variables in Messier 3 for which Bailey considered the results most certain are combined, in order of period, into three equal groups, each of which is given weight 10 in deriving the reduction constant. The photo-visual magnitudes are referred to the Mount Wilson system, a series of polar comparison plates being used to obtain the appropriate correction to Bailey's photographic magnitudes. Black squares in the diagram designate the results for Messier 3.

5. *Messier 13.*—Two Cepheid variables in Messier 13 were discovered by Bailey and Barnard and studied by the latter.[1] Of five others found by the writer among the fainter stars[2] four appear to have periods less than a day. Approximate median photo-visual magnitudes for these four and the earlier two have been obtained from Mount Wilson photographs; but without a complete study of all the light-curves the results have low weight. Carrying through the usual reductions, however, and combining the provisional results into two groups, we obtain the values indicated by open squares in Fig. 1.

TABLE X

CEPHEID VARIABLES IN MESSIER 3

Designation	Log. Per.	Median Pv. Mag.	Mag. from Curve	Difference	Provisional Abs. Mag.
Barnard 7...	1.20	12.3	−3.7	−16.0	−3.4
Bailey 37....	−0.49	15.40	−0.35	−15.75	−0.33
Group 1.....	−0.30	15.29	−0.34	−15.63	−0.44
Group 2.....	−0.27	15.39	−0.36	−15.75	−0.34
Group 3.....	−0.22	15.39	−0.38	−15.77	−0.34
			Mean diff......	−15.73	

6. *Other sources.*—Further numerical results bearing on the luminosity-period relation are not now available, but certain sources may be cited from which quantitative results will come in time and from some of which even now we may infer confirmation of the interdependence of period and luminosity.

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[1] Bibliography and discussion in *Mt. Wilson Contr.*, No. 116, p. 78, 1915.

[2] *Ibid.*, pp. 79 ff.

The median photographic magnitudes of the 808 variables discovered by Miss Leavitt in the Large Magellanic Cloud[1] range from 10.5 to 15.8; there is but one fainter exception, which, by showing that fainter stars are easily visible on the plates, proves the rule that Cepheid variation affects only stars brighter than a certain limit. If we should assume the same color correction as used for the Small Magellanic Cloud, the interval of magnitude in which the variables occur is much the same. There can be little doubt that these are Cepheid variables (the ranges of all but two or three are less than two magnitudes[2]) among which the luminosity-period law holds as elsewhere. Good values of the distance and extent of the star cloud will be given eventually through determinations of the magnitudes and periods of these variable stars.

The 50 variable stars in Messier 15 are being studied by Professor Bailey on a series of 75 Harvard and Mount Wilson plates. The periods so far derived, with one exception, are less than a day. The exceptional star has a period of 1.44 days, a provisional value kindly communicated by Professor Bailey; its median magnitude is about 0.43 brighter than the median magnitude of the other variables, according to measures on a few Mount Wilson plates. The results are preliminary and receive no weight in obtaining the luminosity-period curve, but because of the close agreement with results for other clusters, the following data, derived in the usual manner, are plotted in Fig. 1 as small circles containing crosses:

Variable No. 1, Median Mag. (Absolute) $=-0.82$, Log. Per. $=+0.16$
Variable No. 13, Median Mag. (Absolute) $=-0.40$, Log. Per. $=-0.24$
Variable No. 11, Median Mag. (Absolute) $=-0.37$, Log. Per. $=-0.46$

Nos. 13 and 11 are typical of the two subclasses of variables in Messier 15; the median photographic magnitudes, Bailey finds, are in all cases about 15.7.[3]

Variable stars have now been discovered in 26 globular clusters,[4] and from Mount Wilson plates and from reproductions in *Harvard*

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[1] *Harvard Annals,* **60,** No. IV, 1908; *Harvard Circular,* No. 82, 1904.

[2] See n. 1, p. 20.

[3] *Popular Astronomy,* **25,** 520, 1917.

[4] To the list in *Harvard Annals,* **38,** p. 2, several have been added through recent discoveries by Miss Davis (*Publications of the Astronomical Society of the Pacific,* **29,** 210, 260, 1917).

Annals, 38, it is possible to make a preliminary comparison of the relative magnitudes in each system. Thus we find that there are at least 15 clusters, in addition to those examined in some detail in the foregoing discussion, in which some or all of the variables are among the very brightest stars. Clusters which show also a considerable diversity in the magnitude of variables, and which are therefore of importance for the luminosity-period curve, include N.G.C. 104, 362, 1904, 6266, 6397, and 6626.

The final luminosity-period curve, as drawn in Fig. 1 and given numerically in Table XI, is based upon more than 230 stars, and, except for zero-point uncertainty, is probably correct within one or two hundredths of a magnitude. Ten plotted points lie on the curve, 23 are below, and 24 above. The average unweighted deviation is ±0.13 mag.,[1] an amount so nearly of the same order as the errors of the measured magnitudes that for typical[2] Cepheids of given period a rigorously constant median magnitude may be assumed. Almost all of the large deviations from the curve are of

TABLE XI

Co-ordinates of the Luminosity-Period Curve

Logarithm of Period	Absolute Visual Magnitude	Logarithm of Period	Absolute Visual Magnitude
−0.6	−0.34	+0.8	−2.43
−0.5	0.33	0.9	2.79
−0.4	0.3	1.0	3.15
−0.3	0.34	1.1	3.51
−0.2	0.38	1.2	3.87
−0.1	0.50	1.3	4.23
0.0	0.64	1.4	4.59
+0.1	0.81	1.5	4.95
+0.2	0 99	1.6	5.31
+0.3	1.17	1.7	5.67
+0.4	1.37	1.8	6.02
+0.5	1.58	1.9	6.38
+0.6	1.81	2.0	6.74
+0.7	−2.10	+2.1	−7.1

---

[1] The observational errors in the periods are relatively negligible.

[2] The word "typical" is frequently used to make allowance for such stars as RV Tauri, κ Pavonis, and V Ursae Minoris which show some general Cepheid characteristics, but because of various recognized irregularities or peculiarities may also be irregular in the relation of period to absolute brightness.

low weight, due to uncertain estimates of magnitude or period, peculiarities in light variation, or possible error in the magnitude scale. Above absolute magnitude -5.5 the curve depends only on the longest period variables in the Small Magellanic Cloud; but the resulting uncertainty is not serious, as few Cepheids have periods longer than forty days.[1]

Future work on the periods and magnitudes of variables in clusters is not likely to alter appreciably the form of the luminosity-period curve; but further investigation of the proper motions of Cepheids may contribute to the certainty of the zero-point, which is now defined both as to amount and accuracy by equation (7). The distances of even the nearest Cepheids are so great that little can be expected from direct parallax measures in the way of quantitative confirmation or improvement of the curve.[2]

IV. THE MEDIAN MAGNITUDE OF CLUSTER-TYPE VARIABLES

The flattening of the luminosity-period curve for magnitudes fainter than -0.5 indicates that for the typical cluster-type variable the median brightness is essentially invariable and is independent of the length of period. As we shall presently relate the magnitude of these variables to the maximum brightness attained in clusters, a further examination of the dispersion of median magnitudes is advisable. From preceding tables we derive the data in Table XII.

Thus the absolute *photographic* magnitude[3] for 183 variables is

$$\text{Median} = -0.23 \tag{8}$$

The deviations from this mean value may be due largely to the errors in the magnitudes of the longer-period Cepheids of each cluster. No marked change of photographic brightness with period appears; the change of visual magnitude with period recorded in Fig. 1 and Table XI is due to the small progression of color with period, assumed in the reductions on the basis of Fig. 2.

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[1] Cf. Table II of the eighth paper. Long-period Cepheids are liable to irregularity.

[2] See sec. II, above.

[3] This value, which becomes of much importance in the determination of cluster parallaxes, is independent of the earlier transformations from photographic to photovisual magnitudes with the aid of Table VI.

Much work bearing on the constancy of the median magnitude of cluster-type variables has been done at Mount Wilson; but a full discussion of the data would be too extensive for the present paper, and the results for only two systems are outlined below.

TABLE XII

Cluster	Number of Variables	Mean Period	Mean Absolute Photographic Magnitude	Deviation
ω Centauri..	34	$0^{d}.40$	−0.19	$-0^{M}.04$
	37	0.59	−0.25	+0.02
	19	0.76	−0.26	+0.03
Messier 5...	8	0.27	−0.20	−0.03
	10	0.48	−0.20	−0.03
	10	0.55	−0.20	−0.03
	10	0.63	−0.22	−0.01
Messier 3...	1	0.32	−0.20	−0.03
	18	0.50	−0.31	+0.08
	18	0.54	−0.20	−0.03
	18	0.60	−0.20	−0.03

1. In ω Centauri three subclasses of cluster-type variables are recognized. Omitting those for which the classification is uncertain, we have in Table XIII a summary of the data bearing on the dispersion of median magnitudes. Although the stars of the three groups differ from each other in maximum magnitude and range, as well as in period and form of light-curve, the median values are the same. The distribution of individual deviations agrees closely with the probability curve, as shown in Table XIV.

TABLE XIII

VARIABLE STARS IN ω CENTAURI

Subclass	No. of Variables	Mean Period	Maximum Magnitude	Range of Variation	Median Magnitude	Average Deviation
a...........	33	$0^{d}.586$	12.99	$1^{M}.11$	13.55	$\pm 0^{M}.09$
b...........	15	0.752	13.10	0.87	13.55	±0.10
c...........	28	0.395	13.33	0.56	13.61	±0.09
All.........	76	....	....	....	13.57	±0.10

2. The variables in Messier 3 differ very little from each other in any respect. The comparison stars and some of the variables have been studied by Miss Davis and the writer on a series of 65 photographs; the magnitude scale has been revised and referred to the Mount Wilson system. A sample of the revised data, showing the constancy of the median magnitudes, has been given in *Mt. Wilson Communication* No. 47.[1] For the 54 light-curves selected by Bailey as most definite the median photographic magnitude is 15.49±0.01, the average deviation from the mean value is ±0.07, and the largest deviation is less than two-tenths of a magnitude. If we include all 110 variables for which periods are known, the mean is 15.50±0.006, with an average deviation ±0.08.

TABLE XIV

Number of Residuals between	Theory	Observation
$0^{\mathrm{M}}.00$ and $0^{\mathrm{M}}.03$	18	19
0.04 “ 0.06	14	12
0.07 “ 0.09	13	15
0.10 “ 0.12	10	13
0.13 “ 0.15	7	7
0.16 “ 0.18	5	0
0.19 “ 0.24	6	7
≥0.25	3	3

A further examination of the median magnitudes for the 54 selected stars shows the following small systematic variation, which is definitely connected with the range:

Mean range of variation	$1^{\mathrm{M}}.04$	$1^{\mathrm{M}}.14$	$1^{\mathrm{M}}.26$	$1^{\mathrm{M}}.33$	$1^{\mathrm{M}}.48$
Mean median magnitude	15.58	15.52	15.49	15.47	15.41
Number of variables	10	4	22	9	9

One explanation of this variation is that the duration of exposure was often so long that for some stars the brightness at the top of the sharp-pointed curves was never determined, the measures yielding fainter, more rounded maxima than actually exist. The range deduced for such stars is always too small, and the systematic error goes into the median magnitude with half its weight. The variation may be due, rather, to errors of observation at maximum light, where usually only a few estimates are available and any error will directly correlate range and median magnitude. The first explanation gives 15.4 as the true median magnitude, the second leaves it at 15.5.[1]

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[1] *Proceedings of the National Academy of Sciences*, 3, 480, 1917.

In either case an appropriate and simple correction to the deviations from the mean median magnitude, for the systematic errors in the maxima, leaves the uncertainty in the median magnitude but half as great as given above, and the average deviation for a single star is less than ±0.05. The distribution of the corrected residuals again accords with the law of error as closely as could be expected for a small number of values:

Number of Residuals between	Theory	Observation
$0^{\mathrm{M}}00$ and $0^{\mathrm{M}}04$	29	27
0.04 " 0.08	16	15
0.08 " 0.12	7	9
>0.12	2	3

The magnitudes in Messier 2, 5, 15, and 22 yield results similar to the foregoing. In each cluster, apparently, the total light variation of short-period Cepheids is confined to a narrow interval of brightness; and in all cases, where the observations are sufficient to justify a conclusion, the deviations of the median magnitudes from their mean are far within the errors of observation. Hence we are led to place much confidence in the hypothesis that the parallax of a cluster-type variable (or of any cluster containing such stars) may be derived immediately from the measurement of the median magnitude.

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[1] The error is probably common to all groups of variables. It does not vitiate the work on cluster parallaxes, for the median value as observed is used to obtain both apparent and absolute magnitudes. A systematic error may be introduced into the determination of the distances of some isolated cluster-type variables, but at most it will be only a few per cent and far within the uncertainty of the various magnitude scales.

V. PARALLAXES OF CLUSTERS FROM THEIR BRIGHTEST STARS

The most important use of accurate median magnitudes is to furnish a starting-point for the study of the absolute luminosity of the brightest stars in a cluster. In Messier 3, for instance, where the magnitudes of all the brighter stars[1] have been accurately measured, there are nearly 300 more luminous than the median magnitude of the variables. A few that are more than two magnitudes brighter appear in the cluster, it may be, by projection only, or they may be double or multiple stars. If we limit our study to the region within 9′ from the center and exclude a few, say five, of the very brightest objects, we can feel sure that practically all the remaining bright stars are really typical members of the cluster.

TABLE XV

PHOTOGRAPHIC MAGNITUDES OF 30 BRIGHTEST STARS IN MESSIER 3

Star	Color-Index	Pg. Mag.	Deviation	Star	Color-Index	Pg. Mag.	Deviation
206....	+1.01	11.27	Bright	1449....	+1.40	14.18	−0.05
420....	+0.19	13.59	Bright	463....	+1.13	14.25	+0.02
589....	+0.91	13.71	Bright	238....	+1.79	14.27	+0.04
205....	+1.09	13.81	Bright	334....	+1.13	14.27	+0.04
837....	+1.36	13.83	Bright	265....	+1.14	14.29	+0.06
				925....	+1.34	14.29	+0.06
1127....	+1.30	13.92	−0.31	1397....	+1.69	14.29	+0.06
1219....	+1.40	14.04	−0.19	398....	+1.13	14.32	+0.09
706....	+1.16	14.06	−0.17	237....	+0.42	14.33	+0.10
1000....	+1.25	14.08	−0.15	309....	+1.19	14.37	+0.14
1128....	−0.60	14.08	−0.15	640....	+1.18	14.40	+0.17
417....	+1.15	14.09	−0.14	1203....	+1.32	14.40	+0.17
740....	+0.70	14.70	−0.13	1208....	+1.23	14.40	+0.17
490....	+1.80	14.13	−0.10	1214....	+1.22	14.40	+0.17
297....	+1.32	14.14	−0.09	177....	+1.00	14.45	+0.22
1392....	+1.22	14.14	−0.09				
				Means..	+1.16	14.23	±0.12

In Table XV the sequence of the 30 brightest objects is shown for Messier 3, the data being taken from an unpublished investigation of the magnitudes and colors of nearly a thousand stars.[2] The mean luminosity of the 25 chosen stars is determined with nearly the same accuracy as the median magnitude of the variables. The difference, Med. − Mean Br., is +1.27; hence from equation (8) we find that the brightest stars have an absolute photographic magnitude of −1.5 in the mean, and a maximum just fainter than −2. These bright stars are reddish, however, and the maximum visual absolute brightness exceeds −3 magnitudes in a few cases.

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[1] Stars within a concentric circle of nearly 3′ diameter are excluded because of possible systematic errors arising from crowding of images and Eberhard effect.

[2] The numbers are those of von Zeipel's catalogue, *Annales de l'Observatoire de Paris*, **25**, 1908.

Other clusters which contain short-period Cepheids agree in showing that the maximum photographic luminosity is regularly between 1.5 and 2.0 magnitudes brighter than the median value for the variables. We see, therefore, in this apparent constancy of maximum magnitude, the possibility of an expeditious method of furthering our knowledge of cluster distances. The phenomenon is qualitatively illustrated by an inspection of the photographs reproduced in *Harvard Annals*, **38**; and a quantitative confirmation is readily possible through magnitude measures, even when the variables are few in number and their light-curves are unknown. Thus, for Messier 22 a study of three polar-comparison plates gives for the mean magnitude of the 25 brightest stars[1] the value 13.08, with extremes of 12.51 and 13.55, and an average deviation of ±0.19, while the median magnitude of the variable stars,[2] from the measures given in Table XVI, appears to be 14.45. Hence, Med. − Mean Br. = +1.37. For a few clusters, in which the variables have been extensively studied, more accurate values of this difference can be obtained.

All material now available has been considered in discussing the relation of median to brightest magnitude, summarized in Table

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[1] Five stars brighter than the "25 brightest" are always excluded in using this method.

[2] Sixteen variable stars are listed by Bailey (*Harvard Annals*, **38**, 242, 1902). No. 11 is a very close double and was not measured; Nos. 3 and 14 could not be certainly identified. The variability of Nos. 2, 5, 9, and 16 is not definitely confirmed by the Mount Wilson plates. No. 8 is one of the brightest stars in the cluster and appears to be a long-period Cepheid; similarly, Nos. 5 and 9 are probably bright Cepheids of long period. For the remaining 8 stars a short-period variation of a magnitude or more is fairly well established by these plates. The total interval of brightness is about two magnitudes; the uncertainty of the median magnitude given above is possibly one or two tenths.

XVII.[1] Some of the tabulated material has been adapted from Bailey's published work, but most of it has been derived from Mount Wilson plates, of which the total number entering the discussion for each cluster is noted in the second column. The third column gives the number of cluster-type variables contributing to the mean median magnitude. It should be observed that the final result appears to be independent of the frequency of variable stars.

TABLE XVI

VARIABLES IN MESSIER 22—OBSERVATIONS OF 1917

Number	Plate 3887 Aug. 14.71	Plate 3888 Aug. 14.72	Plate 3892*a* Aug. 14.75	Plate 3892*b* Aug. 14.75	Plate 3940 Sept. 9.70	Plate 3963 Sept. 10.68
1	....	....	....	15.51	14.06	15.37
2	14.96	....	....	15.12	15.15	14.89
3	....	....	....	....	....	16.36?
4	14.80	15.06	....	14.95	15.13	14.38
5	12.72	12.78	12.72	12.75	12.46	12.64
6	15.04	14.79	....	14.37	14.98	14.01
7	15.06	15.10	....	14.78	14.32	15.41
8	12.16	12.17	12.29	12.10	12.76	12.79
9	13.24	13.31	13.21	13.17	13.32	13.15
10	15.14	15.02	....	15.39	15.03	14.38
12	15.01	14.99	....	14.42	14.95	14.93
13	14.20	13.78	13.72	13.58	14.98	13.68
15	14.95	14.99	....	15.36	14.52	14.93
16	14.41	14.54	....	14.52	14.47	14.34

The radius of the concentric circular area in which the bright stars were measured is given in the fourth column of Table XVII. The choice of this quantity is somewhat arbitrary, and small changes in it may affect the mean perceptibly if thereby exceptionally bright stars happen to be included or omitted.[2] The adopted radius depends in general upon both the nature of the photographs and the cluster's angular diameter, but mainly upon the galactic latitude. In rich fields the area is necessarily small in order to exclude bright non-cluster stars. We might have adopted the same radius for all clusters and attempted to allow for foreign objects by varying with galactic latitude the number of excluded stars, but the actual angular dimensions differ so greatly that a more flexible procedure seemed advisable. Every effort has been made in this and subsequent work of the same kind to obtain homogeneous results, in each case so choosing the area for measures of bright stars that, with the five brightest rejected, the resulting mean gives a trustworthy value of the maximum luminosity.[1]

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[1] In at least three clusters (N.G.C. 6266, 6626, and 6723), in which magnitudes have not been quantitatively studied, a considerable number of short-period variable stars are from one to two magnitudes fainter than the brightest stars. Qualitatively, therefore, the relation between the median and brightest magnitudes is verified in 10 clusters and the Magellanic clouds and is nowhere controverted. Its quantitative expression, however, is probably less definite than the differences of Table XVII suggest, the close agreement of several values being partially fortuitous. A later study of Messier 15, for instance, indicates an uncertainty of 0.2 mag. in the provisional value given here; but the adopted probable error of the mean difference amply covers this discrepancy.

[2] The mean magnitude for Messier 3 is 14.30 for a radius of 7′ and 14.17 for a radius of 11′.

TABLE XVII

MEDIAN MAGNITUDES AND THE BRIGHTEST STARS

CLUSTER	NO. OF PLATES	NO. OF VARIABLES	RADIUS	PHOTOGRAPHIC MAGNITUDE			WEIGHT
				Mean Br.	Median	Diff.	
Messier 3..	65	110	9′	14.23	15.50	1.27	8
5..	3	61	4	13.97	15.26	1.29	4
15..	7	48	6	14.31	15.63	1.32	2
2..	7	7	4	14.61	15.71	1.10	1
22..	6	8	5.5	13.08	14.45	1.37	1
13..	15	4?	6	13.75	15.3:	1.5:	0
ω Centauri.	3	90	....	12.3:	13.9:	1.6:	0
				Mean difference....		1.28	

The mean difference, +1.28, combined with (8), gives $M = -1.51$ as the mean absolute photographic magnitude for the bright stars. The probable error of the difference is not likely to be in excess of two-tenths of a magnitude. This estimate makes generous allowance for the real dissimilarities of the clusters (which seem to be small so far as magnitude limits go) and for the uncertainties in excluding peculiar and non-cluster stars. Taking also into consideration the probable error of (7) and the observational errors in the apparent magnitudes, m, we conclude that the probable error for the difference $M-m$, in the relation $5 \log \pi = M-m-5$, certainly does not exceed 0.4 mag., and, therefore, that the absolute parallax of a globular cluster may be obtained from the apparent magnitudes of its brightest stars with a probable error of less than 20 per cent. For relative parallaxes the probable error does not include the error of (7) or the possibility of systematic error in the magnitude scale; its value is 10 per cent or less for parallaxes derived from homogeneous data.

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[1] Central condensations and multiple stars were avoided. Counts of stars on the Franklin-Adams charts were frequently of service in estimating the probable frequency of outside stars in a cluster field.

VI. REMARK ON THE PARALLAXES OF CLUSTERS DERIVED FROM THEIR APPARENT DIAMETERS

The foregoing discussion shows that the mean apparent magnitude of the brightest stars in a globular cluster is a pretty dependable criterion of its distance, thus indicating that all systems are much alike in the maximum luminosity attained by any individual member. In consequence it is a natural assumption that clusters may also be closely comparable in actual size. In fact, the first paper of this series[1] contains a provisional curve correlating decreasing maximum brightness with decreasing angular diameter, and it follows that the apparent size of a globular cluster is also a direct measure of the parallax. As we may obtain the parallaxes of nearly 30 globular clusters by the methods outlined on preceding pages, a curve showing the relation of distance to apparent size can be readily constructed, and using this curve the parallax of any other cluster can be obtained from its diameter. A necessity of such work is homogeneity in the observations, and this is afforded in a highly satisfactory manner by the Franklin-Adams photographic charts, which cover the whole sky and include every known globular cluster. Further discussion of this phase of the work is reserved for the following contribution.

VII. SUMMARY

1. The determination of the distances and distribution in space of globular clusters involves a general treatment of extensive data bearing on the magnitudes, periods, light-curves, proper motions, and radial velocities of Cepheid variables in the Galaxy

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[1] *Mt. Wilson Contr.*, No. 115, p. 12, 1915.

and other systems and on the angular diameters of clusters and the number, magnitudes, and colors of their individual stars. Its successful accomplishment will help somewhat to a better understanding not only of the most remote objects known in the stellar universe, but also of the dimensions and dynamics of cluster systems and of the maximum luminosity attainable in stellar evolution. Suggestions relative to the extent and arrangement of the galactic system and to the sun's position therein will be a natural outcome of the work.

2. From parallactic motions the mean absolute magnitude of eleven isolated Cepheid variable stars has been derived with a relatively small computed probable error (sec. II). The luminosities of the individual stars are shown to be uniquely defined by their periods.

3. An extension of these results gives a relation (Fig. 1) connecting the periods of both the ordinary Cepheids and the cluster-type variables with their absolute magnitudes, which permits the derivation of the distances of all such variable stars as soon as their periods and apparent magnitudes are measured; and when we adopt the plausible hypothesis that Cepheids of a given period are comparable wherever found, the relation also yields the parallax of any cluster containing Cepheid variables. Data for more than 200 individual variables from seven different stellar systems contribute to the determination of the luminosity-period relation. Fainter than a definitely fixed luminosity Cepheid variation probably never occurs.

4. Further investigation makes the derivation of cluster parallaxes practically independent of variable stars by substituting the apparent magnitudes of the brightest stars as the criteria of distance (sec. V). Stars brighter than the absolute photographic magnitude −2 are exceedingly rare in clusters.

5. Angular diameters are next employed in extending the work, until finally for all globular clusters in both hemispheres values of the parallax become possible. The distances are derived and considered statistically in the next paper of this series.

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STUDIES BASED ON THE COLORS AND MAGNITUDES IN STELLAR CLUSTERS

SEVENTH PAPER: THE DISTANCES, DISTRIBUTION IN SPACE, AND DIMENSIONS OF 69 GLOBULAR CLUSTERS

By HARLOW SHAPLEY

I. PARALLAXES FROM VARIABLE STARS, APPARENT MAGNITUDES, AND ANGULAR DIAMETERS

Applying the methods discussed in the preceding *Contribution*,[1] the parallaxes of a few clusters are obtained directly from the periods and magnitudes of Cepheid variables; the parallaxes of a considerably larger number are derived from the mean magnitudes of the brightest cluster stars, and the survey is then made complete through measures of diameters of the photographic images of all globular systems. In Table I are listed the clusters for which the variable stars have received a sufficiently detailed discussion to permit a determination of the parallax directly from the luminosity-period curve of Cepheid variation. The apparent diameters and the adopted parallaxes are taken from tables appearing on following pages; the method of weighting the results is also subsequently described.

The parallaxes for Messier 3, 5, 15, and 22, depending almost entirely on the median magnitudes of numerous variable stars, are the most accurate. The computed probable error of the absolute magnitude is ±0.2; that of the apparent magnitude is estimated to vary from ±0.05 for Messier 3 to possibly ±0.2 for Messier 22. The corresponding limits of the probable error of the parallaxes are $\pm 0''.000007$ and $\pm 0''.000015$, that is, 10 and 13 per cent, respectively.

For most other clusters, of course, the errors are somewhat greater, particularly for those where the parallax depends solely upon either the magnitudes of the brightest stars or the measured diameter on photographic charts. For the former the estimated average probable error is 20 per cent, for the latter 25 per cent.[1] When two or three sources are available, as in Tables I and V, the errors are less; but in Table I the different parallaxes for each cluster are not all completely independent, as they were used in part to determine the reduction constants and curves. The residuals in the last column of Table I are expressed in millionths of a second of arc, and their minuteness indicates the validity of the methods involving diameters and the magnitudes of the bright stars.

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[1] *Mt. Wilson Contr.*, No. 151.

TABLE I

COMPARISON OF CLUSTER PARALLAXES FROM VARIABLES, MAGNITUDES, AND DIAMETERS

DESIGNATION		APPARENT DIAMETER	PARALLAX (UNIT IS 0″000001)				RESIDUALS
N.G.C.	Messier		Adopted	From Variables	From Magnitudes	From Diameters	
5272	3	7′0	72	72	71	72	0, − 1, 0
5904	5	8.6	80	80	80	81	0, 0, +1
6205	13	10.6	90	82:	89	91	−8:, − 1, +1
6656	22	16.0	118	116	121	116	−2, + 3, −2
7078	15	5.0	68	67	69	59	−1, + 1, −9
7089	2	7.0	64	65	60	72	+1, − 4, +8
5139	....	30	153	150	170:	155	−3, +17:, +2
Small Magellanic Cloud		....	52	52	....	....	....

The parallax of the Small Magellanic Cloud, which is given at the end of Table I, is relatively uncertain, for the value from variables can be checked by neither diameter measures nor maximum luminosities, and the zero-point error in the provisional magnitude scale used by Miss Leavitt is unknown.[2]

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[1] These estimates appear to be safely conservative. After the smoothing operation, described on a later page, much smaller average errors are obtained for the final parallaxes in Tables V and VIII; thus, including the 10 per cent probable error in the parallaxes due to uncertainty of the zero-point of the absolute scale, the average probable error for all clusters is estimated to be less than 15 per cent. Cf. Table VII.

[2] See sec. III of the preceding *Contribution*. Previous values for the parallax of the Small Magellanic Cloud are: Hertzsprung, $\pi = 0''0001$, *Astronomische Nachrichten*, **196**, 204, 1913; Kapteyn, $\pi = 0''00004$, *Mt. Wilson Contr.*, No. 82, 71, 1914; Shapley, $\pi = 0''00006$, *Mt. Wilson Contr.*, No. 116, 82, 1915. The new value in Table I is probably an improvement over the others because it allows for diversity in color of the variable stars and is based upon more definite knowledge of their absolute magnitudes.

In the preceding paper we found from the study of the bright stars and variables in several clusters that, after excluding the five brightest, the mean absolute photographic magnitude of the 25 most luminous objects is

$$M_{25} = -1.51 \pm 0.28.$$

Adopting this value, we have derived from measures of apparent magnitude the distances of practically all clusters north of declination −30°, the southern limit reached with the 60-inch reflector.

Of 300 cluster photographs taken during the last three years as a part of the program, about 175 have been measured for the magnitudes of either the bright stars or the variables; but the material is too extensive to describe in detail. Nearly all of the photographs were made with full aperture and on Seed 27 plates of various emulsions. The exposures vary in length from 10 seconds to 2 hours, but are mostly between 1 and 12 minutes in duration. For the clusters south of −20° the altitude was frequently so low that plates with first-class images could not be secured. Mr. Hoge has assisted with all the observational work.

A summary of the work on each cluster is given in Table II. The designation in the first column is followed in the second with numbers indicating the total number of plates used in all phases of the work and those used in the derivation of the mean magnitude. For the latter purpose the plates, with few exceptions, involve direct polar comparisons on two or more nights; the images of between 50 and 100 cluster stars and of between 20 and 50 Polar Standards were measured at least twice on each plate; and the measures were corrected as usual for scale, distance from center, and differential atmospheric extinction. As the measures, reductions, and discussion cannot well be given for the individual clusters, the method of work is illustrated merely with a summary of the final magnitudes for the bright stars in Messier 2 (Table III) and Messier 53 (Table IV).

The radius of the concentric region in which all bright stars were measured, given in the third column of Table II, does not closely indicate the apparent size of the cluster either actually

TABLE II

SUMMARY OF OBSERVATIONS

N.G.C.	No. Plates	Photographic Magnitude 25 Stars				Weight	Angular Diameter			
		Radius	Mean	Av. Dev.	Extremes		Melotte	Davis	Shapley	Mean
288	2, 2	4′	14.81	±0.16	14.38–15.04	*d*	12′	4′.0	4′.4	4′.2
1904	3, 2	2	15.29	0.13	15.01–15.72	*d*	4.5	2.4	2.2	2.3
4147	4, 2	1.5	16.58	0.18	16.23–16.93	*d*	1	0.8	0.8	0.8
5024	2, 2	5	15.07	0.09	14.94–15.26	*b*	5	5.6	5.4	5.5
5139	3, 2	....	(12.3:)	....	....	*d*	45	32	28	30
5272	65, 4	9	14.23	0.13	13.92–14.45	*a*	18	5.8	8.3	7.0
5904	3, 3	4	13.97	0.15	13.74–14.27	*a*	15	9.1	8.2	8.6
6093	2, 2	3	14.88	0.08	14.72–15.09	*d*	5	4.0	3.0	3.5
6121	3, 2	6	13.84	0.35	13.18–14.40	*c*	20	10.0	11.0	10.5
6205	15, 4	6	13.75	0.11	13.45–13.92	*b*	12	10.2	11.0	10.6
6218	2, 2	6	13.97	0.22	13.56–14.31	*b*	9	8.8	8.8	8.8
6229	3, 2	2.5	16.18	0.13	15.90–16.37	*d*	....	1.0	1.2	1.1
6254	2, 2	5	14.06	0.24	13.35–14.38	*c*	10	10.9	10.0	10.4
6333	2, 2	4	15.61	0.16	15.08–15.88	*b*	3	3.0	3.5	3.2
6341	3, 3	7	13.86	0.13	13.60–14.16	*c*	8	8.4	7.6	8.0
6356	2, 2	1.5	17.16	0.14	16.86–17.44	*d*	1.5	2.0	1.8	1.9
6402	2, 2	5	15.44	0.24	14.85–15.86	*b*	6	3.1	3.8	3.4
6626	3, 3	3	14.87	0.16	14.49–15.11	*b*	5	5.2	4.2	4.7
6638	2, 2	2	16.22	0.19	15.90–16.60	*b*	1.5	1.6	1.8	1.7
6642*	3, 2	1	16.07	0.26	15.51–16.46	*c*	1	1.2	1.2	1.2
6656	6, 3	5.5	13.08	0.19	12.58–13.55	*a*	20	14.5	17.5	16.0
6712	5, 2	1.5	16.10	0.19	15.65–16.36	*d*	2.5	2.2	2.0	2.1
6779	6, 2	2	15.31	0.20	14.98–15.70	*c*	1.5	2.2	2.6	2.4
6864	2, 2	1	17.06	0.13	16.76–17.35	*c*	2	1.6	1.5	1.6
6934	5, 3	3	15.78	0.19	15.33–16.11	*c*	1.5	1.4	1.5	1.4
6981	7, 2	3	15.92	0.17	15.53–16.20	*b*	2	2.2	2.4	2.3
7078	7, 3	6	14.31	0.11	14.13–14.55	*a*	6	4.8	5.2	5.0
7089	7, 3	4	14.61	0.09	14.25–14.76	*b*	8	7.3	6.8	7.0
7099	4, 3	2.5	14.63	±0.27	13.77–15.04	*c*	6	4.6	4.7	4.6

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* See n. 1, p. 7.

or relatively. The basis of its choice has been described in the fifth section of the preceding paper.

TABLE III

MAGNITUDES OF BRIGHT STARS IN MESSIER 2 (N.G.C. 7089)

STAR*	PHOTOGRAPHIC MAGNITUDES				RESIDUALS†	DEVIATION
	3883P	3902P	3904P	Means		
1.......	14.09	14.16	14.04	Bright	....	....
2.......	14.68	14.59	14.53	14.60	+ 8, − 1, − 7	0
3.......	14.65	14.64	14.70	14.66	− 1, − 2, + 4	+ 6
4.......	13.54	13.57	13.71	Bright	....	....
5.......	14.87	14.69	14.62	14.73	+14, − 4, −11	+13
6.......	14.77	14.69	14.65	14.70	+ 7, − 1, − 5	+10
7.......	14.73	14.69	14.65	14.69	+ 4, 0, − 4	+ 9
10.......	14.60	14.49	14.51	14.53	+ 7, − 4, − 2	− 7
11.......	14.80	14.72	14.70	14.74	+ 6, − 2, − 4	+14
13.......	14.68	14.69	14.57	14.65	+ 3, + 4, − 8	+ 5
14.......	14.65	14.57	14.61	14.61	+ 4, − 4, 0	+ 1
16.......	14.68	14.66	14.57	14.64	+ 4, + 2, − 7	+ 4
17.......	13.61	14.19	13.92	Bright	....	....
18.......	14.34	14.56	14.31	14.40	− 6, +16, − 9	−20
19.......	14.29	Contact‡	14.17	14.23	+ 6, ..., − 6	−37
20.......	13.54	13.78	13.70	Bright	....	....
21.......	14.68	14.69	14.57	14.65	+ 3, + 4, − 8	+ 5
23.......	14.77	14.69	14.57	14.68	+ 9, + 1, −11	+ 8
25.......	14.57	14.54	14.57	14.56	+ 1, − 2, + 1	− 4
26.......	14.65	14.72	14.57	14.65	0, + 7, − 8	+ 5
27.......	13.60	14.02	13.99	Bright	....	....
28.......	14.33	14.40	14.23	14.32	+ 1, + 8, − 9	−28
29.......	14.73	14.85	14.70	14.76	− 3, + 9, − 6	+16
30.......	14.57	14.72	14.46	14.58	− 1, +14, −12	− 2
31.......	14.73	14.72	14.57	14.67	+ 6, + 5, −10	+ 7
33.......	14.52	14.69	14.40	14.54	− 2, +15, −14	− 6
36.......	14.65	14.69	14.46	14.60	+ 5, + 9, −14	0
39.......	14.73	14.66	14.72	14.70	+ 3, − 4, + 2	+10
40.......	14.73	14.66	14.53	14.64	+ 9, + 2, −11	+ 4
43.......	14.65	14.59	14.49	14.58	+ 7, + 1, − 9	− 2
Mean.................................				14.60	Mean..............	±0.09

* Stars fainter than the brightest thirty are omitted.
† Residuals and deviations from mean expressed in hundredths of a magnitude.
‡ Cluster star in contact with a Polar Standard.

For the average cluster the mean value of the photographic magnitude, in the fourth column, depends upon about 500 measures. Its estimated probable error varies from two to four-tenths of a magnitude, the principal uncertainty coming from possible non-homogeneity of the clusters and from the error in choosing the area to be measured. The average deviation shows the dispersion of the magnitudes entering the mean, but gives little indication of the certainty of the result; the extremes also show the dispersion, and the upper limit records the highest luminosity of the individual stars.

TABLE IV

MAGNITUDES OF BRIGHT STARS IN MESSIER 53 (N.G.C. 5024)

STAR*	PHOTOGRAPHIC MAGNITUDES			RESIDUALS†	DEVIATION
	2357P	2568P	Means		
2	15.03	14.98	15.00	+ 3, − 2	− 8
4	15.25	15.21	15.23	+ 2, − 2	+15
6	14.99	15.01	15.00	− 1, + 1	− 8
7	14.97	14.93	14.95	+ 2, − 2	−12
9	14.95	15.10	15.02	− 7, + 8	− 6
11	15.17	15.26	15.22	− 5, + 4	+14
12	15.10	15.26	15.18	− 8, + 8	+10
13	15.00	14.93	14.96	+ 4, − 3	−12
15	15.03	14.91	14.97	+ 6, − 6	−11
16	15.03	15.21	15.12	− 9, + 9	+ 4
17	15.03	14.88	Bright	....	....
18	14.50	14.57	Bright	....	....
20	15.27	15.21	15.24	+ 3, − 3	+16
22	15.15	14.93	15.04	+11, −11	− 4
24	14.87	14.57	Bright	....	....
26	15.05	15.01	15.03	+ 2, − 2	− 5
27	15.05	14.83	Bright	....	....
28	12.90	13.18	Bright	....	....
29	15.28	15.24	15.26	+ 2, − 2	+18
30	14.95	15.10	15.02	− 7, + 8	− 6
31	15.03	15.07	15.05	− 2, + 2	− 3
32	15.30	15.14	15.22	+ 8, − 8	+14
33	15.05	14.93	14.99	+ 6, − 6	− 9
34	15.15	15.33	15.24	− 9, + 9	+16
35	14.94	15.03	14.98	− 4, + 5	−10
36	15.05	15.10	15.08	− 3, + 2	+ 0
37	15.05	15.13	15.09	− 4, + 4	+ 1
38	15.05	15.03	15.04	+ 1, − 1	− 4
42	14.94	15.01	14.98	− 4, + 3	−10
45	14.94	15.04	14.99	− 5, + 5	− 9
Mean			15.08	Mean	±0.09

* Stars fainter than the brightest thirty are omitted.

† Residuals and deviations from mean expressed in hundredths of a magnitude.

Adding the difference between the brighter extreme and the mean to the adopted absolute magnitude of the latter, we get the maximum brightness in each cluster. Thus we find that the highest photographic luminosity never exceeds magnitude −2.5 (unless some of the excluded five are actually cluster stars), and the maximum usually falls slightly below -2. Contrary to expectation, the mean magnitude is nearer the brighter extreme in nearly one-third of the clusters, suggesting that among the giants the number of stars does not always increase regularly with decreasing luminosity.

In the seventh column of Table II weights are assigned each cluster on the basis of the quality of the plates, their number, the character of the surrounding stellar field, the certainty of the result for the mean magnitude, and other factors. The remaining columns are described later. Except to remark that the plates for N.G.C. 5139 (ω Centauri) were made with the 10-inch refractor and (because of the very low altitude) are of little value except as a check on Bailey's magnitudes, further space will not be taken for the extensive notes compiled relative to the peculiarities of individual clusters, the observations, the measures, and the investigations of errors.

The equatorial and galactic co-ordinates, the parallaxes, and the co-ordinates in space of the 28 globular clusters[1] for which magnitudes have been measured appear in Table V. For a few clusters the values of the parallax in the sixth column are taken from Table I and for the remainder are computed directly from the mean magnitudes of Table II.

If we plot the parallaxes derived from magnitudes against the diameters of the clusters, as recorded by Melotte,[2] a very definite progression of size with parallax is apparent. Melotte's estimates were made directly from the original Franklin-Adams chart plates; but there is no record of what accuracy was sought or what homogeneity may be expected in the results. Accordingly the diameters have been redetermined from the photographic copies of the plates with the special purpose in view of obtaining results as nearly comparable as possible for all parts of the sky. Measures of the diameters of the images were made independently by two observers, using a finely divided scale under low magnification. By estimating the

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[1] N.G.C. 6642 is retained in the table as a twenty-ninth entry. Melotte, with some doubt, classifies it as a globular cluster; Bailey thinks that its stars, few in number, are involved in nebulosity. Mount Wilson plates show a few stars closely crowded, but almost certainly not forming a typical globular cluster. The group is in a rich galactic field.

[2] *Memoirs of the Royal Astronomical Society*, **60**, Part 5, 1915.

TABLE V

PARALLAXES OF 28 GLOBULAR CLUSTERS FROM MAGNITUDES AND DIAMETERS

DESIGNATION		POSITION IN 1900		GALACTIC		PARALLAX (UNIT IS 0″.000001)			DISTANCE (UNIT IS 100 PARSECS)		
N.G.C.	Messier	R.A.	Decl.	Long.	Lat.	From Mag.	From Diam.	Adopted	Radial	Projected on Galactic Plane	From Galactic Plane
288	....	0h47m8	−27° 8′	214°	−88°	55	52	53	189	7	−189
1904	79	5 20.1	−24 37	195	−28	44	36	39	256	226	−120
4147	....	12 5.0	+19 6	227	+78	24	17	19	526	109	+514
5024	53	13 8.0	+18 42	307	+79	43	62	53	189	36	+186
5139	....	13 20.8	−46 47	277	+16	150	155	153	65	62	+ 18
5272	3	13 37.6	+28 53	8	+77	72	72	72	139	31	+135
5904	5	15 13.5	+ 2 27	333	+45	80	81	80	125	88	+ 88
6093	80	16 11.1	−22 44	320	+18	53	48	50	200	190	+ 62
6121	4	16 17.5	−26 17	319	+15	85	90	88	114	110	+ 30
6205	13	16 38.1	+36 39	26	+40	89	91	90	111	85	+ 71
6218	12	16 42.0	− 1 46	344	+25	80	82	81	123	111	+ 52
6229	....	16 44.2	+47 42	41	+39	29	20	23	435	338	+274
6254	10	16 51.9	− 3 57	343	+22	77	89	83	120	111	+ 45
6333	9	17 13.3	−18 25	334	+ 9	38	44	40	250	247	+ 39
6341	92	17 14.1	+43 15	35	+34	84	78	81	123	102	+ 69
6356	....	17 17.8	−17 43	335	+ 9	19	30	26	385	380	+ 60
6402	14	17 32.4	− 3 11	349	+14	41	46	43	233	226	+ 56
6626	28	18 18.4	−24 55	336	− 7	53	57	54	185	184	− 23
6638	....	18 24.8	−25 34	335	− 8	29	28	29	345	342	− 48
6642*	....	18 25.8	−23 32	337	− 7	31	22	26	385	382	− 47
6656	22	18 30.3	−23 59	338	− 9	118	116	118	85	84	− 13
6712	....	18 47.6	− 8 50	353	− 6	30	33	32	312	310	− 32
6779	56	19 12.7	+30 0	30	+ 7	43	37	40	250	248	+ 30
6864	75	20 0.2	−22 12	348	−28	19	26	22	455	401	−213
6934	....	20 29.3	+ 7 4	20	−20	35	24	30	333	313	−114
6981	72	20 48.0	−12 55	3	−34	33	35	34	294	244	−164
7078	15	21 25.2	+11 44	33	−29	68	59	68	147	129	− 71
7089	2	21 28.3	− 1 16	22	−37	60	72	64	156	125	− 94
7099	30	21 34.7	−23 38	356	−48	59	56	58	172	115	−128

*See n. 1 on p. 7.

elongation of similarly situated bright stars corrections have been made for distortion when a cluster is near the edge of the chart;[1] and to counteract the frequently observed actual ellipticity of figure the mean of the diameters in several directions has been determined for all clusters. The last three columns of Table II contain these closely agreeing measures and their mean, thus affording a fairly homogeneous record of relative apparent dimensions. The diameters estimated from the original plates are usually larger, as would be expected, but even they fall short of the actual dimensions for many clusters.

Plotting the adopted diameters of the last column of Table II against the parallaxes derived from magnitudes in Table V, we obtain the curve in Fig. 1—a curve that demands and apparently justifies the hypothesis that all globular clusters are of nearly the same linear dimensions. There is no reason for supposing that the more distant clusters are actually and systematically smaller than nearer ones; hence, as is reasonable a priori, the non-linear form of this curve may be attributed to the characteristic distribution of luminosity in a cluster and its resulting effect on photographic reproduction. Normal points for the data underlying the figure are given in Table VI and are plotted as black circles.[2] The lower part of the curve is somewhat uncertain, depending only on Messier 22 and ω Centauri, but the uncertainty is not important, for only two or three clusters are large enough to make their parallaxes depend on that part of the curve.

Undoubtedly an improvement can be made in at least some of the parallaxes through the process of smoothing with the aid of the

[1] Many of the clusters appear on two or more charts. The differences in quality from plate to plate, which may be greater for the charts, are not nearly so effective in measures of angular diameter as they would be in estimates of magnitudes. Fortunately about 90 per cent of the clusters are on the Johannesburg plates, which attain a fainter limit of magnitude and are more uniform than those made for the northern sky at Mervel Hill (*Memoirs of the Royal Astronomical Society*, **60**, 167, 1914).

[2] Half weight is given to N.G.C. 6642; also to N.G.C. 6712 because of a slight doubt as to its nature and because it is in such a rich region of the galactic clouds that the measured diameter is a little uncertain. Cf. *Publications of the Astronomical Society of the Pacific*, **29**, 186, 1917. Possibly a doubt should also be expressed as to the perfectly normal nature of N.G.C. 4147. Cf. the eleventh paper of this series.

parallax-diameter curve of Fig. 1. Accordingly, parallaxes corresponding to the measured diameters are entered in the seventh column of Table V and are combined with the parallaxes from magnitudes to obtain the smoothed values of the eighth column. The combination is made with due regard for the relative quality of the magnitude work, assigning for convenience weight unity

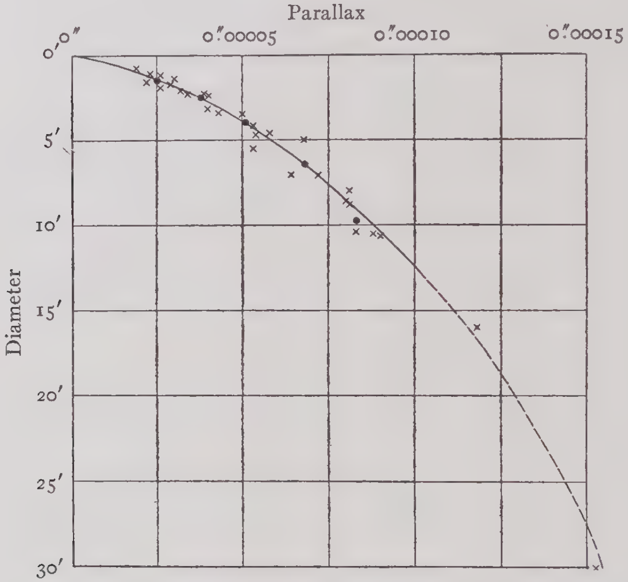

Fig. 1.—The parallax-diameter curve for globular clusters (diameters from Franklin-Adams charts). Dots are normals based upon parallaxes from magnitudes alone; crosses are the finally adopted values for the 29 clusters of Table V.

to all the parallaxes from magnitudes, and, to the corresponding values from angular measurements, the weight zero for group *a*, one-half for group *b*, one for group *c*, and two for group *d*. The adopted parallaxes are plotted in Fig. 1 as crosses.

Before obtaining the parallaxes of other clusters from their diameters alone, we shall note what accuracy may be expected in the results. In Table VII are given the percentage deviations from the parallax-diameter curve, both for the original parallaxes from magnitudes and for the adopted values. Without assigning weights, the arithmetical[1] mean for the first is 14 per cent, and for the second, 7 per cent. The latter shows the average amount of the discrepancy that will affect the parallaxes of the clusters for which no magnitudes are available. The deviations are partly due to real differences in the clusters, but most of the error is within the uncertainty of angular measurement, for the average difference between the estimates of diameter by the two observers is but slightly less than 10 per cent.

TABLE VI

DIAMETERS AND PARALLAXES

Number of Clusters	Mean Parallax	Mean Diameter
7	0″.000025	1′.5
5	0.000038	2.5
5	0.000051	4.0
5	0.000068	6.4
5	0.000083	9.7
1	0.000118	16
1	0.000150	30

TABLE VII

DEVIATIONS FROM THE PARALLAX-DIAMETER CURVE

N.G.C.	$\frac{\pi\text{ Mag.}-\pi\text{ Diam.}}{\pi\text{ Adopt.}}$	$\frac{\pi\text{ Adopt.}-\pi\text{ Diam.}}{\pi\text{ Adopt.}}$	N.G.C.	$\frac{\pi\text{ Mag.}-\pi\text{ Diam.}}{\pi\text{ Adopt.}}$	$\frac{\pi\text{ Adopt.}-\pi\text{ Diam.}}{\pi\text{ Adopt.}}$
288	+0.06	+0.02	6356	−42	−15
1904	+ 20	+ 8	6402	−12	− 7
4147	+ 37	+ 11	6626	− 7	− 6
5024	− 26	− 17	6638	+ 3	+ 3
5139	− 3	− 1	6656	+ 2	+ 2
5272	0	0	6712	− 9	− 3
5904	− 1	− 1	6779	+15	+ 8
6093	+ 10	+ 4	6864	−32	−18
6121	− 6	− 2	6934	+37	+20
6205	− 2	− 1	6981	− 6	− 3
6218	− 2	− 1	7078	+13	+13
6229	+ 39	+ 13	7089	+19	−12
6254	− 14	− 7	7099	+ 5	+ 3
6333	− 15	− 10			

Nearly all of the 41 clusters included in Table VIII are south of declination −30°. The few exceptions will be photographed with the 60-inch reflector, when opportunity permits, and the

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[1] For brevity the terms "algebraic mean" and "arithmetical mean" are used in the sense of *with* and *without* regard to sign.

TABLE VIII

PARALLAXES OF 41 GLOBULAR CLUSTERS FROM DIAMETERS

N.G.C.	R.A. 1900	DECL. 1900	GALACTIC		ANGULAR DIAMETER *D, S*	PARALLAX (UNIT IS 0″.000001)	DISTANCE (UNIT IS 100 PARSECS)		
			Long.	Lat.			Radial	Projected on Galactic Plane	From Galactic Plane
104	$0^h 19^m.6$	−72° 38′	272°	−44°	26′.0, 27′.6	148	68	49	− 47
362	0 58.9	−71 23	268	−46	5.4, 6.6	66	152	106	−109
1261	3 9.5	−55 36	238	−51	2.6, 2.5	39	256	161	−199
1851	5 10.8	−40 9	211	−34	4.8, 5.0	58	172	143	− 96
2298	6 45.4	−35 54	213	−15	3.0, 2.7	41	244	236	− 63
2808	9 10.0	−64 27	249	−11	5.2, 4.9	59	170	167	− 32
3201	10 13.5	−45 54	244	+10	6.6, 6.3	68	147	145	+ 26
4372	12 20.1	−72 7	269	− 9	10.8, 9.3	88	114	113	− 18
4590	12 34.2	−26 12	268	+37	4.8, 5.8	62	161	129	+ 97
4833	12 52.7	−70 20	271	− 8	5.6, 4.9	61	164	163	− 23
5286	13 40.1	−50 52	279	+11	3.6, 4.5	51	196	193	+ 37
5634	14 24.4	− 5 32	309	+49	2.2, 2.0	33	303	199	+229
5897	15 11.7	−20 39	311	+30	6.4, 6.2	67	149	129	+ 75
5986	15 39.5	−37 27	305	+13	3.3, 4.1	48	208	203	+ 47
6101	16 14.4	−71 58	284	−15	3.0, 3.7	47	213	206	− 55
6144	16 21.2	−25 49	319	+15	2.8, 2.8	41	244	236	+ 63
6171	16 26.9	−12 50	331	+22	5.8, 5.0	62	161	149	+ 60
6235	16 47.4	−22 1	326	+13	1.0, 1.2	20	500	487	+112
6266	16 54.8	−29 58	320	+ 7	6.4, 5.9	66	152	151	+ 19
6273	16 56.4	−26 7	324	+ 9	5.6, 5.4	63	159	157	+ 25
6284	16 58.4	−24 37	325	+10	1.4, 1.9	27	370	364	+ 64
6287	16 59.1	−22 34	327	+11	1.3, 1.4	23	435	428	+ 83
6293	17 4.0	−26 26	325	+ 8	2.9, 2.2	38	263	261	+ 37
6304	17 8.2	−29 20	323	+ 5	2.0, 1.9	31	322	320	+ 28
6316	17 10.3	−28 1	325	+ 5	1.0, 1.0	19	526	524	+ 46
6352	17 17.8	−48 19	308	− 7	3.4, 3.0	44	227	226	− 28
6362	17 21.5	−66 58	293	−17	7.3, 8.4	77	130	124	− 38

6388	17 29.0	−44 40	312	− 7	2.5, 2.2	36	278	276	− 34
6397	17 32.5	−53 37	304	−12	18.0, 16.2	120	83	81	− 17
6441	17 43.4	−37 1	321	− 5	1.3, 1.2	22	455	453	− 40
6541	18 0.8	−43 44	316	−11	6.8, 6.2	68	147	144	− 28
6584	18 10.6	−52 15	309	−16	2.6, 2.6	38	263	253	− 73
6624	18 17.3	−30 24	330	− 8	2.4, 2.2	35	286	283	− 40
6637	18 24.8	−32 25	329	−11	3.3, 3.6	47	213	209	− 41
6652	18 29.2	−33 4	328	−12	2.0, 2.0	32	312	305	− 65
6681	18 36.7	−32 23	329	−13	4.4, 4.6	55	182	177	− 41
6715	18 48.7	−30 36	333	−16	5.7, 5.2	62	161	155	− 44
6723	18 52.8	−36 46	327	−18	7.3, 9.2	79	127	121	− 39
6752	19 2.0	−60 8	303	−26	16.8, 14.4	114	88	79	− 39
6809	19 33.7	−31 10	335	−24	11.7, 13.2	100	100	91	− 41
7006	20 56.8	+15 48	32	−20	0.8, 0.7	15	667	626	−228

parallax, as determined from diameters, will receive an independent check from the study of magnitudes.[1] The right ascension, declination, and galactic co-ordinates in Table VIII, as in previous tables, have been taken, whenever possible, from Melotte's catalogue, the results being checked with Bailey's lists and other catalogues. The angular diameters in the fifth column were obtained at the same time as those measured for Table V; their mean value is the basis of the parallaxes in the sixth column. The first cluster of this list is 47 Tucanae.

Tables V and VIII contain all clusters now thought to be definitely globular—a total of 69. The 15 or 20 others[2] frequently

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[1] Since writing the above, one highly satisfactory confirmation has been secured. The object N.G.C. 7006 was noted as a small faint cluster by Curtis (*Lick Observatory Bulletins*, **7**, 84, 1912). On the Franklin-Adams plates and charts it appears as a nebulous star and was excluded from Melotte's catalogue of globular clusters. It is not mentioned by Bailey. The apparent diameter as given in Table VIII is only $0'.75$—the smallest cluster of the two lists. If our hypotheses are right, therefore, it should be the most distant cluster, and the mean magnitude should be fainter than any hitherto measured. Two polar-comparison photographs were secured in December 1917. A seven-minute exposure on a fast plate shows about two hundred stars brighter than magnitude 18.5, and one star near the center is nearly as bright as magnitude 15; but the mean of the 25 brightest, according to the preliminary measures, is 17.7. The corresponding parallax is a little more than $0''.000014$, differing by less than a millionth of a second from the value in Table VIII.

[2] Among the clusters thought by some to be globular are the following:

N.G.C.	Angular Diameter	Galactic		Remarks
		Long.	Lat.	
371........	$2'.1$	268°	−44°	In Small Mag. Cloud. Not typical, Bailey. If globular, it gives for the distance of the cloud $\pi = 0''.000033$. Cf. Table I.
2660........	1.5	234	− 2	Appears to be globular cluster, Melotte. If so, $R \sin \beta = -1400$ parsecs; $R \cos \beta = 39{,}700$ parsecs.
5466........	5.4	10	+72	Apparently an open cluster, Shapley. If globular, $\pi = 0''.00006$.
6496........	1.5	315	−11	Probably a nebula. Bailey does not mention it.
6535........	0.8	354	+10	A group of a few faint stars, Shapley.
6569........	1.3	328	− 7	A nebula according to Bailey. If a globular cluster, $\pi = 0''.00002$.
6760........	1.0	3	− 5	Small cluster of very faint stars, Curtis, Pease. An open cluster, Shapley.
7492........	3.5	21	−64	An open cluster, Curtis, Shapley. Very loose globular cluster, Melotte. If globular, $\pi = 0''.00005$.

Bibliography: Bailey, *Harvard Annals*, **60**, No. 8, 1908; **76**, No. 4, 1915. Curtis, *Lick Observatory Bulletins*, **7**, 81, 1912; **8**, 43, 1913. Melotte, *Memoirs of the Royal Astronomical Society*, **60**, Part V, 1915. Pease, *Publications of the Astronomical Society of the Pacific*, **26**, 204, 1914. Shapley, *Publications of the Astronomical Society of the Pacific*, **29**, 186, 1917. Shaw, *Helwan Observatory Bulletins*, No. 9, 1912; No. 15, 1915; *Monthly Notices*, **76**, 105, 1915.

admitted to the catalogues are rejected on the basis of either the Mount Wilson plates or the published opinions of Bailey, Curtis, Shaw, or Melotte. Two or three of these questioned groups may be admitted later, and other faint objects now considered nebulous stars or open clusters or faint nebulae may be proved by the large reflectors to be globular clusters. But as far as systems containing stars brighter than the sixteenth photographic magnitude are concerned, the present work may be considered exhaustive.

II. DISTRIBUTION IN SPACE

Some striking features of the arrangement of clusters in space are brought to light by a study of the data in Tables V and VIII. Fig. 2 illustrates the distribution in three dimensions, showing on the plane of the Milky Way the galactic longitudes and projected distances, while the distances from the Galaxy are shown by vectors drawn to scale in the plane of the figure. Distances above (north of) the galactic plane are represented by full heavy lines drawn upward from black circular bases, those below by broken lines downward from open circular bases. To visualize the actual positions in space one needs only to imagine the full-line vectors standing erect on their bases and the broken lines hanging vertically from theirs; the arrow points are then at the positions of the clusters.

The most remote of all the clusters[1] is N.G.C. 7006 with a distance of 67,000 parsecs, the equivalent of 220,000 light-years; the clusters N.G.C. 4147, 6229, 6235, 6287, 6441, and 6864 (M 75) are nearly as far away. Fortunately, of these seven most distant systems six are within reach of the Mount Wilson reflectors. One-fourth of all globular clusters appear to be more distant than 30,000 parsecs (100,000 light-years); and one (N.G.C. 4147) is more than 50,000 parsecs from the plane of the Milky Way. ω Centauri and 47 Tucanae, with distances somewhat less than 7000 parsecs, are the clusters nearest to the sun.

The concentration into a limited interval of galactic longitude is conspicuous; the region from 41° to 195° is completely void of globular clusters. The mean value of all longitudes is 316°, or, excluding the five largest and five smallest values, is 318°; but the mean is too much affected by widely divergent values, and the

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[1] See n. 1, p. 14, of this paper and the later discussion in the eleventh paper.

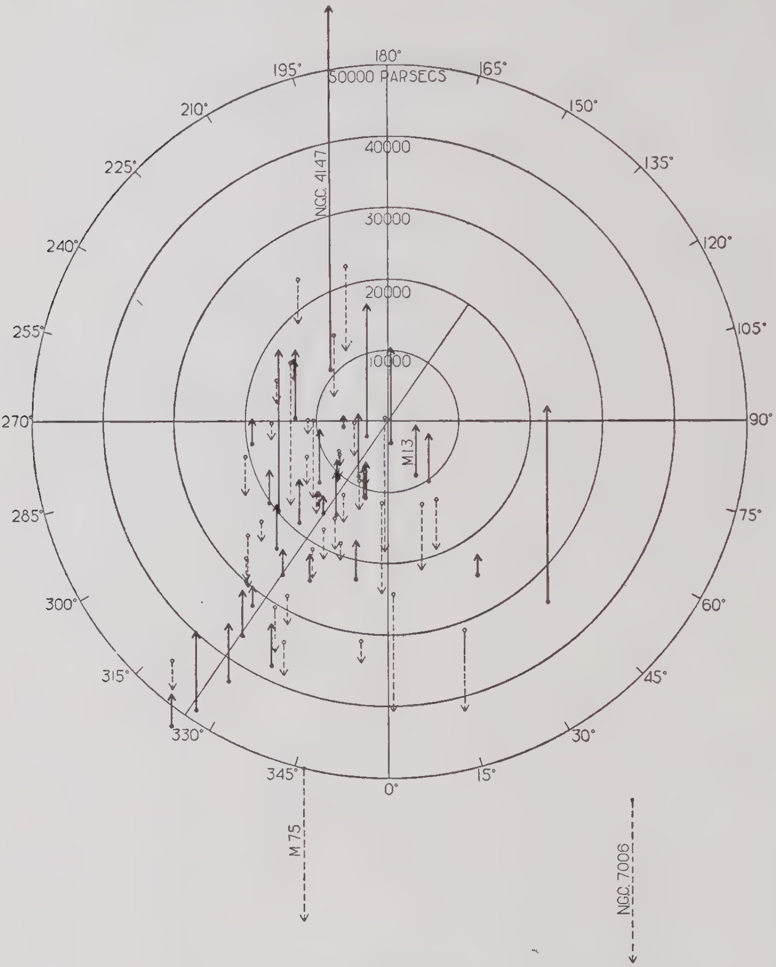

Fig. 2.—Distribution in space of globular clusters. The galactic plane is the plane of the diagram; distances above and below are shown to scale by full-line and broken-line vectors, respectively. Galactic longitudes are indicated in the margin and the scale of distances along the vertical radius. The sun is at the origin of co-ordinates. The diagram illustrates the remarkable distribution in longitude, with a maximum frequency at 325°, and by the absence of very small or zero vectors shows that globular clusters are not found within 1000 parsecs of the plane of the Milky Way.

median value,[1] 325°, is preferable in determining the central line of the system of globular clusters. The frequency of longitudes shown by Fig. 3 agrees in placing the center in longitude 325°, the points for the curve depending on the data of Table IX.

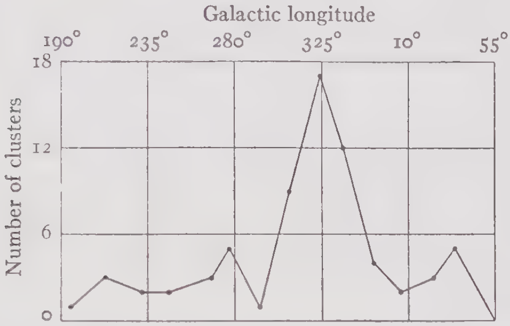

Fig. 3—Distribution of globular clusters in galactic longitude

TABLE IX

Distribution of Clusters in Galactic Longitude

Interval of Longitude	Number of Clusters	Mean Longitude
195° to 210°.....	1	195°
210 to 225	3	213
225 to 240	2	232
240 to 255	2	246
255 to 270	3	268
270 to 285	5	277
285 to 300	1	293
300 to 315	9	308
315 to 330	17	324
330 to 345	12	336
345 to 360	4	352
0 to 15	2	6
15 to 30	3	23
30 to 45	5	34

Projecting all positions on to a plane through the sun perpendicular to the Galaxy and including the circle defined by galactic longitude 325°, we get the diagram in Fig. 4, which represents the apparent distribution as seen from a great distance in the direction of galactic latitude 0° and galactic longitude 235°. Black dots above the central line represent clusters north of the galactic plane; open circles below represent those south. Thus the ordinates are $R \sin \beta$ and the abscissae $R \cos \beta \cos (\lambda - 325°)$, where R, β, and λ are respectively the distance, galactic latitude, and galactic longitude of a cluster.

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[1] That is, the longitude of the thirty-fifth cluster when they are taken in order of increasing longitude, beginning with N.G.C. 1904.

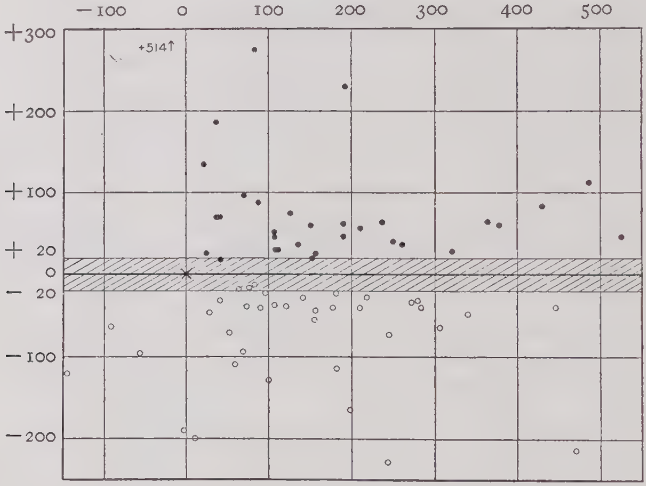

FIG. 4.—Projection of the positions of globular clusters on a plane perpendicular to the Galaxy, illustrating (1) the absence of clusters from the mid-galactic region, (2) their symmetrical arrangement with respect to the Galaxy, (3) the eccentric position of the sun (the cross) with respect to the center of the system of clusters. The ordinates are distances from the galactic plane, $R \sin \beta$; the abscissae are projected distances in the direction of the center, $R \cos \beta \cos (\lambda - 325°)$. The unit of distance is 100 parsecs; the side of a square is accordingly 10,000 parsecs. On this scale the actual diameter of the clusters is about one-fifth the diameter of the circles and dots. The cluster N.G.C. 4147 is outside the boundary of the diagram, as indicated by the arrow.

Fig. 5 differs from the preceding diagram only in having $R \cos \beta$ for abscissae. Hence the sun, as origin, is at the extreme left edge of the figure, and the actual distance of each cluster is represented by the radial distance from the origin.

Figs. 4 and 5 show more clearly than Fig. 2 the remarkable distribution of clusters with respect to the Galaxy. In the first place clusters are impartially distributed above and below, there being 32 north and 37 south of the plane. Although the average value without regard to sign is ±79 (in units of 100 parsecs), the algebraic mean of all distances from the plane is −1; rejection of the very distant cluster, N.G.C. 7006, would change this to +2. Considering the accidental variation and the size of the distances involved, the algebraic mean is vanishingly small, and we may say confidently that the plane of the Milky Way is also a symmetrical plane in the great system of globular clusters. This relation to the Galaxy holds with good approximation at all distances from the

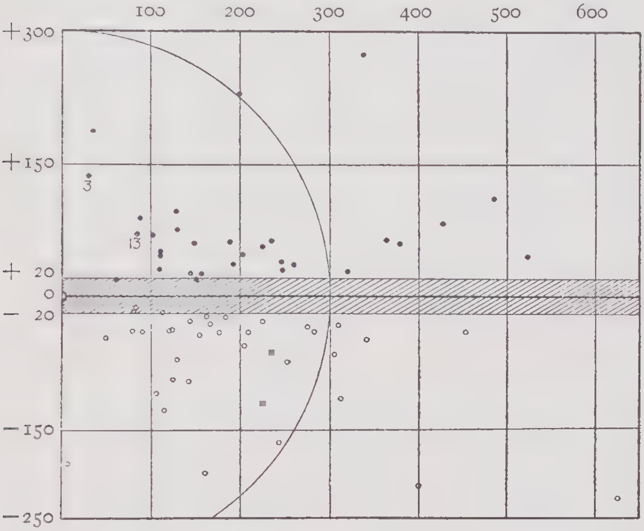

FIG. 5.—Distribution of globular clusters. The ordinates are $R \sin \beta$, as in Fig. 4; the abscissae are distances projected on the galactic plane, $R \cos \beta$. The unit of distance is 100 parsecs. The very small semicircle, with radius corresponding to a parallax of 0″.002, illustrates the region around the sun which contains all but a few of the stars in Charlier's B-type cluster. The large semicircle indicates the distance to which the present results are thought to be complete. Messier 3 and Messier 13 are indicated by numbers; the most distant cluster now known, N.G.C. 7006, is near the lower right-hand corner of the diagram. N.G.C. 4147, with co-ordinates 109 and 514, is not shown.

sun (graphically shown best by Fig. 5), as may be seen from the following tabulation:

	Interval of R cos β				
	0–100	100–200	200–300	>300	All
Number of Clusters	11	29	15	14	69
Mean* R sin β Algebraic	+ 7	+ 9	−23	− 5	− 1
Arithmetical	±70	±84	±59	±100	±79

*See n. 1, p. 11.

A second phenomenon clearly illustrated by the diagrams is the avoidance of the Milky Way—a result that may be of very exceptional significance. There is no cluster within 1300 parsecs of the plane of the Galaxy, and within 2000 parsecs of that plane there are only five, four of which are among the clusters nearest the sun. In Figs. 4 and 5 a shaded region, 13,000 light-years in width, indicates the zone from which globular clusters are practically excluded.

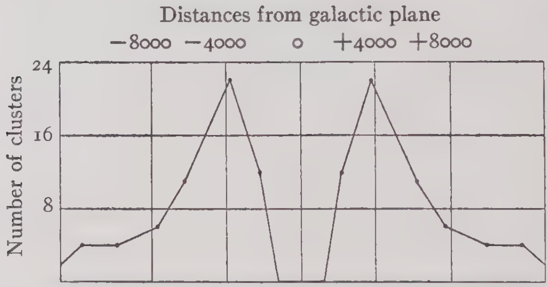

Fig. 6.—Reflected frequency-curve of the distances of globular clusters from the galactic plane, illustrating the equatorial region devoid of globular clusters. The unit of distance is one parsec.

The increasing concentration toward the Galaxy from both sides stops almost abruptly at the boundary of the shaded zone. The frequency of distances from the plane is treated in more detail in Table X, and the undoubted dependence of the clusters on the galactic plane, noted numerically above, is further emphasized by the curve in Fig. 6. The completion of that curve, in a form naturally to be expected for the frequency of objects concentrated toward the

TABLE X

The Frequency of Distances from the Galactic Plane ($R \sin \beta$)

Limits of $R \sin \beta$ (Unit Is 100 Parsecs)	$R \cos \beta < 200$					$R \cos \beta > 200$					All Clusters	
	No. of Clusters		Mean $R \sin \beta$			No. of Clusters		Mean $R \sin \beta$				
	North	South	North	South	All	North	South	North	South	All	Number	Mean $R \sin \beta$
<10........	0	0	....	....	....	0	0	....	....	....	0	....
10– 30.......	4	6	22	20	21	1	1	28	28	28	12	22
30– 50.......	3	8	37	40	39	5	6	40	39	39	22	39
50– 70.......	4	0	61	....	61	4	3	61	61	61	11	61
70– 90.......	3	1	78	71	76	1	1	83	73	78	6	77
90–110.......	1	3	97	100	99	0	0	....	....	....	4	99
110–130.......	0	1	....	128	128	1	2	112	117	119	4	118
130–150.......	1	0	135	....	135	0	0	....	....	....	1	135
150–170.......	0	0	....	....	....	0	1	....	164	164	1	164
170–190.......	1	1	186	189	188	0	0	....	....	....	2	188
190–210.......	0	1	....	199	199	0	0	....	....	....	1	199
210–230.......	1	0	229	....	229	0	2	....	220	220	3	223
>230........	1	0	514	....	514	1	0	274	....	274	2	394
Totals....	19	21	....	....	....	13	16	....	....	....	69	....

Galaxy, would require at least 50 globular clusters within 1500 parsecs of the plane; there is, however, only one, Messier 22, and its distance below the plane corresponds to a parallax of 0."0008. It should be observed, moreover, that not only at great distances, where insufficiency of observations may be urged, do we note this absence of clusters, but also within a distance from the sun of 20,000 parsecs, where the data are quite sufficient (37 clusters) and undoubtedly are complete. Hence we conclude that this great mid-galactic region, which is peculiarly rich in all types of stars, planetary nebulae, and open clusters, is unquestionably a region unoccupied by globular clusters.

To explain this remarkable condition several hypotheses have been considered, such as error in choosing the origin of galactic latitudes,[1] incompleteness of data, general absorption of light in space,[2] clouds or a ring of absorbing matter along the spine of the Milky Way analogous to the dark peripheral rings of spiral nebulae,[3] and finally the actual absence of globular clusters from the regions rich in stars because of the dynamical impossibility of existence. The first three seem clearly impossible, the fourth improbable or at least unquestionably insufficient, and, therefore, without going at present into the meaning and consequences of such a theory, the last hypothesis is tentatively adopted.

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[1] The adopted position of the north galactic pole is that given by Gould (*Uranometria Argentina*): $\alpha = 12^h 41^m$, $\delta = +27°21'$. This position differs by less than a degree from those obtained through other reliable and definitive investigations. The most recent, and probably the best, is based on the Harvard Map of the Sky, by Nort, who finds $\alpha = 12^h 44^m$, $\delta = +27°$ (*Recherches Astronomiques de l'Observatoire d'Utrecht*, VII, 84, 112, 1917). An error of ±1° in the cluster latitudes might slightly displace, widen, or narrow the zone of avoidance, but nothing short of selectively operative errors of several degrees could seriously obscure it. The frequency-curve of galactic latitudes also shows the zone, but naturally to a lesser degree. The values in Tables V and VIII show no latitude less than 5°. See the remarks relative to N.G.C. 2660 in n. 2, p. 14.

[2] See the notes in a following paper (the eleventh of the series) on the color of stars in the two most distant clusters. The dark obstructing nebulae which are frequently found in and near the Milky Way are undoubtedly capable of obliterating or greatly diminishing the light of any cluster involved in the nebulosity or beyond it. N.G.C. 4372, a large southern cluster (almost certainly globular), which is very faint for its angular diameter, falls alongside a vacant space in the sky. N.G.C. 6144 is near the edge of the ρ Ophiuchi dark nebulosity and appears large for the magnitude of its bright stars. This latter nebulosity may also affect the brightness of Messier 4 (N.G.C. 6121) to some extent. But it is interesting to note that in developing a parallax method that is independent of the magnitudes we have escaped from the errors in parallax that such obstructing material might occasionally have imposed.

[3] Possibly a hypothetical wedge-shaped ring might explain some of the divergence from the galactic plane with increasing distance (Figs. 4 and 5); but insufficiency of material for faint clusters would better account for most of it. The phenomenon, of course, may be real—a widening of the zone of avoidance in the direction of the

We have found that the center of the elongated and somewhat irregular system of globular clusters lies in the plane of the Milky Way on a line directed toward galactic longitude 325°. The distance along that line may be estimated from an inspection of Fig. 4, and perhaps obtained more accurately from the following consideration of the frequencies of $R \cos \beta \cos (\lambda - 325°)$, in which clusters more than 15,000 parsecs distant from the plane are excluded:

	INTERVAL OF DISTANCE IN DIRECTION OF CENTER								
	<100	−100 to 0	0 to +100	+100 to +200	+200 to +300	+300 to +400	+400 to +500	>500	All
Number of clusters	1	2	18	20	10	5	3	1	60
Mean of $R \cos \beta \cos (\lambda - 325°)$...	−145	−74	+59	+138	+246	+342	+454	+525	+158

From a plot of these numbers we estimate provisionally the distance of the center to be 13,000 parsecs. Incompleteness of data because of faintness will not materially affect the galactic longitude and latitude of this point, but is likely to make the distance too small. The mean value of $R \cos \beta \cos (\lambda - 325°)$ for the 60 clusters, +158, is probably nearer the true value, but it is also liable to understate the distance. A definitive value is hardly possible, and, at least until the number of very faint clusters can be considerably increased, we may adopt as the center of the general system of globular clusters a point for which the parallax is between $0''.00006$ and $0''.00004$, with equatorial co-ordinates $\alpha = 17^h 30^m$, $\delta = -30°$, and galactic co-ordinates $\lambda = 325°$, $\beta = 0°$. The position

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center of the globular cluster system, combined with a lack of observations for clusters far beyond the center. See n. 2, p. 14; if N.G.C. 2660 is globular, the apparent tendency to widen with distance is somewhat counteracted.

lies in the constellation Sagittarius, a few degrees east of its boundary with Scorpio and Ophiuchus.

III. LINEAR DIMENSIONS OF CLUSTERS

A literal interpretation of the curve in Fig. 1 gives for the dimensions of globular clusters:

	Parallax (Unit Is 0″.000001)						
	20	40	60	80	100	120	150
Angular diameter	1′.1	2′.7	5′.2	8′.3	12′.4	17′.2	27′.5
Linear diameter in parsecs.....	16	20	25	30	36	42	53

As remarked before, however, the diameters from the charts do not give a true measure of the clusters, and the hypothesis that size depends on distance from the sun, irrespective of distance from the Galaxy, is quite untenable.

There are, without doubt, real though relatively inconspicuous differences among globular clusters; and some of the differences, such as frequency of certain spectral types, degree of condensation and ellipticity, and possibly total numbers of stars, are being recognized and evaluated through the Mount Wilson studies. Two properties, however, so far seem to show little variation from cluster to cluster—the actual linear diameter and the mean magnitude of the brightest stars. We shall assume, therefore, on the basis of what observational evidence we now have at hand, that all globular clusters are of practically the same dimensions, explaining the apparent decrease in size with increasing distance (tabulated above) as a natural consequence of a central concentration of luminosity and of an intermingling near the edges with non-cluster stars. On that basis an investigation of the dimensions of one cluster will suffice for all.

Long exposures with the 60-inch reflector upon the outer parts of some of the brightest clusters have confirmed the conclusion, obtained from a study of the distribution of variable stars, that the clusters are much greater in extent than would be inferred from ordinary visual or photographic observation. For instance, on the

Franklin-Adams charts the apparent diameter[1] of Messier 3, the cluster chosen for the present illustration of dimensions, is 7′; on the original plates it is 18′, according to Melotte; but the actual diameter is in excess of half a degree.

The distance of Messier 3 is 13,900 parsecs,[2] corresponding to nearly three thousand million times the distance of the sun from the earth. The distance north of the galactic plane is 13,500 parsecs. The accompanying plate is reproduced from a photograph of several hours' exposure made by Mr. Ritchey with the 60-inch reflector. The original negative shows more than twenty thousand stars outside the central burned-out area, the smallest images being fainter than the twentieth magnitude.

The cluster extends beyond the limits of the photograph in all directions. The most distant variable star (undoubtedly a member of the system) is 17′ from the center, corresponding to a projected distance of fourteen million astronomical units. As ordinarily seen and photographed, the cluster covers an area but little larger than one of the squares, but we may be sure that its actual projected area is at least twenty-five times as great; that is, the diameter is about thirty million astronomical units. To cross the cluster, light must travel 470 years.

If we suppose the sun situated at the center of the cluster, all stars with parallaxes greater than 0″1 would be included within the concentric circle. Sirius would be at the distance indicated by the cross, and the projected distance of the bright triplet near the bottom of the picture equals the distance of the Hyades from the sun.

Inclosed in small circles are a few of the variable stars, chosen at random, the close equality of whose magnitudes is to be noted. In some cases they appear as doubles, but the actual separation in the closest of the indicated pairs is more than half the distance separating the sun from α Centauri. In the small square is the image of an exceptional variable, No. 37, for which the period is less than eight hours.

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[1] The estimates from the charts, listed in Tables V and VIII, refer actually to what appears to be a central core of each system. The scale of the photographs does not permit close differentiation of the outlining members of a cluster from the stars of its surrounding field.

[2] The diagram was made on the basis of a parallax of 0″000074; the final value of Tables I and V indicates that the linear dimensions of the cluster on the plate are too small by 3 per cent, an amount, however, that is far within the probable error.

The cluster variables, in the mean, have the absolute magnitude −0.2, photographically nearly six magnitudes brighter than the sun. A star of the brightness and color of the sun would not appear on the photograph, being nearly two magnitudes too faint (21.5). Sirius, located in this cluster, would be of the seventeenth apparent magnitude, corresponding to the star indicated on the photograph by an arrow-point.

The condensation of stars at the center of the cluster may be readily contrasted with that of stars around the sun. Within the circle, which marks a distance from the center corresponding to a parallax of 0.″1 (approximately two million astronomical units), there are at least 15,000 stars brighter than magnitude 20. (This estimate deducts those stars not within the concentric sphere but appearing by projection within the circular area.) In a sphere of the same radius, with the sun as center, less than twenty stars brighter than the sun are known. But only those which are two magnitudes brighter appear on this photograph of the cluster; there are, accordingly, in the sphere around the sun only four or five stars to compare with the 15,000 in Messier 3.

Finally, we shall make some estimates relative to the probable mass of a globular cluster. We may go astray, to be sure, in assuming similar masses and analogous relations of mass to luminosity for stars in clusters and in the general galactic system. The dynamical conditions in these highly condensed globular systems may conceivably have some important effect upon the amount of matter that goes into a single star as well as upon the speed and nature of subsequent development. Eddington's recent theoretical work on the masses of stars, however, indicates that as long as we deal with typical giants the masses are definitely limited; and, further, the identity of Cepheid phenomena wherever studied—in the galactic system, in the more condensed Magellanic clouds, and in the extremely condensed globular clusters—tends to support the view of the universal comparability of stellar masses.

All that we know of the masses of stars from observation has been summarized recently by Russell.[1] The data come altogether from double stars, but we shall probably not commit serious error in applying the results without alteration to the single stars in the condensed globular clusters, although possibly slightly greater average values for each type would be appropriate for the isolated stars in our general galactic system. For the present approximation we shall simply take a mean value from Russell's data, say an average of four times the sun's mass for every star brighter than the sun. The number of such stars in Messier 3 may be fairly estimated at 40,000, only three-fourths of which have ever been photographed. Hence the entire mass, distributed among the stars which are brighter absolutely than photographic magnitude +5.6, is 160,000 times the solar mass; and something like three-fourths of this amount is within 10 parsecs of the center.

It seems quite futile at present to estimate the total mass of the cluster. Perhaps it is two or three million times the solar mass; probably it is several times the amount estimated for the stars brighter than 5.6—that we must admit on the basis of what little we now know of the relative frequency of dwarfs and giants in the vicinity of the sun; but as a matter of absolute certainty we do not know that there is a single star fainter than the sun, and even our estimate of 160,000 may be 50 per cent too great.

SUMMARY

1. Following the methods outlined in the preceding paper the parallaxes and positions in space are obtained for 69 globular clusters—all that can now be definitely assigned to the globular class. The distances range from 6500 to 67,000 parsecs, the brightest stars in the most distant clusters being fainter than the seventeenth photographic magnitude. The average probable error of a parallax is of the order of 20 per cent, varying considerably with the method used and with the quality of the observational work. Something more than 15,000 measures of magnitudes were made for one phase of the work. Section I contains various

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[1] *Popular Astronomy*, 25, 666, 1917.

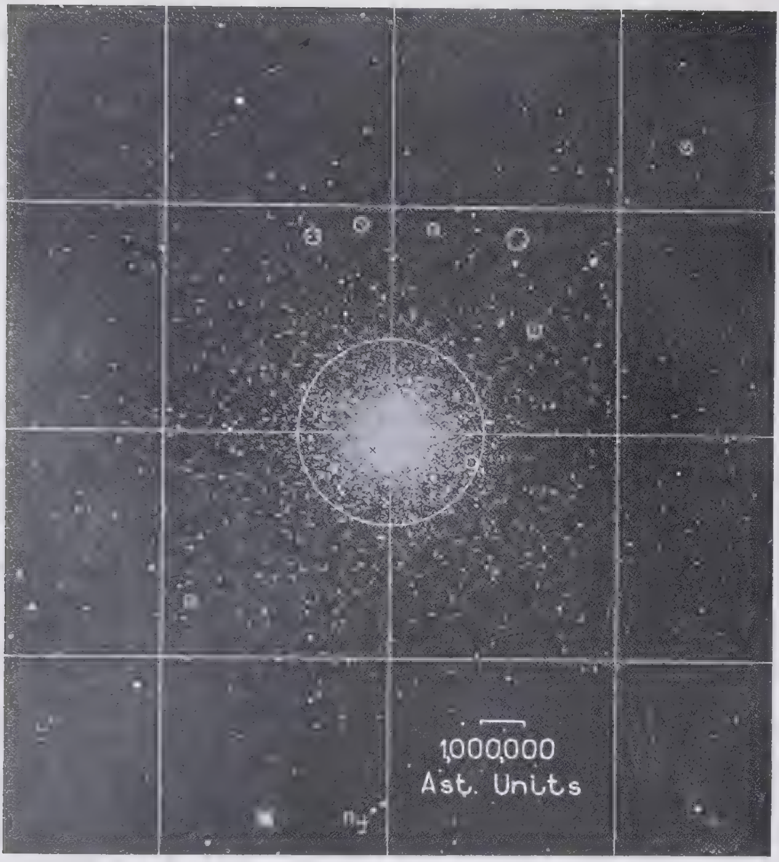
items relative to the method of investigation, comparative accuracy, maximum luminosities, and the frequency of giants.

2. The study of the distribution of clusters in space brings out a number of remarkable features (cf. sec. II), the most significant of which appear to be the absence from the denser stellar regions of globular clusters and the final proof that they are subordinate to the general galactic system. The center of the system of globular clusters is found, with some uncertainty in one co-ordinate. A discussion of the part played in a general theory by the distribution of clusters is postponed to a following paper.

3. The derivation of parallaxes has permitted the discussion of the actual dimensions of clusters and a comparison with familiar distances near the sun. Plate IV sufficiently summarizes the result for a typical system, Messier 3, whose distance from the earth is of the order of 250,000 million million miles. The total mass of a typical globular cluster is estimated to be from a quarter to a half of a million times the solar mass, with much uncertainty as to the upper limit.

MOUNT WILSON SOLAR OBSERVATORY
December 1917

PLATE IV



The Globular Cluster Messier 3 (N.G.C. 5272)

The side of a large square is 5,000,000 times the distance of the earth from the sun. The radius of the concentric circle corresponds to a parallax of $0''.1$ (2,062,650 astronomical units). To cross the circle light must travel for sixty-five years. Small circles contain typical variables; the small square, variable No. 37. If the sun were situated at the center, the Hyades would be at the distance of the triplet at the bottom of the picture; Sirius would be at the distance of the black cross near the center. A star of the luminosity of Sirius is indicated by the arrow point. Stars of our sun's brightness are nearly two magnitudes too faint to appear on the photograph.

Contributions from the Mount Wilson Solar Observatory, No. 153
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STUDIES BASED ON THE COLORS AND MAGNITUDES IN STELLAR CLUSTERS

EIGHTH PAPER: THE LUMINOSITIES AND DISTANCES OF 139 CEPHEID VARIABLES

By HARLOW SHAPLEY

Aside from the error involved in the determination of the zero-point of the luminosity-period curve,[1] most of the error affecting the absolute parallax of an isolated Cepheid variable is due to the uncertainty of apparent magnitude, since the period of light-variation is usually known with an accuracy that for this work is superfluous. Scores of observers, employing numerous methods and systems of magnitudes, have participated in the discovery and observation of these stars; but observations have often been made solely for the determination of periods, little attention being given to light-curves or accurate values of maximum and minimum light. Frequently the magnitudes are referred to the system of the *Bonner Durchmusterung*, and only occasionally, for the brighter or best known stars, to the Potsdam or Harvard photometric scales.

Adding to the uncertainty in the systems of magnitude the occasional difficulty, arising from changes in the form of light-curves, in obtaining accurate mean values of maximum and minimum brightness, we must expect an average probable error in the adopted median magnitude of the order of 0.4 mag. The consequent average probable error computed for the parallaxes of stars that are normal Cepheids is about 20 per cent of the tabulated values; for a number of the brighter stars and for well-observed cluster-type variables the error is only a little more than half that amount, but possibly it attains a maximum of 50 per cent for some of the faintest stars that are scantily observed on uncertain magnitude systems. Accurate determinations of the apparent visual or photovisual magnitude should reduce the probable error of the parallax of any Cepheid to less than 15 per cent.

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[1] *Mt. Wilson Contr.*, No. 151, sec. III.

The estimated range of 10 to 50 per cent in the probable error of the parallaxes allows for a variation in the probable error of apparent median brightness from a tenth to a whole magnitude—an amount which is shown by an examination of the underlying observational material to be sufficient. We note, therefore, that for parallaxes obtained with the period-luminosity curve the accuracy appears to surpass that of direct measures on any object for which the parallax is less than 0″01, and is essentially independent of distance. About two-thirds of the Cepheids now known have parallaxes smaller than 0″001.

The greatest chance for serious error in the work lies in the unintentional inclusion of some stars that are not typical Cepheids. For a star showing periodic continuous variation which simulates certain Cepheid characteristics but exhibits peculiarities such as double maxima or minima, we have found from cluster studies that the absolute brightness usually is less than for typical variables having the same period. The mean periods of such peculiar stars are often long; and, conversely, Cepheids of very long period are frequently abnormal. Until the normality of period and light-curve is established, we must, therefore, look with some doubt upon the enormously great absolute luminosity (and distance) obtained for galactic variables with periods in excess of forty days. Accordingly such stars are relegated to a supplementary table, which also contains provisional parallaxes and luminosities of variables that for various other reasons seem uncertain.

The list of variables in Table I is essentially complete for typical stars with definitely determined periods less than forty days. The various Harvard compilations and Hartwig's annual catalogue are the principal sources of observational data.[1] Names and positions have been taken, whenever possible, from Table VIII of *Harvard Annals*, **56**. For a majority of the stars the median visual magnitude of the eighth column is the mean of the maximum and minimum magnitudes given in this Harvard list, although for a number of variables improved magnitudes are obtainable from recent literature. All photographic magnitudes have been reduced to the visual system with the aid of a mean color-curve. The periods are mainly from Hartwig's catalogues, but occasionally recent publications afford better values. The parallaxes have been computed from the absolute magnitudes, which were read directly from the luminosity-period curve. A representation of the curve appears in Fig. 1 of *Contribution* No. 151. For the sake of uniformity the first decimal place is retained for all the distances in the last three columns, though for the larger values it is generally meaningless.

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[1] *Note added to proof, April, 1918.*—Hartwig's catalogue and ephemeris of variable stars for 1915 was the last number of that annual publication available when the tables for this paper were compiled. The issue for 1918, which has now been received, contains not only numerous revisions of the periods of the older variables but also some additions to the list. The corrections demanded by the revised periods have been applied to the values in all the tables, the text has been modified where necessary, and the statistical results in Tables II–VI now include the data for the twelve additional stars in Table I*a*.

The 35 stars in Table II include: (1) all with periods greater than forty days, some of which, RS Puppis, for instance, seem to show typical Cepheid variation, although others, with M-type spectra, may be classed more correctly with the long-period variables; (2) a few with period or type somewhat uncertain; (3) 7 stars that appear to belong to the RV Tauri type of variation,[1] and (4) a number known to be otherwise peculiar or irregular. Further observation is sure to place some of these stars with those of Table I. While it is very probable that the absolute brightness of all of them is high, in many cases the luminosity-period relation may not hold rigorously. Moreover, the high galactic latitude of some variables with periods in excess of forty days suggests, as strongly as the frequent peculiarities of period and amplitude, that these stars differ too greatly from the typical Cepheid to make the estimated distances of much value.

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[1] The periods of RV Tauri, R Sagittae, and V Vulpeculae are taken as thirty-nine, thirty-five, and thirty-eight days, respectively; these values, representing approximately the cycle of the principal variations, are more likely to give correct absolute luminosities. Cf. van der Bilt, *Recherches Astronomiques de l'Observatoire d'Utrecht*, VI, 1916. On the authority of Enebo similar treatment is accorded TV Andromedae, RY Lacertae, SW Persei, and RX Ursae Majoris. L_2 Puppis is a bright southern variable that may belong to this interesting type; a critical examination of its spectrum is very desirable. The study of the secondary variations of such typical stars as RR Lyrae (*Astrophysical Journal*, **43**, 217, 1916) shows that the difference between the RV Tauri type and the ordinary Cepheids is not so great as appears superficially.

TABLE I

PARALLAXES AND DISTRIBUTION IN SPACE OF 127 CEPHEID VARIABLES*

No.	NAME	POSITION IN 1900		GALACTIC		PERIOD IN DAYS	MED. MAG. VISUAL	ABS. MAG.	PARALLAX	DISTANCE (UNIT IS 100 PARSECS)		
		R.A.	Decl.	Long.	Lat.					Radial	Projected	From Plane
1...	SY Cassiopeiae	$0^{\mathrm{h}}\ 9^{\mathrm{m}}.8$	+57°52′	85°	− 4°	4.071	9.6	−1.8	0″.00052	19.2	19.2	− 1.3
2...	SW Andromedae	0 18.4	+28 51	85	−33	0.442	9.3	−0.3	0.0012	8.3	7.0	− 4.5
3...	TU Cassiopeiae	0 20.9	+50 44	88	−11	2.137	7.9	−1.2	0.0015	6.6	6.5	− 1.3
4...	*a* Ursae Minoris	1 25.4	+88 46	91	+26	3.968	2.12	−1.81	0.016	0.6	0.5	+ 0.3
5...	RR Ceti	1 27.0	+ 0 50	114	−59	0.553	8.6	−0.4	0.0016	6.3	3.2	− 5.4
6...	RW Cassiopeiae	1 30.7	+57 15	97	− 4	14.80	9.5	−3.8	0.00022	45.4	45.3	− 3.2
7...	U Trianguli	1 49.7	+33 17	106	−27	0.447	11.6	−0.3	0.00042	23.8	21.2	−10.8
8...	UX Persei	2 6.1	+57 37	102	− 4	4.6	10.6	−2.0	0.00030	33.3	33.2	− 2.3
9...	UY Persei	2 27.1	+58 26	104	− 2	5.5	9.6	−2.2	0.00044	22.7	22.7	− 0.8
10...	SU Cassiopeiae	2 43.0	+68 28	101	+ 9	1.950	5.9	−1.2	0.0038	2.6	2.6	+ 0.4
11...	TU Persei	3 1.8	+52 48	111	− 5	0.607	11.7	−0.4	0.00038	26.3	26.2	− 2.3
12...	RW Camelopardalis	3 46.2	+58 22	112	+ 5	16.402	8.8	−3.9	0.00029	34.5	34.4	+ 3.0
13...	SX Persei	4 10.2	+41 29	126	− 5	4.290	10.8	−1.9	0.00029	34.5	34.4	− 3.0
14...	SZ Tauri	4 31.4	+18 20	147	−17	3.149	7.1	−1.6	0.0018	5.5	5.2	− 1.6
15...	SV Persei	4 42.8	+42 7	130	0	11.128	8.9	−3.3	0.00036	27.8	27.8	0
16...	RX Eridani	4 45.2	−15 54	182	−33	0.587	9.2	−0.4	0.0012	8.2	6.9	− 4.5
17...	SU Aurigae	4 49.6	+30 24	140	− 7	0.470	8.7	−0.3	0.0016	6.3	6.2	− 0.8
18...	U Leporis	4 52.0	−21 23	188	−33	0.581	9.5	−0.4	0.0010	9.5	8.0	− 5.2
19...	RX Aurigae	4 54.5	+39 49	134	0	11.626	7.6	−3.4	0.00063	15.9	15.9	0
20...	SY Aurigae	5 5.5	+42 43	133	+ 4	10.138	9.2	−3.2	0.00033	30.3	30.2	+ 2.1
21...	Y Aurigae	5 21.5	+42 21	134	+ 6	3.859	9.6	−1.8	0.00052	19.2	19.1	+ 2.0
22...	RZ Geminorum	5 56.6	+22 15	155	+ 1	5.529	9.4	−2.2	0.00048	20.8	20.8	+ 0.4
23...	RS Orionis	6 16.5	+14 44	164	+ 2	7.566	8.6	−2.7	0.00055	18.2	18.2	+ 0.6
24...	T Monocerotis	6 19.8	+ 7 8	171	− 1	27.012	6.2	−4.7	0.00066	15.2	15.2	− 0.3
25...	RT Aurigae	6 22.1	+30 34	151	+11	3.728	5.3	−1.8	0.0038	2.6	2.6	+ 0.5
26...	RZ Camelopardalis	6 23.7	+67 6	115	+23	0.480	12.0	−0.3	0.00035	28.6	26.3	+11.2
27...	W Geminorum	6 29.2	+15 24	165	+ 5	7.916	7.1	−2.8	0.0010	9.5	9.5	+ 0.8
28...	ζ Geminorum	6 58.2	+20 43	164	+13	10.154	4.0	−3.2	0.0036	2.8	2.7	+ 0.6
29...	TZ Aurigae	7 4.6	+40 56	143	+23	0.392	11.8	−0.3	0.00038	26.3	24.2	+10.3
30...	RR Geminorum	7 15.1	+31 4	155	+21	0.397	10.2	−0.3	0.00079	12.7	11.9	+ 4.5

31...	X Puppis	7 28.4	−20 42	204	0	25.953	8.5	−4.6	0.00024	41.7	41.7	0
32...	V Carinae	8 26.7	−59 47	242	−12	6.695	7.8	−2.5	0.00087	11.5	11.2	− 2.4
33...	T Velorum	8 34.4	−47 1	233	− 3	4.639	8.0	−2.0	0.0010	10.0	10.0	− 0.5
34...	V Velorum	9 19.2	−55 32	244	− 4	4.371	7.8	−1.9	0.0012	8.7	8.7	− 0.6
35...	*l* Carinae	9 42.5	−62 3	251	− 7	35.523	4.3	−5.1	0.0013	7.6	7.5	− 0.9
36...	RR Leonis	10 2.1	+24 29	176	+54	0.452	9.6	−0.3	0.0010	9.5	5.6	+ 7.7
37...	UW Carinae	10 23.3	−59 10	254	− 1	5.346	9.0	−2.2	0.00058	17.2	17.2	− 0.3
38...	YZ Carinae	10 24.6	−58 51	253	− 1	18.158	8.6	−4.1	0.00029	34.5	34.5	− 0.6
39...	UX Carinae	10 25.4	−57 6	252	+ 1	3.682	8.2	−1.7	0.0010	9.5	9.5	+ 0.2
40...	UY Carinae	10 28.5	−61 16	255	− 3	5.544	8.8	−2.2	0.00063	15.9	15.9	− 0.8
41...	Y Carinae	10 29.4	−57 59	253	0	3.640	8.4	−1.7	0.00096	10.4	10.4	0
42...	UZ Carinae	10 32.6	−60 30	255	− 2	5.205	9.3	−2.1	0.00052	19.2	19.2	− 0.7
43...	SV Velorum	10 40.9	−55 46	253	+ 3	14.097	8.7	−3.7	0.00033	30.3	30.3	+ 1.6
44...	VY Carinae	10 40.9	−57 2	254	+ 2	18.984	7.3	−4.1	0.00052	19.2	19.2	+ 0.7
45...	WW Carinae	10 47.6	−58 51	256	0	4.676	9.6	−2.0	0.00048	20.8	20.8	0
46...	WZ Carinae	10 51.4	−60 24	257	0	23.00	7.8	−4.4	0.00036	27.8	27.8	0
47...	XX Carinae	10 53.4	−64 36	258	− 4	15.725	8.2	−3.8	0.00040	25.0	25.0	− 1.8
48...	U Carinae	10 53.7	−59 12	256	0	38.740	7.4	−5.3	0.00029	34.5	34.5	0
49...	XY Carinae	10 58.3	−63 43	258	− 3	12.434	8.9	−3.5	0.00033	30.3	30.3	− 1.6
50...	XZ Carinae	11 0.1	−60 26	258	0	16.644	8.4	−3.9	0.00035	28.6	28.6	0
51...	ST Centauri	11 5.5	−51 57	256	+ 8	3.151	10.0	−1.6	0.00048	20.8	20.6	+ 2.9
52...	SU Draconis	11 32.2	+67 53	100	+49	0.660	9.2	−0.4	0.0012	8.3	5.4	+ 6.3
53...	UZ Centauri	11 36.3	−62 8	262	0	3.334	9.2	−1.6	0.00069	14.5	14.5	0
54...	S Muscae	12 7.4	−69 36	266	− 7	9.657	6.8	−3.1	0.0010	9.5	9.4	− 1.2
55...	SW Draconis	12 12.8	+70 4	94	+48	0.570	10.0	−0.4	0.00083	12.0	8.0	+ 8.9
56...	SX Centauri	12 15.9	−48 39	266	+14	16.50	8.2	−3.9	0.00038	26.3	25.5	+ 6.4
57...	T Crucis	12 15.9	−61 44	267	+ 1	6.732	7.2	−2.5	0.0012	8.7	8.7	+ 0.2
58...	R Crucis	12 18.1	−61 4	267	+ 1	5.825	7.3	−2.3	0.0012	8.3	8.3	+ 0.1
59...	R Muscae	12 36.0	−68 52	270	− 7	0.882	7.0	−0.6	0.0030	3.3	3.3	− 0.4
60...	Z Canum Venat.	12 45.0	+44 19	88	+73	1.890	10.2	−1.1	0.00055	18.2	5.3	+17.4
61...	SS Hydrae	13 25.0	−23 8	284	+38	8.20	7.8	−2.8	0.00076	13.2	10.4	+ 8.1
62...	VW Centauri	13 27.1	−63 33	275	− 2	15.037	8.9	−3.8	0.00029	34.5	34.5	− 1.2
63...	RV Ursae Majoris	13 29.4	+54 31	74	+62	0.468	9.8	−0.3	0.00096	10.4	4.9	+ 9.2
64...	ST Virginis	14 22.4	− 0 26	315	+53	0.411	10.8	−0.3	0.00060	16.7	10.0	+13.3
65...	V Centauri	14 25.4	−56 27	284	+ 3	5.494	7.1	−2.2	0.0014	7.2	7.2	+ 0.4
66...	RS Boötis	14 29.2	+32 11	18	+66	0.377	10.3	−0.3	0.00076	13.2	5.3	+12.0

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* Table I*a*, which is appended to this table, contains twelve additional Cepheids. Cf. n. 1, p. 2.

TABLE I—*Continued*

No.	Name	Position in 1900		Galactic		Period in Days	Med. Mag. Visual	Abs. Mag.	Parallax	Distance (Unit Is 100 Parsecs)		
		R.A.	Decl.	Long.	Lat.					Radial	Projected	From Plane
67...	RU Boötis	$14^h41^m.5$	+23°44′	358°	+63°	0.493	13.5	−0.3	0".00017	58.8	26.7	+52.4
68...	RY Boötis	14 45.2	+23 27	356	+62	9.0	7.2	−3.0	0.00091	11.0	5.2	+ 9.7
69...	R Trianguli Aust.	15 10.8	−66 8	285	− 9	3.389	7.0	−1.7	0.0018	5.5	5.4	− 0.9
70...	U Normae	15 34.6	−54 59	293	− 1	12.641	8.9	−3.5	0.00033	30.3	30.3	− 0.5
71...	S Trianguli Aust.	15 52.2	−63 30	290	− 9	6.323	6.9	−2.4	0.0014	7.2	7.1	− 1.1
72...	U Trianguli Aust.	15 58.4	−62 38	291	− 9	2.568	8.1	−1.4	0.0013	7.9	7.8	− 1.2
73...	RW Draconis	16 33.8	+58 1	54	+40	0.443	10.4	−0.3	0.00072	13.9	10.6	+ 8.9
74...	RV Scorpii	16 51.8	−33 27	319	+ 4	6.062	7.1	−2.4	0.0013	7.9	7.9	+ 0.6
75...	SW Herculis	16 54.2	+21 42	9	+33	0.493	13.5	−0.3	0.00017	58.8	49.3	+32.0
76...	ST Ophiuchi	17 28.8	− 1 0	350	+15	0.450	11.6	−0.3	0.00042	23.8	23.0	+ 6.2
77...	X Sagittarii	17 41.3	−27 48	329	− 1	7.012	4.7	−2.6	0.0035	2.9	2.9	− 0.0
78...	RY Scorpii	17 44.3	−33 40	324	− 4	20.32	8.2	−4.2	0.00033	30.3	30.2	− 2.1
79...	Y Ophiuchi	17 47.3	− 6 7	347	+ 8	17.121	6.3	−4.0	0.00087	11.5	11.4	+ 1.6
80...	S Arae	17 51.4	−49 25	311	−13	0.452	9.3	−0.3	0.0012	8.3	8.1	− 1.9
81...	W Sagittarii	17 58.6	−29 35	330	− 5	7.595	4.7	−2.7	0.0033	3.0	3.0	− 0.3
82...	W Serpentis	18 4.1	−15 34	342	+ 2	14.153	9.0	−3.7	0.00029	34.5	34.5	+ 1.2
83...	WZ Sagittarii	18 11.1	−19 6	340	− 2	21.7	8.4	−4.4	0.00028	35.7	35.7	− 1.2
84...	Y Sagittarii	18 15.5	−18 54	340	− 4	5.773	5.8	−2.3	0.0024	4.2	4.2	− 0.3
85...	XX Sagittarii	18 19.0	−16 51	343	− 4	6.43	9.0	−2.7	0.00046	21.7	21.7	− 1.5
86...	U Sagittarii	18 26.0	−19 12	341	− 6	6.745	6.9	−2.5	0.0013	7.6	7.6	− 0.8
87...	Y Scuti	18 32.6	− 8 27	351	− 3	10.347	9.0	−3.2	0.00036	27.8	27.8	− 1.4
88...	Y Lyrae	18 34.2	+43 52	40	+20	0.503	11.8	−0.3	0.00038	26.3	24.7	+ 9.0
89...	RU Scuti	18 36.7	− 4 13	356	− 1	19.7	8.9	−4.2	0.00024	41.7	41.7	− 0.7
90...	RZ Lyrae	18 39.9	+32 41	29	+15	0.511	10.4	−0.3	0.00072	13.9	13.4	+ 3.6
91...	YZ Sagittarii	18 43.7	−16 50	346	− 8	9.553	7.4	−3.1	0.00079	12.7	12.6	− 1.8
92...	RT Scuti	18 44.0	−10 30	351	− 5	0.496	9.4	−0.3	0.0012	8.7	8.7	− 0.8
93...	SZ Aquilae	18 59.6	+ 1 9	3	− 3	17.136	8.7	−4.0	0.00029	34.5	34.5	− 1.8
94...	TT Aquilae	19 3.2	+ 1 9	4	− 5	13.753	7.6	−3.6	0.00058	17.2	17.1	− 1.5
95...	RR Lyrae	19 22.3	+42 36	43	+12	0.567	7.25	−0.35	0.0030	3.3	3.2	+ 0.7
96...	U Aquilae	19 24.0	− 7 15	359	−13	7.024	6.6	−2.6	0.0014	6.9	6.7	− 1.6
97...	XZ Cygni	19 30.4	+56 10	56	+16	0.467	9.7	−0.3	0.0010	10.0	9.6	+ 2.8

98...	U Vulpeculae	19 32.3	+20 7	24	− 2	7.990	7.0	−2.8	0.0011	9.1	9.1	− 0.3
99...	SU Cygni	19 40.8	+29 1	32	+ 2	3.846	6.6	−1.8	0.0021	4.8	4.8	+ 0.2
100...	η Aquilae	19 47.4	+ 0 45	8	−14	7.176	4.05	−2.62	0.0046	2.2	2.1	− 0.5
101...	S Sagittae	19 51.5	+16 22	23	− 7	8.382	5.8	−2.9	0.0018	5.5	5.5	− 0.7
102...	X Vulpeculae	19 53.3	+26 17	32	− 3	6.319	8.8	−2.4	0.00058	17.2	17.2	− 0.9
103...	XX Cygni	20 1.3	+58 40	60	+13	0.135	11.8	−0.4	0.00036	27.8	27.0	+ 6.2
104...	RW Aquilae	20 7.3	+15 46	24	−11	7.87	8.8	−2.8	0.00048	20.8	20.4	− 4.0
105...	SZ Cygni	20 29.6	+46 16	52	+ 3	15.113	9.2	−3.8	0.00025	40.0	40.0	+ 2.1
106...	X Cygni	20 39.5	+35 14	44	− 5	16.385	6.5	−3.9	0.00083	12.0	12.0	− 1.0
107...	T Vulpeculae	20 47.2	+27 52	40	−11	4.436	5.8	−2.0	0.0028	3.6	3.5	− 0.7
108...	UY Cygni	20 52.3	+30 3	43	−10	0.561	10.0	−0.4	0.00083	12.0	11.8	− 2.1
109...	VX Cygni	20 53.6	+39 48	50	− 5	20.125	9.7	−4.2	0.00017	58.8	58.6	− 5.1
110...	RV Capricorni	20 55.9	−15 37	1	−37	0.468	10.0	−0.3	0.00087	11.5	9.2	− 6.9
111...	TX Cygni	20 56.4	+42 12	52	− 3	14.726	9.1	−3.7	0.00028	35.7	35.7	− 1.9
112...	VY Cygni	21 0.4	+39 34	51	− 5	7.859	9.1	−2.8	0.00042	23.8	23.7	− 2.1
113...	SW Aquarii	21 10.2	− 0 20	20	−33	0.459	10.4	−0.3	0.00072	13.9	11.7	− 7.6
114*..	β Cephei	21 27.4	+70 7	75	+14	0.190	3.3	−0.4	0.018	0.6	0.5	+ 0.1
115...	VZ Cygni	21 47.7	+42 40	59	− 9	4.864	8.8	−2.0	0.00069	14.5	14.3	− 2.3
116...	Y Lacertae	22 5.2	+50 33	66	− 5	4.325	9.0	−1.9	0.00066	15.2	15.1	− 1.3
117...	δ Cephei	22 25.4	+57 54	73	+ 1	5.366	4.14	−2.19	0.0054	1.8	1.8	0
118...	Z Lacertae	22 36.9	+56 19	73	− 1	10.89	8.9	−3.3	0.00036	27.8	27.8	− 0.5
119...	RR Lacertae	22 37.4	+55 55	73	− 2	6.412	9.0	−2.4	0.00052	19.2	19.2	− 0.7
120...	V Lacertae	22 44.6	+55 48	74	− 3	4.983	9.0	−2.1	0.00060	16.7	16.7	− 0.9
121...	SW Cassiopeiae	23 2.9	+58 1	77	− 1	5.44	9.4	−2.2	0.00048	20.8	20.8	− 0.4
122...	RS Cassiopeiae	23 32.6	+61 53	82	+ 1	6.295	9.6	−2.4	0.00040	25.0	25.0	+ 0.4
123...	UU Cassiopeiae	23 45.8	+60 21	83	− 2	4.314	9.6	−1.9	0.00050	20.0	20.0	− 0.7
124...	RY Cassiopeiae	23 47.1	+58 12	83	− 2	12.328	9.8	−3.5	0.00022	45.4	45.4	− 1.6
125...	VW Andromedae	23 59.6	+34 12	80	−28	0.517	10.3	−0.3	0.00076	13.2	11.7	− 6.2
126†..	—Hydrae	12 25.2	−25 30	266	+37	0.479	10.9	−0.3	0.00057	17.5	14.0	+10.5
127†..	—Herculis	16 26.2	+18 36	3	+37	0.365	10.4	−0.3	0.00072	13.9	11.1	+ 8.4

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* The possible uncertainty in the length of period suspected by Guthnick (*Astronomiche Nachrichten*, **196**, 357, 1913; **205**, 97, 1917) does not affect the adopted absolute magnitude appreciably.

† These stars were added to the tables after receiving (February 1, 1918) the list of new variables in *Harvard Circular* No. 201.

TABLE I*a*

ADDITIONS TO TABLE I

No.	NAME	GALACTIC		PERIOD IN DAYS	MED. MAG. VISUAL	PARALLAX	DISTANCE (UNIT IS 100 PARSECS)		
		Long.	Lat.				Radial	Projected	From Plane
128	169. 1907 Leonis	149°	+55°	0.685	12.0	0″.00033	30.3	17.4	+24.8
129	RR Canum Venaticorum	110	+82	0.558	11.0	0.00052	19.2	2.7	+19.0
130	S Comae	181	+88	0.587	10.9	0.00055	18.2	0.6	+18.2
131	SX Ursae Majoris	77	+60	0.307	10.8	0.00060	16.7	8.4	+14.5
132	RU Canum Venaticorum	20	+73	0.364	11.1	0.00052	19.2	5.6	+18.4
133	W Canum Venaticorum	37	+70	0.552	10.3	0.00072	13.9	4.8	+13.1
134	172. 1907 Boötis	28	+66	0.622	10.8	0.00058	17.2	7.0	+15.7
135	X Scuti	346	− 3	4.187	9.8	0.00046	21.7	21.7	− 1.1
136	Z Scuti	355	− 2	12.9	9.2	0.00029	34.5	34.5	− 1.2
137	SX Aquarii	26	−35	0.536	11.8	0.00036	27.8	22.8	−15.9
138	VV Pegasi	46	−32	0.488	11.2	0.00050	20.0	17.0	−10.6
139	RZ Cephei	77	+ 6	0.309	9.4	0.0012	8.7	8.6	+ 0.9

The period of any star in Table II may be found, if desired, by reading its logarithm from the luminosity-period curve for the corresponding tabulated absolute magnitude; and the adopted median apparent magnitude can be readily computed from the parallax and absolute magnitude. The last five stars of the table were added from Hartwig's 1918 catalogue; see note 1, p. 2. The variables of Table II are not used in the diagrams or in the following discussion.

TABLE II

Supplementary List of Variables

No.	Name	Absolute Magnitude	Parallax
1	RX Andromedae	−5.5	0″.00010
2	SZ Cassiopeiae	−5.6	0.00008
3	SW Persei	−6.5	0.00008
4	SW Aurigae	−4.7	0.00004
5	RV Tauri	−5.3	0.00009
6	SS Geminorum	−5.5	0.00018
7	V Lyncis	−6.2	0.00009
8	RU Camelopardalis	−4.4	0.00019
9	RS Puppis	−5.3	0.00029
10	Z Cancri	−6.2	0.00010
11	RX Ursae Majoris	−6.0	0.00003
12	S Antliae	−0.3	0.0042
13	Z Leonis	−5.8	0.00012
14	S Crucis	−2.0	0.0016
15	W Virginis	−4.0	0.00014
16	V Ursae Minoris	−6.2	0.00015
17	UV Draconis	−6.3	0.00010
18	TX Scorpii	−0.6	0.0021
19	κ Pavonis	−3.0	0.0032
20	S Vulpeculae	−6.1	0.00009
21	TX Aquilae	−5.1	0.00010
22	R Sagittae	−5.1	0.00013
23	V Vulpeculae	−5.2	0.00017
24	TW Pegasi	−6.3	0.00018
25	RY Pegasi	−4.6	0.00010
26	RY Lacertae	−6.0	0.00003
27	W Cephei	−2.4	0.00091
28	X Lacertae	−2.2	0.00076
29	TV Andromedae	−6.0	0.00007
30	RU Aquarii	−6.1	0.00009
31	TZ Persei	−4.0	0.00004
32	U Monocerotis	−5.5	0.00044
33	UU Herculis	−6.0	0.00010
34	TX Ophiuchi	−6.2	0.00005
35	AP Sagittarii	−3.3	0.00083

There can be no doubt of the intimate relationship of cluster-type variables to the longer-period Cepheids, but in a few characteristics there appear wide differences—discontinuities in the usual progressive connections. One conspicuous discontinuity appears in the frequency of periods, which is illustrated in Table III for the 139 variables of Tables I and I*a*. Attention has been previously called to this matter by Hertzsprung and others. The reason for the two maxima in the frequency-curve must be sought in the dynamics of the stars themselves.

TABLE III

Frequency of the Periods of 139 Cepheid Variables

Period in Days	Number of Stars	Period in Days	Number of Stars	Period in Days	Number of Stars
0.0–0.2	2	5.0– 5.5	5	13.0–14.0	1
0.2–0.4	7	5.5– 6.0	5	14.0–15.0	4
0.4–0.6	31	6.0– 6.5	6	15.0–16.0	3
0.6–0.8	4	6.5– 7.0	3	16.0–17.0	4
0.8–1.0	1	7.0– 7.5	3	17.0–18.0	2
1.0–1.5	0	7.5– 8.0	6	18.0–19.0	2
1.5–2.0	2	8.0– 8.5	2	19.0–20.0	4
2.0–2.5	1	8.5– 9.0	0	20.0–25.0	1
2.5–3.0	1	9.0– 9.5	1	25.0–30.0	2
3.0–3.5	4	9.5–10.0	2	30.0–35.0	0
3.5–4.0	6	10.0–11.0	4	35.0–40.0	2
4.0–4.5	7	11.0–12.0	2		
4.5–5.0	5	12.0–13.0	4.		

Two other notable differences between the two groups are the space distribution and the space velocities. Hertzsprung has maintained in several notes that the cluster-type stars must be of low luminosity because of the large proper motion of their brightest representative, RR Lyrae ($\mu = 0''.25$), and because of the position of many such stars in high galactic latitude. The closely comparable luminosity of the two groups, however, is established by the luminosity-period curve. Moreover, RR Lyrae has a much larger radial velocity than the ordinary Cepheids, $V_0 = 50$ km/sec., and its proper motion may be an indication of great peculiar velocity rather than of large parallax.

This supposition is strongly supported through spectroscopic observations by Adams of four other cluster-type variables. The smallest radial velocity is -52 km/sec. for RS Boötis; the greatest is -196 km/sec. for XZ Cygni, which has an annual proper motion of $0''.1$, according to a determination kindly made at the writer's request by Professor Tucker. If we adopt the parallaxes in Table I, the space velocities of RR Lyrae and XZ Cygni, 400 km/sec., are among the greatest known. The average radial velocity for the longer-period Cepheids is less than 10 km/sec. It is possible, therefore, that the marked contrast between the wide dispersion of isolated cluster-type stars and the galactic concentration of normal Cepheids arises solely from the fact that the velocities of the former are enormous while those of the latter are moderate, in keeping with the motions of most other red and yellow giant stars.

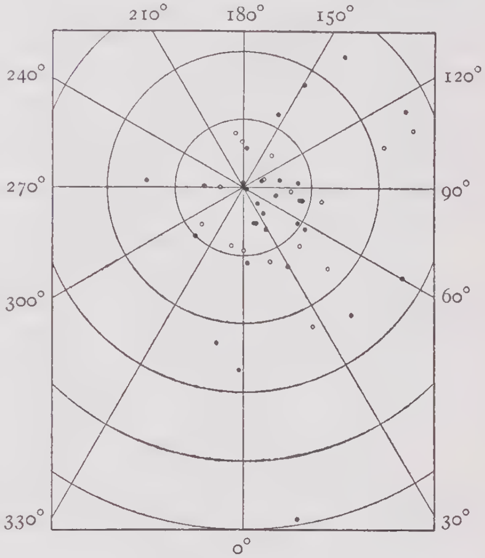

Fig. 1.—Projection on galactic plane of the position of Cepheid variables with periods shorter than one day. The circles are heliocentric, with radii of 1000 parsecs, 2000 parsecs, etc.; galactic longitudes are indicated in the margin. Many of these variables are so far above or below the plane that the diagram does not well represent the distribution in space.

Table IV contains a summary of the distribution of the variables with periods less than one day. The number of such stars is not large; the data are without doubt incomplete and there may be a preference for certain parts of the sky; hence no very definite quantitative conclusions should be based upon the material. The projection of these stars on the galactic plane is illustrated in Fig. 1; the distance from the plane is shown in Fig. 3. The great distances of RU Boötis and SW Herculis, and in particular the significance of the former's position so far from the galactic plane, have been remarked upon previously[1] and will be referred

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[1] *Publications of the Astronomical Society of the Pacific*, **29**, 183, 1917.

to again in a subsequent paper. Six cluster-type variables, with $R \sin \beta$ greater than 1750 parsecs, are beyond the bounds adopted as defining the equatorial segment of the Galaxy (Fig. 3). The mean distance from the galactic plane is 960 parsecs for these 45 variables of Table IV; for Cepheids with periods greater than a day it is 150 parsecs—less than one-sixth as much. The parallaxes of RS Boötis, RU Boötis, XX Cygni, XZ Cygni, and RR Lyrae are the most accurate.

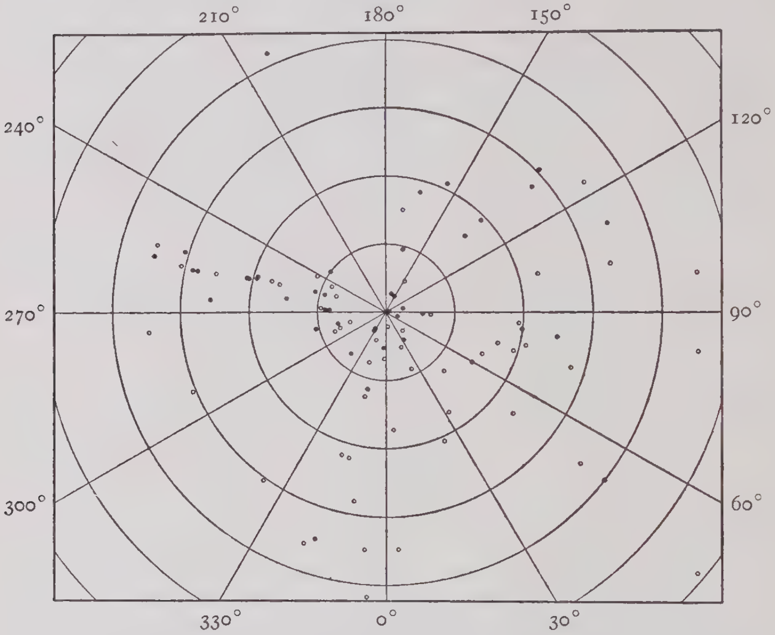

FIG. 2.—Projection on the galactic plane of the positions of Cepheid variables with periods greater than one day. The restriction to the Galaxy is so marked for these stars (see Fig. 3) that the diagram closely represents the distribution in space. The concentration of stars near longitude 255° reflects the systematic study at Harvard of the periods of all Cepheids in a restricted region (*Harvard Circular*, No. 170); it suggests the incompleteness of our data in other longitudes.

The distribution in the galactic plane of the 94 variables with periods longer than a day is shown in Fig. 2. The space co-ordinates are treated in some detail in Table V, showing the close restriction of ordinary Cepheids to the galactic plane. As a graphical test of

TABLE IV

DISTANCE FROM GALACTIC PLANE OF 45 CEPHEIDS WITH PERIODS LESS THAN A DAY*

Variables North of Galactic Plane		Variables South of Galactic Plane	
Limits of Distance	Number	Limits of Distance	Number
>+1500 parsecs....	7	<−1000 parsecs	3
+1500 to +1000...	7	−1000 to −500	5
+1000 to + 500...	9	− 500 to 0	8
+ 500 to 0....	6		
Total..........	29	Total.........	16

* Allowance for the position of the sun north of the Milky Way plane would alter these values slightly but would not change the actual dispersion.

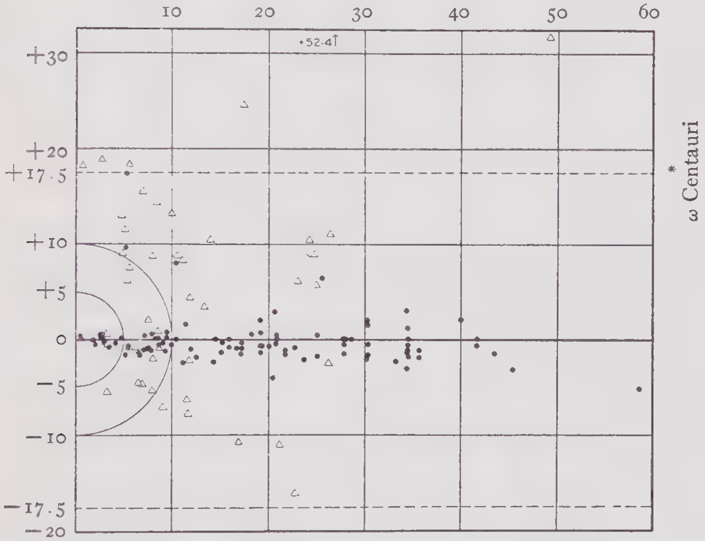

FIG. 3.—Distribution of Cepheid variables. The unit of distance is 100 parsecs. Ordinates are distances from the galactic plane; abscissae are projected distances in the plane; cf. Fig. 5 of the seventh paper. Open triangles and black dots designate respectively cluster-type variables and Cepheids with periods in excess of a day. The nearest globular cluster, ω Centauri, is just outside the boundary of the diagram on the right; RU Boötis, indicated by an arrow, is too far above the plane to fall within the figure. The semicircles with radii of 500 and 1000 parsecs ($\pi=0''.002$ and $0''.001$) indicate how very distant most of these variables are as compared with the average star of the tenth magnitude or brighter ($\pi>0''.004$, Kapteyn). Between the broken horizontal lines, ±1750 parsecs, lies the equatorial galactic region devoid of globular clusters.

the relation of galactic concentration to distance from the sun, the values of $R \sin \beta$ are plotted in Fig. 3, against $R \cos \beta$, triangles for cluster-type variables, and dots for those with periods in excess of a day. It is important to note that for the latter the greatest distance from the plane is attained by the star of shortest period, Z Canum Venaticorum, with $R \sin \beta = +17.4$ and period $= 1.890$ days. Its space velocity may be high.

Table V and Fig. 3 reveal no conspicuous divergence from the galactic plane with increasing distance. The indications of a contrary result, obtained in the earlier study,[1] was due partly to the inclusion of a few stars of the RV Tauri type; these stars are probably of lower luminosity than typical Cepheids of like period, and in consequence the computed distances, both radial and from the plane, were much exaggerated.

TABLE V

SPACE DISTRIBUTION OF 94 CEPHEIDS WITH PERIODS GREATER THAN A DAY*

		LIMITES OF $R \cos \beta$ (IN PARSECS)			
		0 to 1000	1000 to 2000	2000 to 4000	>4000
North of Galaxy	No. of stars.......	14	6	11	2
	Mean $R \sin \beta$	+ 220	+220	+160	+100
	Max. $R \sin \beta$	+1740	+810	+640	+210
	Av. dev. from mean	± 320	±200	±140	±100
South of Galaxy	No. of stars.......	17	17	23	4
	Mean $R \sin \beta$	− 80	−100	−140	−260
	Max. $R \sin \beta$	− 160	−240	−400	−510
	Av. dev. from mean	± 40	± 60	± 80	±150
Total No. of Stars.................		31	23	34	6
Mean $R \sin \beta$	Without regard to sign............	150	130	150	210
	With regard to sign	+ 60	− 20	− 40	−140

* Distances are given only to the nearest ten parsecs.

The distribution in longitude (Table VI) of variables with periods greater than one day shows some peculiarities which may be due to incompleteness of the data. There are, for instance, at least 50 stars thought to be Cepheid variables for which periods have not been determined, and probably a great number for which types are not yet known also belong to this class.

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[1] *Astrophysical Journal*, **40**, 432, 1914.

The greater number of Cepheids south of the galactic plane is easily accounted for by assuming the sun to be slightly to the north. In fact the Cepheids should afford a good determination of the distance of the sun from the galactic plane when a more complete list becomes available.

TABLE VI

Distribution in Galactic Longitude—Number of Stars

	0° to 30°	30° to 60°	60° to 90°	90° to 120°	120° to 150°	150° to 180°	180° to 210°	210° to 240°	240° to 270°	270° to 300°	300° to 330°	330° to 360°	Total
North....	0	2	3	3	3	5	1	0	10	2	1	3	33
South....	6	7	9	3	3	1	0	1	13	5	2	11	61
Total....	6	9	12	6	6	6	1	1	23	7	3	14	94

SUMMARY

1. Through the use of the luminosity-period curve the absolute magnitudes and parallaxes have been determined for 174 variables, 139 of which are believed to be perfectly typical members of the Cepheid class (Tables I and I*a*). The average probable error is estimated at 20 per cent.

2. Forty-five variables belong to the cluster type, with absolute luminosities a little more than one hundred times the brightness of the sun. Ninety-four are ordinary Cepheids with periods longer than a day and with luminosities ranging from two hundred to ten thousand times that of the sun. For 35 stars either the type of variation, or the period, or the magnitude is not certain or regular enough to yield final parallaxes (Table II).

3. The distances of Cepheid variables are considerably greater than have been obtained heretofore for individual stars. Less than one-third of them have parallaxes greater than a thousandth of a second. The most distant Cepheids now known are nearly 20,000 light-years from the sun. The nearest globular cluster is at a distance of about 21,000 light-years. (Cf. ω Centauri in Fig. 3.)

4. The numerical evaluation of the distribution in space confirms both the well-known concentration toward the galactic plane of Cepheids with periods greater than a day and the indifference of cluster-type variables to that plane. A plausible explanation of the wide dispersion of the latter is to be found in their relatively very high velocities in space.

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STUDIES BASED ON THE COLORS AND MAGNITUDES IN STELLAR CLUSTERS

NINTH PAPER: THREE NOTES ON CEPHEID VARIATION

By HARLOW SHAPLEY

At least three classes of stars may be independently used to ascertain the distances of globular clusters through luminosity correlations—Cepheid variables, B-type stars, and the giants of the redder spectral classes. The giant red stars in local parts of the galactic system, as well as in globular clusters, are now known to have a limited dispersion of absolute luminosity; and if we should assume the absolute magnitude in the mean to be the same in both regions, it is readily shown that the resulting parallaxes of the clusters would be closely comparable with the values obtained in the present investigation.[1] The stars of spectral type B (color-class *b*) form a second class of objects that are generally recognized as valuable criteria of distance, not only because of a fairly small dispersion of intrinsic brightness, but also because much is known of the actual absolute magnitudes through studies of community motion and parallactic drift. Apparently, however, there is a tendency for B-type stars to fall into groups that may differ in real brightness by one or two magnitudes,[2] thus introducing a large percentage uncertainty into the distances depending on magnitude correlations. If, as seems necessary from related investigations, the most luminous *b*-class stars in clusters are equated in luminosity with the galactic B stars that have an average absolute magnitude near zero, the distances of globular clusters (and of all Cepheids) are essentially as now adopted. On the other hand, if the brightest *b*-class stars are equated in luminosity with the most luminous of galactic B's, or if much weight be placed upon a direct application of the fragmentary luminosity-curves of B stars,[1] the distances and all the dependent dimensions would be considerably greater, and all giant red stars and Cepheid variables would be systematically much brighter in clusters than in the general galactic system.

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[1] For example, see Division III of the parallax determinations of Messier 13 listed in Table XXVII of *Mt. Wilson Contr.*, No. 116, 1915. The parallax there derived from red giants is 0".00010; that now adopted is 0".00009. Applying this method we would readily obtain also the absolute magnitudes of all Cepheids, and then derive practically the same individual parallaxes for them as given in the eighth paper.

[2] Kapteyn, *Mt. Wilson Contr.*, No. 82, 1914; Charlier, *Meddelanden från Lunds Astronomiska Observatorium*, Series 2, No. 14, 1916.

The present study has shown that for the third class of bright stars common to clusters and the general system—the Cepheid variables—the luminosity is simply a function of the period of light-variation, and, except for some uncertainty in the zero-point of the absolute scale, may be very accurately known. In consequence the Cepheids become of so much greater weight in the estimation of the parallaxes of globular clusters that other classes of highly luminous stars can best be used only to check the results, or as secondary standards.[2]

Although the distances obtained independently of the Cepheids would, as suggested above, be much the same as we have found, and the same conclusions as to the size and structure of the general system of clusters would result, the computed certainty of the distances and dimensions would be of a much lower order. The adopted numerical values therefore depend mainly upon the Cepheid variables, and the investigation of all phenomena related to the luminosity of these stars becomes appropriate. In particular it is important to see if there is any evidence that the variables of a given period in different systems are unlike in median magnitude. In the present communication three notes are brought together that bear on the question of Cepheid variation.

I. THE RELATION OF PERIOD TO LUMINOSITY

In the second and third sections of the sixth paper of this series we have determined a very definite observational relation between the period of light-variation of a Cepheid variable and its total light emission. The luminosity-period curve, based on observations from seven different stellar systems, is reproduced in the lower part of Fig. 1, where absolute magnitude is plotted against the logarithm of the period in days. The curve is defined by more than 330 stars, of which nearly 40, having periods longer than a day, fall along the curve at points where the variation of magnitude with period is rapid. Considering the uncertainties in the periods and magnitudes for some of the stars, the average deviation from the smooth curve is remarkably small—there is hardly a Cepheid known to be discordant.

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[1] *Mt. Wilson Contr.*, No. 116, p. 81, 1915; cf. also Eddington, *Monthly Notices*, **77**, 608, 1917.

[2] Through the method of measuring distance by means of the "25 brightest" stars, the red giants have been used in the rôle of secondary standards to determine the parallaxes of nearly all the clusters of Table V in the seventh paper.

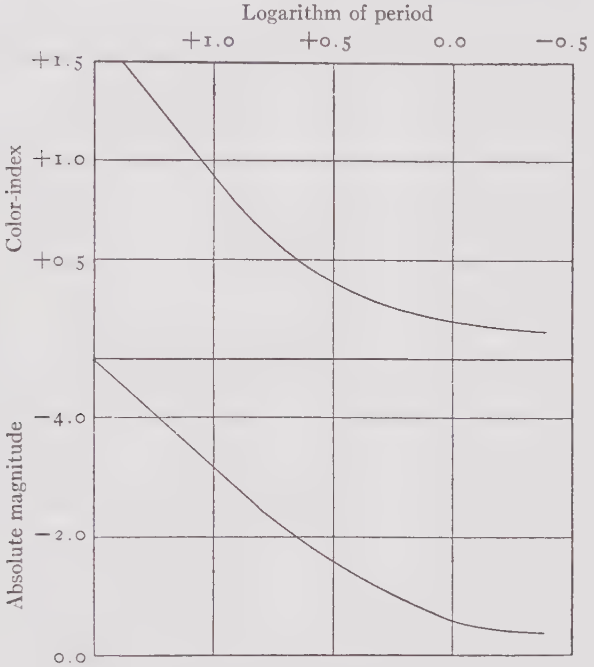

FIG. 1.—*Above:* Relation of period to color-index for Cepheid variables. *Below:* Relation of period to median absolute magnitude for Cepheid variables.

From the curve we see first that the decrease of median brightness with decreasing period is essentially linear until a period of less than three days is reached, and second that the absolute magnitude decreases no farther after the period has become a little shorter than one day. Referring to another curve in the same paper, which is reproduced in the upper part of Fig. 1, we observe that the change of median color-index with period is of much the same general character. A few remarks may be made on the possible interpretation of these curves.

It has been shown in an earlier paper[1] that the relation of period to spectral type and color-index can be provisionally explained (if we adopt a pulsation hypothesis of Cepheid variation) as a result of increasing density with decreasing color-index for giant stars. Thus a general relation, which is known to have a wide application in astronomical and physical problems and which holds without doubt in the present case, gives the period, P, of an oscillation in a gaseous star as a function of the radius and the surface gravity; or, if $\bar{\rho}$ is the mean density,

$$P^2 \propto \frac{1}{\bar{\rho}} \tag{1}$$

For giant stars the dependence of mean density on spectrum is approximately known from eclipsing binaries, and the application of (1) to Cepheid variables of known period gives for each spectral type a mean density of quite the same order as that found from eclipsing stars. Thus, through the intermediary of the mean density, the observed connection of median color-index and period is readily understood.

The reason for the dependence of luminosity on period is not so definite, but the following discussion seems to indicate the physical interpretation. If L be the total luminosity of a giant star, we may write[2]

$$L = \pi r^2 J$$

where r is the radius and J the luminosity per unit of apparent surface. Now $J = f\,(T, \rho_s, i)$; that is, the light intensity per unit area depends not only upon the effective surface temperature, T, but probably also upon the density at the stellar surface, ρ_s, and the degree of ionization, i—the last involving, it may be, the chemistry of the star as well as its ordinary physical properties. But with good cause we believe that the effective temperature (and hence the color-index, C.I.) is by far the most important factor in surface brightness. Denoting mass by μ, we may write

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[1] *Mt. Wilson Contr.*, No. 92; *Astrophysical Journal*, **40**, 448, 1914.

[2] Neglecting darkening at the limb.

$$L \propto r^2 f_1(T) \propto \left(\frac{\mu}{\bar{\rho}}\right)^{\frac{2}{3}} \phi(\text{C.I.}).$$

Introducing (1),

$$L \propto (\mu P^2)^{\frac{2}{3}} \phi(\text{C.I.}) \qquad (2)$$

and we have a physical explanation of the interdependence of the luminosity and period in Cepheid variation.

For the absolute magnitude, M, of stars differing little in spectral type (such as the Cepheid variables for which proper motions are known) we derive from (2)

$$M \propto 2 \log P + \log \mu .$$

It appears from Eddington's theoretical work on giant stars as well as from observational data relative to binaries of various types that the range in the masses of stars is small, so that the approximate theoretical relation

$$M = k \log P \qquad (3)$$

is of the same form as that observed for the Cepheid stars of longer period (Fig. 1).

The inclusion of log μ in (3) would not in the mean disturb the linear form of the relation, for the dispersion in mass is, so far as we know, comparable for giants of different spectral types redder than A. But over a wide range of spectrum ϕ (C.I.) becomes important in maintaining the luminosity in the face of contraction. The increase of surface intensity, with the progression toward bluer spectrum, may wholly counterbalance the tendency to diminish the total light-emission through shortening periods and smaller stars—especially near the blue end of the giant series where critical and asymptotic conditions are more likely to prevail[1]—and in consequence the luminosity-period curve theoretically should flatten out as observed. If, however, the counterbalance is maintained throughout the whole giant series and luminosity depends only on the mass, as some important researches suggest,[1] then (1) the mass apparently determines when in a star's history Cepheid variation may occur, (2) the period never changes materially during the life of the variation except possibly for cluster-type variables, and (3) the luminosity-period law may more appropriately express the mass-period or mass-density relation of Cepheid variation.

---

[1] "The concurrent progression toward a definite limit of luminosity, spectrum, and period suggests that, in the evolutionary sequence of stars, Cepheid variation is abruptly limited at or near the blue end of the giant series because of the changing physical conditions in the interiors of the gaseous masses."—*Mt. Wilson Contr.*, No. 151, p. 21, 1917.

It appears from the foregoing consideration that, granting the pulsation hypothesis of Cepheid variation, there is a definite reason for the dependence of total luminosity on period and a fairly plausible interpretation of the almost linear relation between the logarithm of the period and absolute magnitude. Quantitative evaluation of the theoretical equations should be possible eventually, and in fact has already been made tentatively. The computed results roughly conform with observation, but little weight can be attached to this agreement because of the present necessity of rather vague assumptions as to the progression, with spectral type, both of J and of darkening at the limb.

Eliminating the logarithm of the period from the two curves of Fig. 1, we obtain in Fig. 2, in the plot of absolute magnitude as a function of color-index, a curve that may be of considerable importance in Cepheid theories. For comparison with this curve the relation between absolute magnitude and color-index for the giant stars in four globular clusters is plotted along a broken line in the same figure.[2] We note that the change of luminosity with advancing spectral type is about the same for Cepheid variables of the galactic system as for all giant stars of globular clusters. Perhaps our more extensive knowledge of the former can throw light on the general problem of the relationship of successive spectral types.

The constant difference between the curves of Fig. 2, which is certainly larger than any systematic error involved, indicates that for a given color Cepheids are more than twice as bright as other stars; or rather, since the largest uncertainty is probably in the colors of the Cepheids, it indicates that for a given magnitude the variables appear to be nearly 0.4 mag. bluer than other giant stars. If minimum color and magnitude of the Cepheids be substituted for the median values, the two curves become nearly coincident.

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[1] Various papers by Eddington and Jeans in the *Monthly Notices*, *Nature*, and *The Observatory* during 1916 and 1917. Cf. the tenth paper of this series, p. 8.

[2] Adapted from the mean of the results given in Table II of *Mt. Wilson Communications*, No. 34; *Proceedings of the National Academy of Sciences*, **2**, 525, 1916.

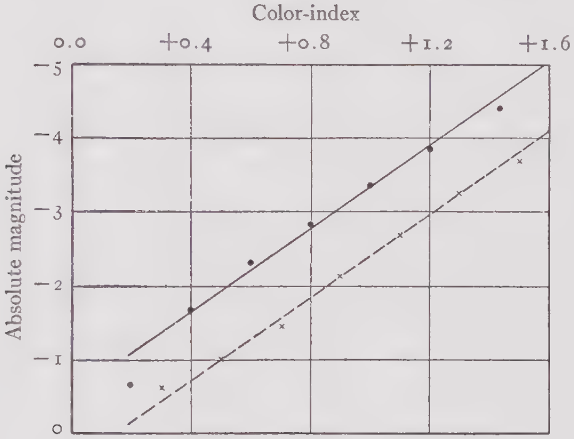

FIG. 2.—Relation of color-index to absolute magnitude for Cepheid variables. The broken line represents the corresponding relation for the giant stars of Messier 3, 5, 13, and 15.

II. THE COMPOSITE COLOR-CURVE OF 103 CLUSTER-TYPE VARIABLES

The determination, from plates made with the 60-inch reflector, of the magnitudes of a sequence of reference stars in Messier 3, permits the transformation to the Mount Wilson system of the light-curves derived by Bailey.[1] Since details of the derivation of magnitudes for the comparison stars will appear elsewhere, it will suffice for the present note to give only the final photographic and photo-visual results. The first two columns of Table I contain the designation of the comparison star by Bailey and the corresponding number in von Zeipel's catalogue.[2] The Harvard and Mount Wilson photographic magnitudes, in the following columns, differ in part because of the well-known systematic difference between the two systems, but mainly because of the somewhat provisional nature of the published Harvard magnitudes for this sequence of stars.[1] By plotting against each other the numbers of the third and fourth columns a fairly smooth curve is obtained which can be used to change the observed photographic magnitudes from one system to the other.

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[1] *Harvard Annals*, **78,** Part 1, 1913.

[2] *Annales de l'Observatoire de Paris*, **25,** 1, 1908.

TABLE I

COMPARISON STARS IN MESSIER 3

Designation		Harvard Photographic Magnitude	Mt. Wilson Photographic Magnitude	Mt. Wilson Photo-Visual Magnitude	Color-Index
Harvard	von Zeipel				
a...........	740	13.50	14.10	13.40	$+0^{M}.70$
b*..........	238	13.58	14.27	12.48	+1.79
c...........	640	13.98	14.40	13.22	+1.18
d...........	263	14.22	14.53	13.41	+1.12
e...........	250	14.50	14.81	13.91	+0.90
f...........	218	14.98	15.04	14.51	+0.53
g...........	227	15.28	15.25	14.60	+0.65
h...........	258	15.70	15.52	15.54	−0.02
k...........	609	16.00	15.66	15.70	−0.04
l...........	1131	16.23	15.81	15.48	+0.33
m...........	1055	16.49	15.95	15.26	+0.69
n...........	1327	16.82	16.17	15.49	+0.68
o...........	....	17.15	16.43	15.78	+0.65
p...........	....	17.63	16.79	16.38	+0.41

* Variability suspected by Bailey.

The close similarity of the photographic light-curves of the typical variables in Messier 3 makes possible the general representation of them all by the single mean curve[2] which is tabulated in the first two columns of Table III and drawn as a full line in Fig. 3. Whatever irregularities may exist in the individual variations are smoothed out, of course, in this mean; and in particular the range may be slightly too small because the rapid variation at maximum is concealed to some extent by the length of exposures necessary on the Harvard plates. Also it is not likely, except in the mean curve, that the light actually remains constant for three hours at minimum. It is to be noted in this mean result, however, that both the form of the curve and the photographic amplitude of 1.2 magnitudes are much the same as those usually observed for isolated cluster-type variables.

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[1] Cf. *Mt. Wilson Contr.*, No. 115, p. 13, n. 1. In a recent letter Professor Bailey writes that the magnitudes are being revised on the basis of the Harvard Polar Sequence.

[2] Derived from Bailey's Table XIX, *op. cit.*, p. 89.

The derivation of visual or photo-visual curves for each of these variables would entail much work, but a mean photo-visual curve can be derived with considerable certainty and without difficulty through the determination of a composite color-curve. Making use of the remarkable similarity of the variables and noting that the slightly differing periods are not commensurable, we may, if we choose, interpret the hundred magnitudes which are derived for variable stars from a single photograph of the cluster as a hundred different observations distributed at random along the light-curve of one variable—that is, along the mean light-curve. Similarly, the determination of the color at a given instant for the hundred variables gives immediately a hundred observations for the determination of a mean color-curve. Some of the periods, however, are not known accurately enough to allow the computation of phases for the time of observation, and the curve, therefore, gives color as a function of photographic magnitude rather than of time. To distinguish this property it will be called a composite color-curve.

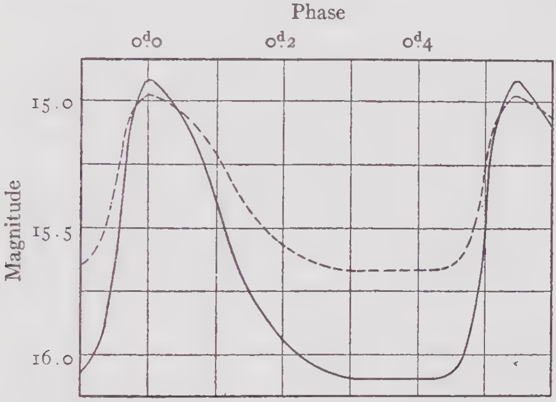

FIG. 3.—Mean photographic and photo-visual light-curves of 103 cluster-type variables in Messier 3.

For the measures of color, two pairs of photographs of Messier 3 were made on December 20, 1917, each pair consisting of a

TABLE II

Variable	Plates 4309 and 4310		Plates 4311 and 4312	
	Photographic Magnitude	Color-Index	Photographic Magnitude	Color-Index
173	15.06	$-0^{\mathrm{M}}15$	14.68	$-0^{\mathrm{M}}37$
181	16.33	....	16.30	+0.18
214	15.54	−0.01	15.68	+0.08
221	16.28	+0.36	16.34	+0.47
222	15.91	+0.19	15.68	+0.37
223	15.41	+0.29	15.54	+0.21
226	15.70	+0.06	15.92	+0.19
234	15.70	+0.35	15.87	+0.47
249	15.06	+0.03	15.64	+0.39
252	15.70	+0.12	15.95	+0.27
253	15.77	+0.44	15.51	+0.17
264	16.23	+0.44	16.24	+0.23
278	15.20	+0.08	15.39	+0.12
279	15.77	+0.22	15.95	+0.52
296	15.33	+0.17	15.61	+0.39
305	15.70	+0.28	15.95	+0.52
318	13.95	+1.61	13.88	+1.52
321	16.20	+0.43	16.05	+0.24
328	15.91	+0.22	15.99	+0.52
338	15.20	−0.08	15.39	+0.05
351	15.87	+0.29	15.97	+0.36
353	15.65	+0.21	15.61	+0.21
355	15.49	+0.32	15.47	+0.34
361	16.20	+0.38	16.12	+0.38
362	15.83	+0.39	15.86	+0.39
366	15.91	+0.33	15.51	+0.36
374	15.14	+0.21	15.34	+0.12
378	16.24	+0.42	16.05	+0.15
379	15.61	+0.32	15.68	+0.25
384	14.47	−0.29	14.83	−0.09
391	16.20	....	16.18	+0.38
396	15.91	+0.22	16.09	+0.29
397	15.94	+0.31	16.09	+0.75
404	16.46	+0.64	16.03	+0.53
411	16.20	+0.46	16.09	+0.35
425	15.98	+0.34	16.09	+0.59
451	15.41	+0.20	15.64	+0.30
455	15.77	+0.01	15.84	+0.17
479	15.36	+0.20	15.61	+0.30
537	15.93	+0.43	16.12	+0.50
541	14.88	+0.14	15.08	−0.05
616	14.62	−0.16	15.01	+0.09
623	14.97	−0.03	15.34	+0.18
644	16.35	+0.53	16.05	+0.55
650	15.91	+0.06	16.03	+0.08
656	14.97	−0.16	15.61	+0.34
679	15.41	+0.25	15.68	+0.22
700	15.91	+0.24	15.92	+0.42
710	16.46	+0.72	16.34	+0.49
713	16.20	+0.67	15.87	+0.13
759	16.28	+0.59	16.13	+0.63
765	15.14	+0.21	15.61	+0.40

TABLE II—*Continued*

Variable	Plates 4309 and 4310		Plates 4311 and 4312	
	Photographic Magnitude	Color-Index	Photographic Magnitude	Color-Index
775	16.33	$+0\overset{\mathrm{M}}{.}66$	16.23	$+0\overset{\mathrm{M}}{.}37$
776	15.40	+0.22	15.68	+0.25
818	15.21	+0.27	15.25	+0.22
840	15.91	+0.09	16.10	+0.44
873	15.14	+0.14	15.44	+0.26
922	15.77	+0.27	15.83	+0.37
948	15.91	+0.56	15.08	+0.16
965	15.91	+0.33	16.05	+0.50
999	15.70	+0.33	15.92	+0.34
1003	15.55	+0.38	15.54	+0.32
1041	16.26	+0.62	16.06	+0.21
1057	15.91	+0.28	16.05	+0.40
1090	15.91	+0.63	15.94	+0.48
1105	15.87	+0.37	15.97	+0.49
1122	15.94	+0.48	16.09	+0.59
1126	15.33	+0.41	15.28	+0.12
1164	16.38	+0.66	16.12	+0.36
1181	16.28	+0.44	16.24	+0.41
1184	15.06	+0.02	15.34	+0.04
1187	15.66	+0.38	15.68	+0.35
1201	16.09	+0.46	16.23	+0.49
1202	15.26	+0.15	15.61	+0.27
1207	15.05	+0.12	15.39	+0.26
1222	15.77	+0.56	15.54	+0.27
1231	15.91	+0.22	16.09	+0.55
1234	15.55	+0.22	15.73	+0.39
1250	14.90	+0.16	15.08	+0.12
1253	15.91	+0.63	15.92	+0.44
1259	15.11	−0.07	15.73	+0.27
1265	15.11	+0.07	15.28	+0.15
1268	15.70	+0.27	15.01	−0.15
1277	15.87	+0.11	16.05	+0.20
1291	15.83	+0.20	15.92	+0.44
1292	15.91	+0.47	15.50	+0.56
1298	15.02	+0.10	15.21	+0.08
1307	15.83	+0.62	16.03	+0.57
1308	15.91	+0.28	16.05	+0.19
1317	15.33	+0.20	15.24	−0.10
1320	15.87	+0.20	15.76	+0.30
1321	15.70	+0.13	15.87	+0.24
1330	15.66	+0.34	15.76	+0.33
1353	15.87	+0.24	16.06	+0.30
1357	15.83	+0.39	16.05	+0.32
1373	15.14	+0.07	15.39	+0.23
1380	15.98	+0.31	15.73	+0.13
1384	15.41	+0.09	15.61	+0.34
1386	15.91	+0.26	15.97	+0.12
1388	15.40	+0.08	15.76	+0.42
1400	15.60	+0.18	15.92	+0.52
1406	15.41	+0.08	15.64	+0.37
1409	16.26	+0.74	16.23	+0.17
1422	15.10	−0.06	15.47	+0.25

photographic and a photo-visual plate. The photo-visual plates were interrupted when half exposed to obtain the photographic exposures, thus insuring the essential coincidence of the mean times of the two observations. Very rarely will an observation occur at such a phase of the light-variation that appreciable error in the color can arise from this method of observing. The results of the measures are given separately for the two pairs of plates in Table II. The first column contains the number of each variable in von Zeipel's general catalogue, the second and fourth contain its observed photographic magnitude on the Mount Wilson system, and the third and fifth the difference between this magnitude and the corresponding photo-visual result. No. 318, a bright variable to which no type or period has been assigned, is not included in the discussion.

A noticeable diversity of color for a given magnitude is evident in the plots of the individual results of Table II. This may be attributed to accidental error, or to real differences in the variables, or perhaps to actual differences in the color for a given magnitude on the ascending and descending branches. But taking the mean of the color-indices, grouped in order of photographic magnitude, a very definite composite color-curve is obtained for each pair of plates. As no material difference appears in the results from the two pairs, the normals for each are combined into the following means, which are graphically shown in Fig. 4:

Mean Photographic Magnitude	Number of Variables	Mean Color-Index
14.78	7	−0.14
15.11	23	+0.05
15.38	26	+0.19
15.64	39	+0.27
15.89	58	+0.34
16.12	38	+0.41
16.33	13	+0.55

From the mean photographic curve and this composite color-curve we obtain immediately for the variables of Messier 3 a mean photo-visual curve. The co-ordinates appear in the first and fourth columns of Table III, and are plotted as a broken line in Fig. 3. Again the form of the curve, its amplitude, and the color variation are all sensibly the same as for the isolated cluster-type variables of the galactic system. The quantitative agreement of these distant stars with the local variables in phenomena of color is also

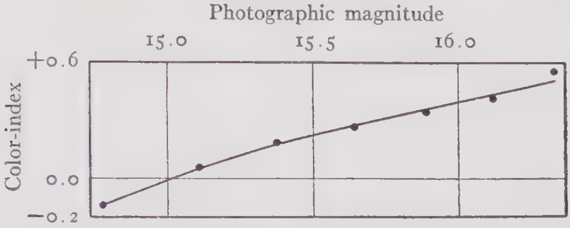

FIG. 4.—Composite color-curve of 103 cluster-type variables in Messier 3

TABLE III

MEAN PHOTOGRAPHIC AND PHOTO-VISUAL LIGHT-CURVES OF CLUSTER-TYPE VARIABLES IN MESSIER 3

Phase	Mt. Wilson Photographic Magnitude	Color-Index	Photo-Visual Magnitude	Phase	Mt. Wilson Photographic Magnitude	Color-Index	Photo-Visual Magnitude
0.000...	14.90	$-0^{\mathrm{M}}07$	14.97	0.271...	16.07	$+0^{\mathrm{M}}42$	15.65
0.011...	14.93	−0.06	14.99	0.283...	16.08	+0.42	15.66
0.023...	14.96	−0.04	15.00	0.294...	16.09	+0.43	15.66
0.034...	15.00	−0.01	15.01	0.305...	16.09	+0.43	15.66
0.045...	15.04	+0.01	15.03	0.317...	16.10	+0.43	15.67
0.057...	15.11	+0.05	15.06	0.328...	16.10	+0.43	15.67
0.068...	15.17	+0.07	15.10	0.339...	16.10	+0.43	15.67
0.079...	15.23	+0.11	15.12	0.350...	16.10	+0.43	15.67
0.090...	15.32	+0.16	15.16	0.362...	16.10	+0.43	15.67
0.102...	15.39	+0.18	15.21	0.373...	16.10	+0.43	15.67
0.113...	15.48	+0.22	15.26	0.384...	16.10	+0.43	15.67
0.124...	15.58	+0.26	15.32	0.396...	16.10	+0.43	15.67
0.136...	15.67	+0.29	15.38	0.407...	16.10	+0.43	15.67
0.147...	15.73	+0.31	15.42	0.418...	16.09	+0.43	15.66
0.158...	15.79	+0.33	15.46	0.430...	16.09	+0.43	15.66
0.170...	15.84	+0.34	15.50	0.441...	16.08	+0.42	15.66
0.181...	15.87	+0.36	15.51	0.452...	16.06	+0.42	15.64
0.192...	15.91	+0.37	15.54	0.464...	16.01	+0.40	15.61
0.204...	15.94	+0.38	15.56	0.475...	15.93	+0.38	15.55
0.215...	15.98	+0.39	15.59	0.486...	15.77	+0.33	15.44
0.226...	16.01	+0.40	15.61	0.497...	15.53	+0.24	15.29
0.237...	16.03	+0.41	15.62	0.509...	15.22	+0.11	15.11
0.249...	16.04	+0.41	15.63	0.520...	15.06	+0.02	15.04
0.260...	16.06	+0.42	15.64	0.531...	14.96	−0.04	15.00

important and favorable evidence of the accuracy of the photovisual and photographic magnitude scales relative to each other, and of the absence of light-scattering in space.

III. ON THE RELATION OF COLOR TO PERIOD FOR CLUSTER-TYPE VARIABLES

In the third and fourth sections of the sixth paper of this series[1] attention has been called to certain properties of the different kinds of cluster-type variables. In ω Centauri, for instance, the three subclasses defined by Bailey differ from each other in the form and amplitude of the mean light-curve, and most clearly in length of period. In Messier 3 and Messier 5, where nearly all variables are of the most typical kind, a second subclass is only sparingly represented, but in Messier 15 the variables fall into two nearly equal groups. It appears definitely that the median intrinsic luminosity in all subclasses is photographically the same—an important phenomenon that a full interpretation of these variables must explain.

The differences in period of the subclasses naturally suggest corresponding differences in the effective density; or it may be that the character of the underlying pulsation differs somewhat in the various groups. On the basis of the first alternative, if the mean densities increase with shortening periods[2] while the photographic magnitudes do not change, we must expect the change in the surface area to be compensated by greater intensity of radiation per unit of surface, and investigations of the color phenomena for the different subclasses should afford a partial test.[3]

In Messier 15 the faintness of the variables makes a detailed study of individual color-curves inadvisable at present, and accordingly the method used in the preceding note is employed to obtain composite color-curves for the two groups of variables. Professor Bailey has kindly communicated, especially for the present investigation, the preliminary results of his study of the periods and light-curves of the 53 variables in this cluster.[4] One or two have periods longer than a day, for several the periods have not yet been found, and for others accurate photometric measures are not possible.

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[1] *Mt. Wilson Contr.*, No. 151, 1917.

[2] See sec. I of this *Contribution.*

[3] ω Centauri is too far south for detailed color investigations at Mount Wilson. Color-curves are being derived for the variables of Messier 5.

[4] A preliminary report on his work is abstracted in *Popular Astronomy*, **25**, 520, 1917.

TABLE IV

Composite Color-Curves of Variables in Messier 15

Variable	Approximate Period	Plates 4284–4285		Plates 3986–3987	
		Photographic Magnitude	Color-Index	Photographic Magnitude	Violet-Index
2	$0^{d}.66$	16.13	$+0^{M}.51$	16.21	$-0^{M}.10$
3	0.39	16.18	+0.44	15.90	+0.03
4	0.31	16.27	+0.44	15.85	−0.12
5	0.38	15.90	+0.16	16.43	−0.29
6	0.67	16.13	+0.39	16.11	+0.05
7	0.35	16.27	+0.50	15.99	−0.16
8	0.65	15.42	+0.06	16.03	−0.01
9	0.72	15.49	+0.08	16.36	−0.10
10	0.40	16.27	+0.42	16.32	−0.12
11	0.34	16.18	+0.23	16.03	−0.07
12	0.59	16.22	+0.28	16.34	−0.14
13	0.57	16.38	+0.36	16.42	−0.02
14	0.38	16.10	+0.35	16.08	−0.20
15	0.58	16.13	+0.19	16.26	−0.24
16	0.67	15.53	−0.20	15.85	−0.10
18	0.37	15.49	−0.16	15.90	−0.07
19	0.57	16.47	+0.65	16.47	....
20	0.69?	15.98	+0.33	16.03	−0.19
22	0.72	15.98	+0.26	15.52	+0.13
23	0.67	16.13	+0.36	16.31	....
24	0.37	16.52	+0.68	15.78	+0.02
25	0.67	16.27	+0.33	16.42	−0.16
26	0.40	16.18	+0.26	16.37	....
29	0.57	16.10	+0.30	16.37	−0.11
30	0.41	15.56	−0.09	16.03	−0.02
31	0.44	15.90	+0.19	16.58	....
32	0.61	16.13	+0.31	15.75	−0.08
35	0.38	15.90	+0.19	16.44	−0.28
38	0.38	15.53	−0.23	16.44	−0.20
39	0.39	16.06	+0.32	15.85	−0.10
40	0.38	16.10	+0.33	15.90	+0.02
41	0.42?	16.27	+0.56	16.03	−0.01
42	0.36	15.49	−0.18	16.32	−0.27
43	0.29??	15.66	−0.05	15.96	−0.03
44	0.60	15.82	+0.08	16.35	−0.19
45	0.66	16.30	+0.54	15.96	−0.08
48	0.38	15.56	−0.16	15.82	0.00
50	0.30	16.22	+0.28	16.43	−0.17
51	0.42	16.30	+0.36	16.22	−0.06

The 39 variables with periods less than a day, for which photovisual and photographic magnitudes have been obtained from a pair of plates made on December 19, 1917, are listed in Table IV.

As a preliminary to the derivation of the magnitudes and colors of the third and fourth columns, it was necessary to study the comparison stars by means of a series of polar comparisons. At the same time colors were determined for a considerable number of other stars in the cluster. Since the discussion of magnitudes follows the customary lines, all details may be omitted from this note.

The color-index of each variable is plotted against photographic magnitude in the upper part of Fig. 5, the dots indicating variables with periods between $0\overset{d}{.}57$ and $0\overset{d}{.}71$ and the crosses indicating those with periods between $0\overset{d}{.}29$ and $0\overset{d}{.}44$. As in the preceding note, all dots may be interpreted as representing different observations on a single mean variable of period about $0\overset{d}{.}64$, and the crosses as representing different observations on a typical variable in the second group, with mean period of about $0\overset{d}{.}36$. The usual absolute values of the color and the usual increase of redness with decreasing brightness are shown by the observations; but no material difference appears in color for the two groups, except, perhaps, near maximum, where there is a suggestion that the stars with longer period are redder.[1]

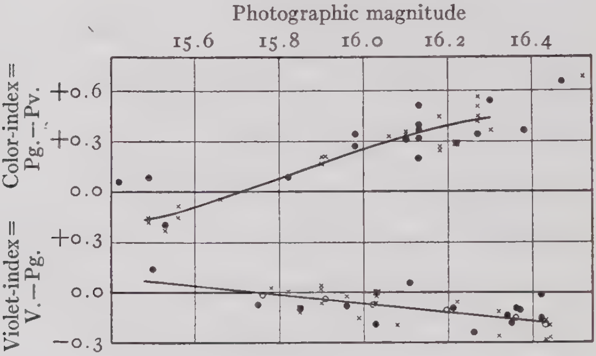

FIG. 5.—Composite color-curves of cluster-type variable stars in Messier 15; the dots refer to the longer-period subclass, crosses to the shorter-period subclass, and open circles to means of six values.

The test for difference in color has been carried into the region of shorter wave-lengths by a pair of plates made September 11, 1917, in which, as a companion to an ordinary photographic plate, a photograph was made on a Seed 27 plate through a violet-color screen that transmits very little to the red of λ 4000 and has a maximum transmission at about λ 3700. The results are tabulated in the last two columns of Table IV and plotted in the lower part of Fig. 5, dots and crosses being used as before and open circles indicating means of six values taken in order of photographic magnitude.

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[1] This would be in keeping with results for Cepheids with periods longer than a day. Cf. Fig. 1.

TABLE V

RELATION OF VIOLET-INDEX TO COLOR-INDEX FOR GIANT STARS IN MESSIER 15

Star	Photographic Magnitude	Color-Index (Pg. − Pv.)	Violet-Index (V. − Pg.)
a	14.14	$+1\overset{\mathrm{M}}{.}32$	$+0\overset{\mathrm{M}}{.}70$
b	14.51	+0.79	+0.15
c	14.58	+1.18	+0.42
d	14.81	+1.18	+0.33
e	14.76	+1.06	+0.46
f	14.98	+1.04	+0.14
g	15.01	+0.75	+0.13
h	15.23	+0.93	+0.28
k	15.49	+0.70	+0.16
l	15.68	+0.68	−0.03
m	15.97	+0.29	+0.01
n	16.02	+0.21	0.00
t	16.10	+0.18	−0.05
u	16.60	+0.66	−0.06
x	16.05	+0.64	0.00
y	14.51	+1.23	+0.55
z	15.07	+0.08	−0.09
100	14.17	+1.40	+0.78
103	13.09	+0.45	+0.36
104	14.11	+0.61	+0.36
105	12.81	+0.61	+0.34
106	14.17	+1.15	+0.63
107	14.51	+1.14	+0.46
109	14.69	+1.28	+0.36
110	14.18	+1.40	+0.65
108	14.15	+1.41	+0.97
112	14.23	+1.52	+0.82
113	14.15	+0.60	+0.01
115	14.10	+1.29	+0.87
155	14.42	+1.66	+0.65
173	14.55	−0.37	−0.43
201	16.08	+0.46	+0.04
222	16.00	+0.27	−0.02

A full consideration of the decrease of violet-index, V. − Pg., with decreasing photographic brightness, which indicates a smaller range in violet magnitude than in photographic, must be deferred until more definite information is at hand relative to the scale of violet magnitudes. As it now stands the violet-index for these cluster-type variables decreases as color-index increases—a result quite the contrary of the relation of the two indices shown in Table V for a number of typical invariable giant stars in this same cluster.

SUMMARY

1. The redder a Cepheid variable the greater is its intrinsic luminosity; a nearly linear relation connects absolute magnitude and color-index.

2. The general characteristics of the luminosity-period curve of Cepheid variation (Fig. 1) appear to have a reasonable explanation on the pulsation hypothesis.

3. For giant stars in at least four globular clusters the decrease of brightness with decreasing color-index parallels the same phenomenon for Cepheids. When median brightness and color are considered, the variables of a given absolute brightness are nearly four-tenths of a magnitude bluer than other giant stars (Fig. 2).

4. The composite color-curve of more than 100 cluster-type variables has been determined from two pairs of photographs of Messier 3 (Fig. 4).

5. In range of variation, form of light-curve, and phenomena of color, the variables of a globular cluster apparently are exactly comparable with the average isolated cluster-type variable (Fig. 3). No evidence has been found that in different stellar systems Cepheids differ systematically either in median luminosity or in nature of variation.

6. In Messier 15 two subclasses of cluster-type variables which have distinctly dissimilar mean periods show neither a difference in median photographic magnitude nor a difference in color at the median magnitude. At maximum, stars with shortest periods may be slightly bluer.

7. Preliminary violet magnitudes of cluster-type variables indicate a smaller amplitude than in photographic light; for the giant invariable stars of clusters the violet-index, V.−Pg., increases with color-index and hence with increasing photo-visual brightness.

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STUDIES BASED ON THE COLORS AND MAGNITUDES IN STELLAR CLUSTERS

TENTH PAPER: A CRITICAL MAGNITUDE IN THE SEQUENCE OF STELLAR LUMINOSITIES

By HARLOW SHAPLEY

The investigations reported in the preceding *Contributions* point to the close equality of the median apparent magnitudes of cluster-type variables as a significant phenomenon in Cepheid variation. The absolute brightness corresponding to this median value marks the limit fainter than which variability of the Cepheid type evidently does not occur. The discussion in the present paper tends to show that the same absolute magnitude, or one nearly the same, also marks a critical point in the dynamical history of stars in general. The result might have been anticipated, for the concentration of the median luminosities of cluster-type variables to a very narrow interval would require a similar concentration for other stars of like mass and color, if Cepheid variation is regarded as accidental rather than inevitable in a star's development. We must keep in mind, however, the possibility that the variation itself might operate to hold a star indefinitely long at the critical magnitude.

From an unpublished study of Messier 3 (N.G.C. 5272) the frequency of photo-visual magnitudes for various classes of colors is given in Table I. The catalogue from which this table was constructed is based upon about ten thousand measures of magnitude. Much care has been taken to avoid errors, and the results are believed to be dependable, both as to actual colors and as to relative frequencies. Only stars between 2′ and 11′ from the center of the cluster and brighter than the seventeenth photo-visual magnitude are included in Table I. All known variables are omitted, as well as stars with doubtful colors or magnitudes. The general discussion of the table we reserve for the paper on Messier 3, using for the present investigation only the relation of star numbers to magnitude.

TABLE I

Frequency of Photo-Visual Magnitudes in Messier 3 for Each Color-Class (Distance from Center 2′.0 to 11′.3)*

Limits of Photo-Visual Magnitude	<*b*5	*b*5 to *a*o	*a*o to *a*5	*a*5 to *f*o	*f*o to *f*5	*f*5 to *g*o	*g*o to *g*5	*g*5 to *k*o	*k*o to *k*5	>*k*5	<*f*5	>*f*5	All Colors
12.20–.39	......	......	......	......	......	......	......	......	......	1	......	1	1
.40–.59	......	......	......	......	......	......	......	......	1	1	......	2	2
.60–.79	......	......	......	......	......	1	......	1	......	2	......	4	4
.80–.99	......	......	......	......	......	1	......	......	4	......	......	5	5
13.00–.19	......	......	......	......	......	......	......	5	2	......	......	7	7
.20–.39	......	......	......	......	......	......	......	1	2	......	......	3	3
.40–.59	......	......	......	......	......	1	......	4	1	......	......	6	6
.60–.79	......	......	......	......	......	......	4	3	......	......	......	7	7
.80–.99	......	......	......	......	1	......	7	6	......	......	1	13	14
14.00–.19	......	......	1	......	......	2	2	5	1	......	1	10	11
.20–.39	......	......	......	......	......	1	3	3	......	......	......	7	7
.40–.59	......	1	......	1	3	16	6	......	......	......	5	22	27
.60–.79	1	......	......	1	1	11	11	2	......	......	3	24	27
.80–.99	......	......	......	1	1	3	7	4	......	......	2	14	16
15.00–.19	......	......	1	2	......	7	17	3	......	......	3	27	30
.20–.39	......	1	4	4	15	19	13	1	......	......	24	33	57
.40–.59	......	9	23	12	15	26	1	......	......	......	59	27	86
.60–.79	......	8	6	4	2	28	8	2	......	......	20	38	58
.80–.99	......	2	6	......	7	17	5	......	......	......	15	22	37
16.00–.19	2	2	2	......	12	14	6	......	......	......	18	20	38
.20–.39	1	1	......	4	16	13	1	1	......	......	22	15	37
.40–.59	......	1	......	9	21	19	......	......	......	......	31	19	50
.60–.79	......	......	2	20	26	11	1	......	......	......	48	12	60
.80–.99	......	2	4	24	38	1	......	......	......	......	68	1	69
All magnitudes	4	27	49	82	158	191	92	41	11	4	320	339	659

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*To save space in the table two tenth-magnitude red stars and one of eleventh magnitude with color-class *a* are omitted. Probably they are not physical members of the cluster.

Plotting the data of the last column, we obtain, as in the upper part of Fig. 1, the luminosity-curve for all colors, which shows a conspicuous, sharp maximum at photo-visual magnitude 15.50. This combines the luminosity-curve of the reddish giants with an abnormal or at least irregular luminosity-curve of the bluer stars. In the lower half of Fig. 1 the two are given separately. The curve for the redder giants is as smooth and complete as could be expected for a small group of giant stars.[1] The other, though otherwise a regular curve for which the fainter limit is set by the

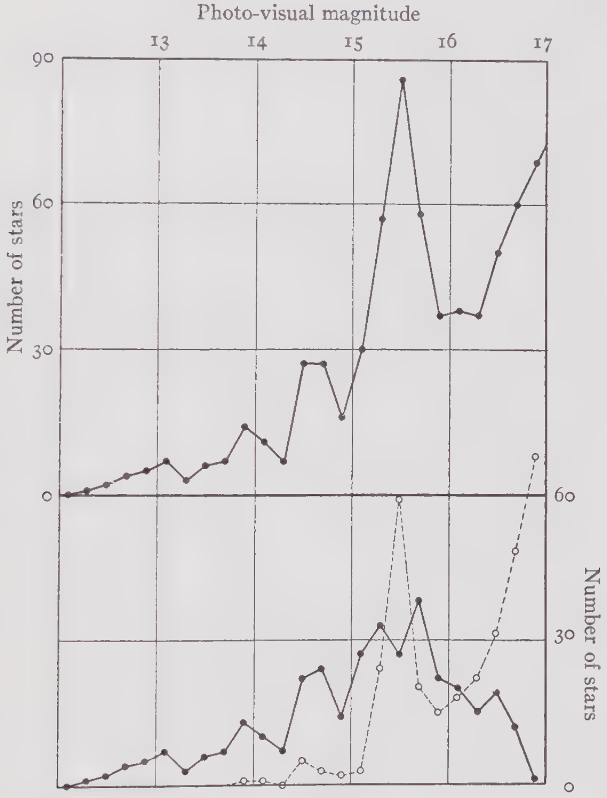

FIG. 1.—Frequency of photo-visual magnitudes in Messier 3. *Above:* All colors. *Below:* Full line, color-index ≧ +0.60; broken line, color-index < +0.60.

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[1] Actually this curve is also a composite of several distinct luminosity-curves, as may be inferred from the columns of Table I.

limit of the catalogue, shows a remarkable gathering of stars between magnitudes 15.20 and 15.60. A smooth curve would allow less then 15 stars to this interval; the actual number is 83, exclusive of known variable stars.

The median photo-visual magnitude of the variables has not been directly determined, but it is known[1] to lie between 15.25 and 15.35. Therefore, if all the known variables within the measured area were included, with median values of their magnitudes and colors, the excess of stars between 15.20 and 15.60 would be about 1200 per cent.

The observed luminosity-curve for the blue stars is not of the regular form originally expected and most readily explained. If, with the variables excluded, a smoothly increasing curve had resulted, we would have assumed that the variables are peculiar stars intermingled with a normal aggregation. On the other hand, if a smoothly increasing curve resulted only when all the variables were included at their median, maximum, or minimum brightness, we would have assumed that the variables are normal stars undergoing a stage in stellar evolution that is common to nearly all stars with sufficient mass to attain this critical luminosity. The actual condition, however, is accounted for less easily.

A question naturally arises concerning the constancy of the light of the stars that form the excess over the number required for a smooth luminosity-curve. In the area covered by the manuscript catalogue of stars in Messier 3 there are about 100 typical cluster-type variables—all nearly identical in every known characteristic;[2] and there are about 70 of these additional stars, which, though closely resembling the variables in magnitude and color, do not in a single case appear to undergo a definite variation of light. Are they incipient variables, or stars in which a variation once existed but is now disappearing or has completely vanished? Or, through the fortunes of position and motion, have they avoided the cause of variation or, by relatively slight differences in physical structure, have they been able to withstand it?

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[1] See Figs. 3 and 4 in the second part of the preceding *Contribution*.

[2] Except, of course, in distance from the center, and possibly, therefore, in space velocity.

The stars in question may be readily selected from the catalogue, and a future detailed study of their magnitudes may help to solve a problem which is of much significance in the full interpretation of Cepheid variation, and which may be of some importance in getting at the internal structure of a star. Meanwhile we have an indication of the possible result through the following tabulation of the means of the photographic residuals:[1]

Photo-visual magnitude 15.20 to 15.60:
  Color-index 0.00 to +0.20, Mean Pg. residual ±0.103 (33 stars)
  Color-index +0.20 to +0.40, Mean Pg. residual ±0.083 (19 stars)
  Color-index +0.40 to +0.60, Mean Pg. residual ±0.071 (37 stars)
All magnitudes and colors, Mean Pg. residual ±0.082 (800 stars)

The interval of brightness near median magnitude is the most comfortable of all for measurement, being neither too bright nor too faint, and the accidental errors should be less than for the average. The mean residual for all stars includes a considerable number of values referring to objects which later were rejected because of uncertainty; therefore this mean is slightly greater than it should be for a fair comparison.

It is evident then that the 33 stars most nearly like the variables in median magnitude and color are subject to uncommonly large magnitude deviations. Moreover, one or two uncertain results are not responsible for the large average residual, as one-half of the deviations exceed ±0.10. While awaiting further study our provisional conclusions are that Cepheid variation affects some of these stars to some extent, and that it may be possible to witness the beginning or end of this type of phenomenon. If so, the initial and final stages must be short compared with the whole duration of mature variability, for of all the variables in this cluster none is yet known to have an intermediate amplitude or period.[2]

Since the mean photographic median magnitude is definitely known from Bailey's extensive studies, we shall be able to examine more closely the relationship of the variables to these 33 stars if we put the frequency analysis on a photographic basis. Rearranging the data of Table I and plotting the numbers for color-class *a*0–*a*5, we obtain the photographic luminosity-curve given in Fig. 2*a*. The maximum frequency corresponds to a brightness fainter than the median, 15.50, by less than a tenth of a magnitude.

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[1] The residuals will appear in the forthcoming catalogue of Messier 3. The magnitude of each star is the mean of from two to six determinations.

[2] Cf. sec. IV of the sixth paper. Variable No. 37 belongs to a distinctly different subtype.

A similar investigation of the photographic luminosity-curve for Messier 13—the only other globular cluster for which extensive color data are available[1]—results in the curve (Table II) illustrated in Fig. 2*b*, showing a similarly close coincidence of maximum frequency and median magnitude (15.03).

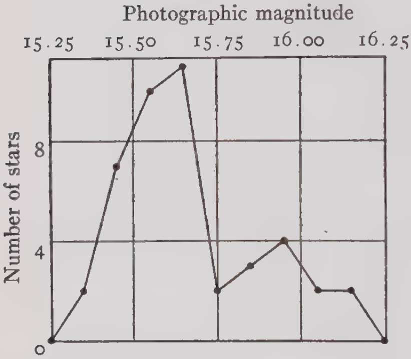

FIG. 2*a*.—Photographic luminosity-curve of blue stars in Messier 3.

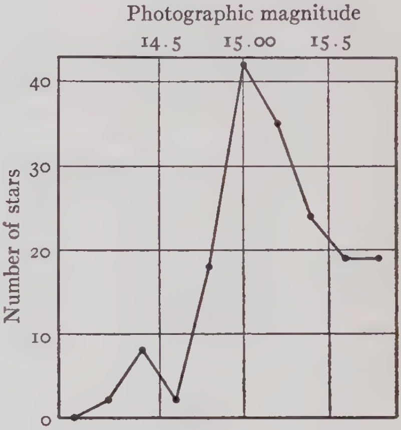

FIG. 2*b*.—Photographic luminosity-curve of blue stars in Messier 13.

In Messier 13 the few faint variable stars have not been thoroughly studied, and it is necessary to obtain a hypothetical median magnitude by adding 1.28 to the mean magnitude of the 25 brightest stars.[2] And also, since the number of stars of color-class *a* is not large, the interval of color is extended in both directions, thus including numerous stars with negative color-indices. In fact, the maximum is chiefly due to stars with color-class *b7* to *a0*—a comparatively infrequent type[1] in Messier 3.

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[1] *Mt. Wilson Contr.*, No. 116, p. 51, 1915. Instead of photographic, as erroneously printed, the limits of magnitude in this table should read photo-visual. The change to the photographic system for the present curve is made with sufficient accuracy by assuming mean colors and magnitudes for each tabulated interval. For Messier 3, however, the transformation was carried out rigorously.

[2] See sec. V of the sixth paper and Table II of the seventh paper.

TABLE II

FREQUENCY IN MESSIER 13 OF THE PHOTOGRAPHIC MAGNITUDES OF HIGH LUMINOSITY BLUE STARS

Interval of Photographic Magnitude	*b0* to *b5*	*b5* to *a0*	*a0* to *a5*	*a5* to *f0*	*f0* to *f5*	Color-Index $<+0.60$
12.70–12.89	1	....	....	....	....	1
.90–13.09	....	....	....	1	....	1
13.10– .29	....	....	....	....	1	1
.30– .49	....	....	....	....	....	0
.50– .69	....	....	....	....	....	0
.70– .89	....	3	....	....	....	3
.90–14.09	....	....	....	....	....	0
14.10– .29	....	....	....	2	....	2
.30– .49	....	1	1	....	6	8
.50– .69	....	....	1	....	1	2
.70– .89	....	5	6	7	....	18
.90–15.09	10	24	3	2	3	42
15.10– .29	4	21	9	....	1	35
.30– .49	1	11	7	3	2	24
.50– .69	....	5	6	5	3	19
.70– .89	....	....	3	4	12	19

The luminosity-curves of Figs. 2 render probable the hypothesis that the median magnitude is an epochal point in the luminosity of a cluster star whether or not its light is periodically variable. As a partial and provisional interpretation of the congestion of blue stars at this magnitude, we may refer to the luminosity-period curve of Cepheid variation and in particular to the constancy of absolute magnitude for all periods less than a day. If we adopt the theory of pulsations for these variables, it is clear that a closely analogous curve would connect luminosity with mean density, and accordingly a progression through the whole range of densities possessed by cluster-type variables would demand no change in median brightness. Now, if a more or less uniform progression of density with time is a property of all stellar evolution (a natural and almost necessary assumption, if stars are either contracting or expanding), there may come an era in the history of every massive star when no change of magnitude will occur throughout a long interval of time, although the variation of the mean density is making conspicuous progress. Such a condition will result in a maximum in the frequency-curve of luminosity, while it would produce no irregularity in a graph of the frequency of density.[1] Therefore, the perfect counterbalance, at the median magnitude of cluster-type variables, of decreasing surface area with increasing intensity of light-emission, which can account for the flattening out of the luminosity-period curve, may explain at the same time the accumulation of invariable stars about this critical point.[2]

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[1] The small difference in the mean color at the maxima of the two curves is probably larger than the combined zero-point errors in the two color scales.

Assuming, however, on the basis of the theoretical studies of Eddington and Jeans, that the brightness of a giant star is independent of the density and is essentially a simple function of the mass, then the congestion of stars at the median magnitude would indicate a sharp maximum in the frequency of stellar masses. Such an assumption suggests interesting and unexpected consequences in the interpretation of the luminosity-period curve[3] and in the study of the relative ages of stellar systems.

Of the two clusters discussed above, one is unusually rich in typical cluster-type variables but poor in *b*-class stars of the median magnitude; the other is poor in variables but remarkably rich in the number of stars of the median magnitude with negative color-indices. There may be much significance in this compensatory difference, but our material is still too scanty to warrant further hypothesis. Future work must also decide to what extent the cluster phenomena may be modified in their translation to the general stellar system.

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[1] In the third note of the preceding paper we find in Messier 15 neither a change of color nor an appreciable difference in magnitude for the two groups of variable stars which differ decidedly in period and presumably in mean density. The mean periods are about three and five-eighths of a day, respectively, and the mean densities must differ in the ratio of three to one unless two different classes of vibration are involved. How such a change in the effective density could occur (with its supposedly necessary change in surface area) without a distinct alteration either in the total light-emission or in the median color is not obvious. A detailed study of the two groups of variables in red or violet light may supply the answer.

[2] There are large numbers of fainter blue stars in both Messier 3 and Messier 13, as is shown by Table I of this paper and by Tables XXI, XXIV, and XXV of the second paper, *Mt. Wilson Contr.*, No. 116. One is reminded of the differences in mean absolute magnitude found by Kapteyn and Charlier for various groups of galactic B-type stars.

[3] See the first note of the preceding *Contribution.*

Meanwhile, through investigations of composite luminosity-curves for other clusters, we may look into the generality of the maximum at the median magnitude. Lacking color-indices, we have not the power to determine quantitatively the influence of the giant red stars on the composite curves, but we shall be able to extend the work to fainter magnitudes and to learn something of the relative importance of this maximum frequency near absolute magnitude zero.[1]

A summary of the photographic magnitudes in four clusters appears in Table III. Messier 13, which affords a good example of a cluster with few variables, has been remeasured for this table because the data of Table II are too incomplete for the fainter stars. For none of the four systems do the measures extend over the whole photographic plate, the condensed central regions, for instance,

TABLE III

FREQUENCY OF THE MAGNITUDES OF 4015 STARS IN FOUR CLUSTERS

Corrected Scale Reading	Messier 2	Messier 5	Messier 13	Messier 15
<10	2	1	1	17
10.5	0	0	2	12
11.5	1	0	6	17
12.5	3	2	13	39
13.5	6	10	21	62
14.5	19	26	41	100
15.5	46	45	51	151
16.5	76	79	59	148
17.5	95	109	53	161
18.5	94	126	62	148
19.5	84	110	69	161
20.5	79	89	97	186
21.5	92	79	147	212
22.5	122	90	228	266
Total	719	766	850	1680
Median Pg. Scale	17.2	17.7	14.1	15.4
Median Pg. Mag	15.90	15.26	15.03	15.63
Number of variables included	5	70	3	50

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[1] The adopted absolute value of the photographic median magnitude is −0.23; cf. sec. IV of the sixth paper.

being necessarily avoided. Over large representative areas, however, all stars are included so that correct relative frequencies are obtained. Care has been taken that peculiarities which might be associated with galactic planes should not unduly prejudice the results. After correcting for distance from the center and irregularities of the measuring scale, the measures are retained in the original scale readings, the exact evaluation for fainter stars being impossible from the present material. Except for a possible divergence near the ends of the scale, the average value of an interval is one-third of a magnitude.

The relative luminosity-curves are plotted in Fig. 3, each adjusted so that the median magnitude is on the same vertical line. For both Messier 2 and 13 the median photographic brightness is the hypothetical value based on the brighter stars. The sharpness of all the maxima has probably

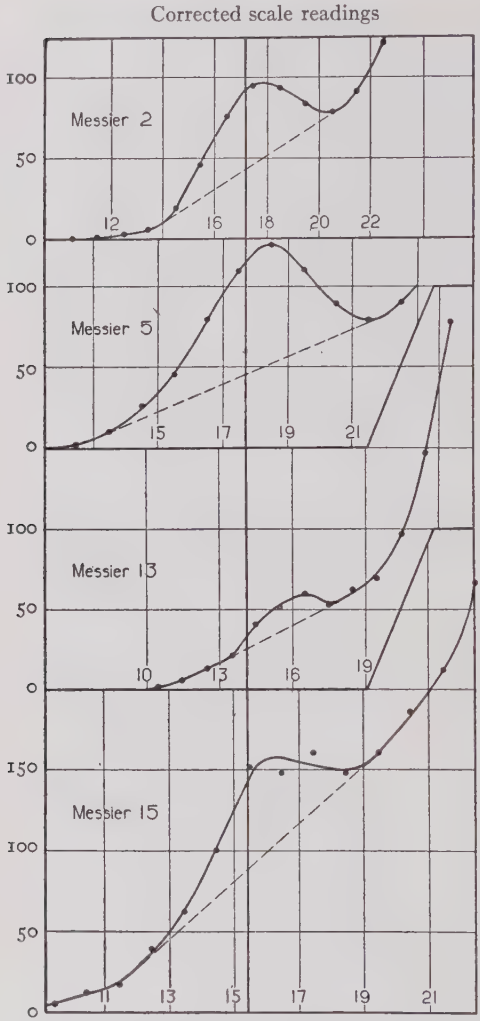

Fig. 3.—Photographic luminosity-curves of stars of all colors in four globular clusters, illustrating the maximum frequency near zero absolute magnitude. The median magnitude for each cluster is indicated by the heavy vertical line. The number of stars measured in different clusters (ordinates) is not proportional to the total number in the cluster.

been somewhat lessened through the process of smoothing out scale irregularities.

In comparing these maxima with the median magnitude it should be remembered that the latter is subject to some uncertainty of measurement and the former to a displacement of unknown amount and direction through the inclusion of the giant red stars. The possible effect of the superposition of the luminosity-curves of the red giants is illustrated in the curve of Fig. 4, where, for Messier 3, the total numbers of Table I are plotted, after transforming them to the photographic system. This curve should be compared with that of Fig. *2a*.

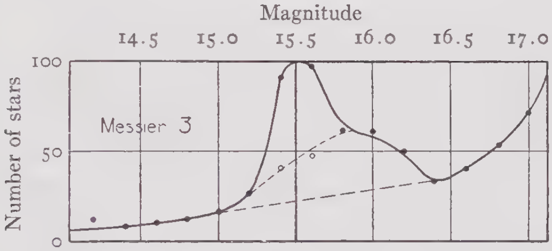

Fig. 4.—Photographic luminosity-curve of stars of all colors in Messier 3. Excluding variable stars we have the broken line with circles.

From the evidence of this series of diagrams there can be little doubt that in all these clusters we have a concentration of the magnitudes of high-luminosity blue stars about the median magnitude of cluster-type variables.

SUMMARY

The absolute photographic magnitude −0.2 defines a limit of stellar luminosity at which the liability to Cepheid variation suddenly stops. All known variables of the class appear to surpass this limiting brightness, which is probably coincident with a turning-point in the development of the internal structure of a star. Whether or not analogous dynamical changes occur, regardless of variability, at the same epoch in the evolution of luminosity for all giant stars, has been discussed for several globular clusters with the aid of luminosity-curves derived from the magnitudes of 5000 stars.

For a graphical representation of the results reference may be made to the figures on the foregoing pages.

Table I affords the complete luminosity-curves for all giant stars in Messier 3 redder than color-class 90. Confirming earlier work, the final results for this cluster show that the highly luminous stars are red, that the fainter ones are blue. The infrequence of cluster-type variables in the Hercules cluster (Messier 13) is compensated for by the richness of extremely blue stars at the median magnitude.

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STUDIES BASED ON THE COLORS AND MAGNITUDES IN STELLAR CLUSTERS

ELEVENTH PAPER: A COMPARISON OF THE DISTANCES OF VARIOUS CELESTIAL OBJECTS

By HARLOW SHAPLEY

The bearing that parallaxes of clusters and variable stars may have on the general structural problems of the sidereal system is well illustrated through a tabulation and diagrammatic representation of the distances of various remote celestial objects. Part of the data compared has been obtained directly or indirectly from the statistical investigations of Kapteyn, Charlier, and Dyson, and from recent trigonometric parallaxes, mainly by van Maanen, but the greater part is from the observations discussed or summarized in the preceding contributions or from similar work that will be reported more fully at another time.

Table I summarizes the distances illustrated in Fig. 1. On the scale of the diagram objects as near as the Hyades or the brighter naked-eye stars can be shown only with difficulty, even when the lines representing distances of clusters are folded many times. The accuracy of the values in the second column of the table differs considerably, and for only a few, which are averages for many objects, is there any meaning in the third significant figure. For some of the results the relative certainty is indicated in the last column, but for most of them reference must be made to the brief discussion of the individual entries to which the remainder of this paper is devoted. Special attention may be called to Section J, which contains a brief discussion of pre-giant stars and the variables of the Orion nebula.

A, 1. *The most distant globular cluster.*—The four most distant clusters now known are N.G.C. 7006, 4147, 6266, and 6316; the adopted parallaxes in millionths of a second are 15, 19, 19, and 20, respectively, corresponding in the mean to a distance of 55,000 parsecs (180,000 light-years). For N.G.C. 4147 the adopted value depends upon measures of both magnitudes and diameter, for the others on diameter alone; but subsequent to the preparation of the seventh paper the parallax of N.G.C. 7006 has been studied by means of the magnitudes of the brightest stars (p. 14, n. 1, of seventh paper), and for the present, at least, its distance seems

TABLE I

COMPARISON OF THE DISTANCES OF VARIOUS STARS, CLUSTERS, AND NEBULAE

Object	Distance (Unit is 100 parsecs)	Remarks
A. Globular clusters:		
1. Most distant cluster	670	N.G.C. 7006
2. Diameter of typical system	1.5	Messier 3
3. Nearest cluster	65	ω Centauri
4. Mean distance	230	69 clusters
5. Maximum $R \sin \beta$	500	N.G.C. 4147
6. Minimum $R \sin \beta$	13	N.G.C. 6656 (M 22)
B. Distance covered in a million years by constant velocity of 1180 km/sec	12	
C. Cepheid variables:		
1. Most distant	60	Three stars
2. Maximum $R \sin \beta$, periods less than a day	29.3	Mean of five
3. Mean $R \sin \beta$, periods less than a day	9.63	45 variables
4. Mean $R \sin \beta$, periods greater than a day	1.48	94 variables
D. Eclipsing binaries:		
1. Most distant	31	Mean of five
2. Mean distance	7.6	90 variables
E. Galactic clouds near Messier 11	50	Four fields
F. Messier 37	40	Preliminary estimate
G. Small Magellanic Cloud:		
1. Radial distance	190	Provisional values from variable stars
2. Approximate diameter	15	
3. $R \sin \beta$	130	
H. Three galactic novae	1.1	Direct measures
I. Nebulae (van Maanen):		
1. Two spirals	2	Direct measures with 60-inch reflector
2. Three planetaries	0.7	
J. Orion nebula (Kapteyn)	1.9	*Mt. Wilson Contr.*, No. 147
K. Hyades (Boss, Kapteyn)	0.4	
L. Most distant naked-eye star	10	
M. B-type stars (Charlier):		
1. Most distant	8.5	Mean of five
2. Dispersion in galactic plane	1.8	751 stars
3. Diameter of system	10.0	Including 95 per cent of stars
N. Distance of sun from center:		
1. Of system of globular clusters	200	69 clusters
2. Of system of B-type stars	0.9	751 stars of Charlier's cluster
O. Width of equatorial segment	35	Arbitrarily chosen; see note 1, p. 5

definitely to be the greatest known. Further work on the colors and magnitudes, to test if this remote system is actually comparable to the nearer globular clusters, is described in the following paragraphs.

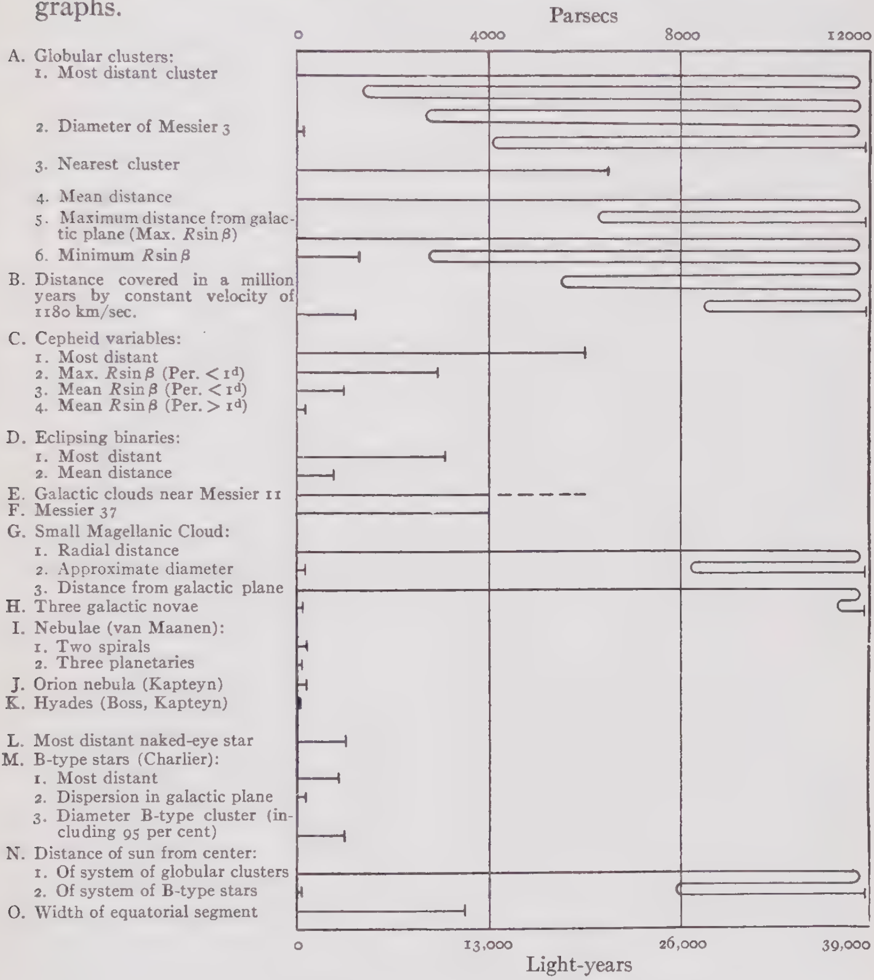

FIG. 1.—Comparison of distances in the galactic system

Since in N.G.C. 7006 the stars of zero absolute magnitude are probably of the nineteenth apparent magnitude, it is very difficult to extend the investigation of photo-visual brightness to any but the most luminous red giants. The extremely small apparent diameter also introduces difficulties of measurement and liability to systematic error if the central part is not carefully avoided.

Table II contains a list of all the stars between 0′.3 and 1′.0 from the center that could be measured on a photo-visual plate of 45 minutes exposure. Probably not more than two or three non-cluster stars are included.

TABLE II

MAGNITUDES OF THE BRIGHT STARS IN N.G.C. 7006

Star	Pg. Mag.	Pv. Mag.	Star	Pg. Mag.	Pv. Mag.
17	17.06	15.76	37	17.19	16.27
18	17.15	15.64	38	17.53	16.51
19	17.13	16.27	39	16.85	15.76
20	17.40	16.42	40	17.97	16.71
21	17.93	16.98	41	18.01	17.37
22	18.01	16.79	42	16.89	15.90
23	17.80	16.95	44	17.89	16.97
24	17.89	16.86	45	17.76	16.76
25	18.01	16.97	46	17.93	16.60
26	17.53	16.32	48	17.89	16.46
27	17.69	16.79	49	17.83	16.57
28	17.31	15.77	50	18.39	17.32
29	16.29	15.26	51	17.49	16.90
30	16.99	16.42	52	17.62	16.51
31	17.58	16.26	53	17.76	16.46
32	16.21	15.71	54	17.26	16.19
33	17.31	15.71	55	18.21	17.01
34	16.87	15.45	56	18.04	16.77
35	17.61	16.90	57	16.99	16.45
36	17.76	16.46			

The results are grouped in Table III according to photo-visual magnitude in order to show, as far as the data go, the typical decrease of color with absolute brightness. Star No. 32, being at some distance from the center and peculiarly blue for its magnitude, is probably not a member of the cluster and has been excluded from the table. Were it retained, the first mean magnitude would be 13.59, with a corresponding color-index of +1.23. The table also contains analogous data[1] for Messier 3 and 13. The frequency of stars of high luminosity is much the same in the three systems; further, the mean color-index is the same for the brighter stars and shows for all clusters a similar progressive decrease. Hence this very distant faint cluster of small angular diameter apparently

[1] One bright star in Messier 13, with color-index −0.52, is excluded.

differs no more in constituency and stage of development from the nearer systems than they differ among themselves. But as far as our present photometric study is concerned, N.G.C. 7006 is nearly two hundred thousand years younger than Messier 13—a significant phenomenon that will be considered further in the following paper.

The evidence of Table III bearing on the question of the scattering of light in space is of considerable interest. Granting the physical similarity of the three clusters, we find no appreciable difference in color on account of the much greater distance of N.G.C. 7006. The total effect of interstellar media on the color-indices of stars in this most distant system apparently does not much exceed a tenth of a magnitude. Therefore the "absorption coefficient," expressed as change of color-index for each parsec of distance, is

TABLE III

Comparison of N.G.C. 7006 with Messier 3 and 13

N.G.C. 7006			Messier 3			Messier 13		
Mean Pv. Mag.	No. of Stars	Mean Color-Index	Mean Pv. Mag.	No. of Stars	Mean Color-Index	Mean Pv. Mag.	No. of Stars	Mean Color-Index
15.56..	5	+1.37	12.59..	7	+1.30	12.11..	6	+1.31
16.02..	6	+1.14	12.90..	5	+1.18	12.47..	7	+1.14
16.41..	7	+1.04	13.10..	7	+1.61	12.72..	5	+0.94
16.55..	6	+1.16	13.43..	9	+1.12	12.87..	6	+0.82
16.82..	7	+0.96	13.70..	7	+0.99	13.05..	6	+0.92
17.08..	7	+0.95	(13.90..	13	+0.96)	13.14..	6	+0.93
16.46..	38	+1.09	13.17..	35	+1.15	12.72..	36	+1.02

$$d < 0.000002 \text{ mag.},$$

a value but one-fifth the upper limit found from the study of nearby globular clusters and a hundredth the smallest value derived from the parallaxes and colors of isolated bright stars. We note, however, that the galactic latitude of N.G.C. 7006 is $-20°$, so that only 8 per cent of the light-path lies within the equatorial segment[1] devoid of globular clusters. If we assume that all the diminution of light through scattering (if there be any at all) occurs within this region where stellar matter appears to be greatly concentrated, the value of d is still sufficiently small to be safely ignored in all ordinary stellar problems.

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[1] The adopted width of the segment is 3500 parsecs; in Figs. 4 and 5 of the seventh paper the shaded area representing the mid-galactic segment is 4000 parsecs wide, but includes five clusters. For Messier 13 one-fourth of the light-path is within this region.

The magnitudes of Table III can be used for a new determination of the parallaxes of Messier 13 and N.G.C. 7006. The parallax of Messier 3, being one of the best determinations for any globular cluster, is adopted for the computation. Assuming that stars of the same absolute magnitude are involved in the final means for each cluster, we derive readily:

Messier 3, $\pi = (0.''000072)$
Messier 13, $\pi = 0.000089$
N.G.C. 7006, $\pi = 0.000016$

The previously adopted parallaxes of the last two clusters are 0.″000090 and 0.″000015, respectively. It is to be noted that the present result is based upon photo-visual magnitudes, and that the five brightest stars of each cluster have not been excluded. The almost exact agreement with the former values merely shows that the similarity in the amount and frequency of color among the giant stars in clusters makes photo-visual as well as photographic magnitude a fairly definite criterion of distance.

A, 2. *The diameter of Messier 3.*—In the seventh paper of this series appears a discussion of the size of globular clusters. The occurrence of cluster stars far beyond the limits shown by the usual photograph has been inferred by Bailey and others from the distribution of cluster-type variables. We have recently found, however, that among galactic stars the variables of this type are peculiar in their occasionally great distance from the galactic plane, and for the five so far investigated the radial velocity is abnormally high. Is it possible that high velocity or some other cause has also scattered the short-period variables of clusters beyond the domain occupied by other cluster stars? To answer this question long-exposure photographs of the brighter systems are being made, with the clusters 15′ or 20′ out of center. The counts so far available show many faint cluster stars as far from the center as any of the variables. The angular diameter of Messier 15, for instance, appears to be not less than 35′, corresponding to a linear diameter of 150 parsecs. These counts indicate, first, that the stars of the outer parts of clusters may be systematically fainter, smaller, and less massive than those close to or at moderate distances from the center; secondly, that the space outside the equatorial galactic region may be occupied not only by occasional cluster-type variables but also by other isolated stellar bodies that possibly are relatively faint; for the time being, however, both suggestions must be regarded as tentative.

A, 3. *The nearest globular cluster.*—The parallax of ω Centauri ($\pi = 0\overset{''}{.}00015$) is uncertain because of the provisional nature of the magnitudes of the variables and bright stars. The adopted parallax of 47 Tucanae (N.G.C. 104) is nearly as large, but it is also lacking in precision, being derived from an uncertain part of the parallax-diameter curve.[1] The distances of both clusters can be very accurately determined when the variables have been further studied, but for the present an uncertainty of some 30 per cent may affect the adopted values. In general the largest as well as the very smallest cluster parallaxes are of relatively low accuracy, depending completely or in part upon the extremities of the curve relating parallax to diameter.

A, 5. *Maximum distance from the plane of the Galaxy.*—Five globular clusters are more than 20,000 parsecs from the galactic plane, but the small faint cluster N.G.C. 4147 in galactic latitude +78° is nearly twice as remote from the dense stellar regions as the next most distant system. Its peculiar position in space suggests the possibility of peculiarity in structure and content—perhaps in this case the methods of determining distance may not be strictly applicable. The adopted parallax appears to be of the right order, however, as the survey of the colors of the brightest objects in the cluster (Table IV) indicates that we are dealing with typical giants; but a deficiency in the number of giant stars is indicated by this table, and long exposures on fast plates suggest a deficiency of faint stars as well. Further, Table II of the seventh paper contains evidence that the angular diameter is somewhat small for the magnitude of the bright stars.

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[1] *Mt. Wilson Contr.*, No. 152, Fig. 1.

This discrepancy between the values of the parallax of N.G.C. 4147 as based on the provisional magnitudes and on the estimates of diameter was considered unimportant until a verification of the earlier value of the mean magnitude was obtained through a recent study of additional plates. It now appears that the adopted parallax may be a few millionths of a second too small, and the distances given in the diagram and in Table I of this paper may be somewhat too large. Because of its high galactic latitude, however, the cluster will probably remain the most isolated of known systems, even when further work on the colors permits a definite revision of the parallax.

TABLE IV

COLORS AND MAGNITUDES OF GIANT STARS IN N.G.C. 4147

Star	Pv. Mag.	Color-Index	Star	Pv. Mag.	Color-Index
1	14.66	+0.99	18	15.74	+0.51
2	15.31	+0.71	22	13.59	+1.83
3	15.62	+0.76	23	15.92	+0.41
5	15.89	+0.08	24	15.33	+0.97
6	14.74	+1.08	29	15.95	+0.70
11	15.92	+0.69	30	15.81	+0.42
13	15.76	+0.52	35	15.74	+0.73
17	15.53	+0.76			

B. *Distance traversed in a million years by maximum velocity.*—The highest speed of translation so far recorded for any celestial object is 1180 km/sec.—the mean radial velocity of the spiral nebula N.G.C. 4594.[1] The corresponding entry in Fig. 1 permits an easy consideration of the possible past and future relationship of the various sidereal bodies. The greatest radial velocity recorded for a globular cluster is about one-third of the foregoing value and for isolated stars about one-half.[2] For discussions of this nature it is convenient that a velocity of a kilometer a second is very nearly equivalent to a velocity of a parsec in a million years.[3]

C. *Cepheid variables.*—The data for Cepheid variables of the galactic system are from Tables I and I *a* of the eighth paper of this series. The three most distant are VX Cygni, RU Boötis, and SW Herculis, the last two being cluster-type variables. The difference in the dispersion with respect to the galactic plane is well illustrated by C. 3 and C. 4 for Cepheids of short and long period, respectively.

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[1] Pease, *Mt. Wilson Communications*, No. 32; *Proc. Nat. Acad. Sci.*, **2**, 517, 1916.

[2] See Table I of the following paper.

[3] More exactly it is 1.02 parsecs.

The maximum $R \sin \beta$ for Cepheids with periods less than a day is the mean of the values for RU Boötis, SW Herculis, 169.1907 Leonis, RR and RU Canum Venaticorum; it is much greater than the adopted semi-width of the equatorial segment that is devoid of globular clusters. The corresponding value of maximum $R \sin \beta$ for periods greater than a day (excluding Z Canum Venaticorum) is 6.70.

D. *Eclipsing variables.*—With a few revisions based upon later orbital computations, the data for eclipsing binaries are taken from a paper by Russell and Shapley.[1] The five most distant variables are W Crucis, SS Carinae, UZ Cygni, RV Persei, and VV Cygni. The quantity designated as the average distance in the diagram in the *Publications of the Astronomical Society of the Pacific* for February, 1918, is the distance corresponding to the mean parallax—a quantity that, because of the wide range of values, is less than half the mean distance now computed.[2]

The parallaxes of eclipsing binaries, even of those with well-determined light-curves and spectra, are somewhat less accurately known than the parallaxes of average Cepheid variables. They depend upon the apparent magnitude, mass, surface-brightness, and radius of the brighter component of the eclipsing pairs, and in the evaluation of some of these quantities approximations are necessary. For the most favorable cases, however, the estimated error in the hypothetical parallaxes is not in excess of 25 per cent, and is essentially independent of the distance.

Direct measures of the parallax of several eclipsing stars, while yielding results of the same order of magnitude as the probable errors, serve to confirm the hypothetical values. Nearly all of the direct determinations have been made by various observers at the

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[1] *Astrophysical Journal*, **40**, 417, 1914.

[2] In addition to the inclusion of more objects, the present diagram differs from the earlier one in several minor details.

Yerkes Observatory;[1] a summary of their results is given in Table V.

On the average the hypothetical parallaxes differ from the trigonometric values (corrected for comparison-star parallax by means of the Greenwich table)[2] by less than the probable error of the direct measures; systematically they are 0.″002 larger than the trigonometric results. The mean trigonometric parallax of β Persei, using all sources, is +0.″034±0.″011. For β Aurigae, an eclipsing binary not on the Yerkes list, the mean value of the parallax by three observers is +0.″034±0.″015; the hypothetical value is +0.″030, and if the star is a member of the Ursa Major group its parallax is +0.″025.[3] By entering these two mean values in Table V the small systematic difference disappears.[4]

TABLE V

COMPARISON OF HYPOTHETICAL AND TRIGONOMETRIC PARALLAXES OF ECLIPSING BINARIES (YERKES RESULTS)

Star	Measured by	Trigonometric Parallax (Abs.)	Probable Error	Hypothetical Parallax	Difference
β Persei	Lee	+0.″025	±0.″014	+0.″026	−0.″001
W Ursae Majoris	Lee	+0.011	±0.010	+0.020	−0.009
U Ophiuchi	Joy	−0.008	±0.004	+0.006	−0.014
Z Herculis	van Biesbroeck	+0.031	±0.009*	+0.009	+0.022
RX Herculis	van Biesbroeck	−0.006	±0.016	+0.006	−0.012
Y Cygni	Mitchell	+0.004	±0.010	+0.004	0.000
Mean			±0.0105		±0.0097

* The probable error is given as ±0.″010 in the collected results, p. 57, of *Yerkes Publications* 4, Part I.

As a class the eclipsing variables are too remote for direct measures of parallax, but the foregoing comparison for the nearer ones shows that the method based upon orbital data is adequate to locate them in space with considerable certainty.

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[1] *Publications of the Yerkes Observatory*, **4**, Part I, 1917.

[2] Dyson and Thackeray, *Monthly Notices*, **77**, 14, 1916. The Greenwich values should all be decreased by 10 per cent if instead of 19.5 km/sec. we adopt for the velocity of the sun the value given in Sec. 2 of the sixth paper. The correction is unimportant in the present case.

[3] Erroneously printed as +0.″019 in *Astrophysical Journal*, **40**, 425, 1914.

[4] *Note added to proof.* An overlooked parallax of u Herculis by Miller: Trig. π (abs.) = −0.″026, hyp. π = +0.″005.

E. *Galactic clouds.*—The study of the colors of stars near Messier 11 indicates the great extent of the galactic clouds in the line of sight;[1] the maximum distance so far estimated depends only on the limit of magnitude attainable on the photo-visual plates. Photographs of the field of Messier 11 are now available for the extension of this work to the fainter and more remote stars. It appears very probable, from the preliminary results, that isolated stars as distant as 30,000 parsecs may be found near the galactic plane in these southern star clouds.

F. *Messier 37 (N.G.C. 2099).*—This northern object, one of the richest of the bright open clusters, is situated near the galactic plane, almost diametrically opposite the point of concentration of globular clusters. A catalogue, now in manuscript, of the magnitudes and colors of several hundred of its stars will be published later as a paper of this series.

G. *Magellanic Clouds.*—The adopted parallax of the Small Magellanic Cloud is that given in Table I of the seventh paper—a value undoubtedly of the right order but one which will be replaced by a value of much greater weight when the magnitudes of the variables have been referred to a standard system. The adopted mean angular diameter is $4\overset{\circ}{.}5$, representing the region throughout which the variable stars belonging to the cloud are distributed. We infer from the similarity of the provisional magnitudes of the variables that the distance of the two Magellanic Clouds is of the same order, but the Larger Cloud covers more than double the area.

H. *Galactic novae.*—The three objects for which directly determined parallaxes are available are Nova Persei No. 2, Nova Lacertae, and Nova Geminorum No. 2. The derivation of the adopted mean value of $+0\overset{''}{.}009$ will be discussed by van Maanen in a forthcoming contribution.

I. *Spiral and planetary nebulae.*—The direct measurement of the parallaxes of nebulae will be considered by van Maanen in the same paper. He attaches little weight to the measured results for the spirals because of the non-stellar character of the central nuclei. One of the spirals is the Great Andromeda Nebula (N.G.C. 224);

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[1] The fifth paper of this series, *Mt. Wilson Contr.*, No. 133, 1917.

the other is Messier 51 (N.G.C. 5194). The planetaries are N.G.C. 2392, 6720, and 7662. The adopted parallax of N.G.C. 6720, the ring nebula in Lyra, is the mean of the values by van Maanen and by Newkirk.

J. *The Orion nebula.*—On the basis of his exhaustive study of the motions of the B-type stars, Kapteyn adopts as the parallax of the "nebula-group" in Orion:

$$\pi = 0\overset{''}{.}0054 \pm 0\overset{''}{.}0009.$$

That the same parallax is the best available for the nebula itself can hardly be doubted.

It does not seem to have been pointed out that if the numerous variables of the Orion nebula are a part of the nebula-group we have in them a remarkably large assemblage of stars of low luminosity. Of more than 100 stars within two or three degrees of the trapezium that are supposed to be definitely variable, nearly all are fainter than the fourteenth photographic magnitude at maximum, and many never become brighter than the fifteenth magnitude. With a parallax of $0\overset{''}{.}005$ the average absolute brightness at maximum is only about a tenth that of the sun, and at minimum less than a twentieth. Apparently they are dwarfs, while all other variable stars—Cepheids, eclipsing binaries, long-period variables (at maximum)—are typically objects of high luminosity.

The possibility that in the faint variable stars of Orion, which appear to be intimately associated with diffuse nebulosity, we have examples of stars in the pre-giant stage of stellar evolution, led to the initiation, nearly four years ago, of systematic observations of their magnitudes and colors. Such variables might be of considerable importance, for if Russell's theory of the evolutionary sequence of spectral types be accepted and we desire to find a record of the parentage and earliest development of the relatively young, highly luminous, low-temperature and low-density red giant stars, it seems obvious that we should look for (1) objects varying in light secularly or irregularly rather than with definite period and amplitude; (2) objects closely associated with the matter from which typical stars originate—nebulosity, we usually say, that is formless or in a quite chaotic state; (3) objects of faint absolute magnitude and low-surface temperature, therefore very red and possibly with striking peculiarities of spectrum. At least some of these properties are naturally thought of as attributes of the pre-giant stage; and the Orion variables appear upon first examination to be promising subjects for inquiry.

If, however, we cannot find satisfactory evidence of pre-giants among such stars as the Orion variables, the typical long-period variables, or the occasional irregular variables that are associated with variable nebulosity, there remain at least three ways (before we need abandon Russell's theory) in which we may attempt to account for the absence from our present records[1] of recognized visible forerunners of the low-density red giants. We may suppose (1) that the normal transformation of an invisible nebular mass to a highly luminous star is infrequent as compared with the rapidity of the change; astronomical history is so brief, on that hypothesis, that it contains as yet no definite record of a secularly brightening red star or of the occurrence of a red nova that maintains its maximum; or (2) that the phenomenon of rising luminosity, and hence of the birth of a typical giant star, no longer occurs, signifying that the stellar material is essentially exhausted in the sidereal universe we know; or (3) that the part of space where the phenomenon now occurs is remote from the region we ordinarily observe. Or stated more briefly we may suppose that the (1) frequency, (2) epoch, or (3) place of star-birth makes difficult or impossible our observation of the pre-giant stages.

The photographic observation of the magnitudes of the variables in Orion is very difficult because of the irregularity of the nebulosity and the uncertainties associated with the photographic study at Mount Wilson of southern objects in the winter season. Whether definite periodicity obtains for any of the variables has not yet been determined. Some of the stars previously suspected of variability may owe their apparent variations entirely to photographic phenomena; others at one time definitely variable now appear constant. The results so far derived for thirty variable stars near the trapezium are mainly negative: (1) the variables do not belong to the cluster-type; (2) they are not blue (apparently they are mainly red, but the nebulosity introduces systematic error of undetermined amount into measures of color); (3) they do not appear to show regularity, or great range, or close similarity to each other in color or range; (4) they are, of course, not undergoing a secular increase of brightness apparent within this short interval of time; (5) the variables as well as other faint stars are concentrated in the central part of the nebula; (6) in the dense nebulosity the light of very few faint stars appears to be actually constant—perhaps a spurious photographic effect.

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[1] The records of some globular clusters, and probably of the solar neighborhood as well, are sufficiently complete to show that such antecedents are not present as normal stars.

It seems impossible to say definitely as yet whether the variables are really associated with the nebula-group of bright stars and undergo intrinsic variations of light or are for the most part at a greater distance than the Orion stars and owe their variations to occultations by nebular matter. Most of the evidence, however, supports the view that the variables are typical dwarf stars within the nebula, deriving their irregular light-variations from contact with the irregular nebulosity, which recent spectroscopic work has shown to be moving differentially. Some strong points favoring this view may be inferred from the preceding paragraph, and together with some possible objections they will be more fully discussed when a detailed report is made on the magnitudes and colors. The similar, commonly accepted hypothesis of nebular friction seems to account satisfactorily for the novae, where a much greater range of variation is involved.

L. *The most distant naked-eye star.*—The adopted lower limit of the parallax, $\pi = 0\overset{''}{.}001$, of a hypothetical most distant naked-eye star depends upon the fairly safe assumption that the difference between apparent and absolute brightness, $m-M$, does not exceed ten magnitudes; that is, we assume that no first-magnitude star is brighter absolutely than -9, that no fifth-magnitude star is brighter absolutely than -5. The study of the giants in clusters, Kapteyn's work on B-type stars, and the results of direct measures of parallax support this assumption. The most relevant contribution to this question, however, is the very important statistical discussion by Dyson of the proper motions of one-sixteenth of all the stars brighter than the ninth magnitude.[1] He finds that brighter than the sixth magnitude there are in the whole sky only 15×16=240 stars more distant than 200 parsecs. The extreme limit[2] of 260 parsecs, to which naked-eye stars extend according to Dyson's formula, is only one-fourth of the limit assumed for the present illustration. We may note, however, that if a naked-eye Cepheid variable fainter than the fifth apparent magnitude should be found with period in excess of 32 days, its parallax would be somewhat smaller than the lower limit adopted above.

M. *Charlier's "Galaxy of B stars"*.—A discussion of the distribution in space of all B-type stars in the Harvard catalogues has been made by Charlier,[3] who finds that they form a large cluster, the center of which is nearly 100 parsecs distant from the sun. As practically all the stars of all spectral types for which we know the position in space are within the bounds of this B-type cluster, it is chosen, in this comparison with the system of globular clusters, to represent the stellar system as ordinarily conceived.

N. *The eccentric position of the solar system.*—In the seventh paper it has been noted that the adopted distance to the center of the system of globular clusters is tentative, but probably of the correct order of magnitude.

Confirming Charlier's determination, either of the general direction or of the amount of the sun's displacement from the center of the local aggregation of stars, we have the results of Russell and Shapley on eclipsing binaries and Cepheids, of Walkey on 30,000 stars of the Harvard catalogues, and of Strömberg on second-type stars of known parallax and radial velocity.

SUMMARY

This paper contains a tabular and diagrammatic comparison of various sidereal distances. In addition to the material already available from the study of clusters and variables and from the results of other investigators, a considerable amount of new observational and computational work has been necessary for the discussion. The following statement summarizes the more important results, some of which are not specifically discussed in the text of the paper.

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[1] *Monthly Notices*, **77**, 212, 1917.

[2] Dyson notes that the limit is too small; *ibid.*, p. 217.

[3] *Meddelanden från Lunds Astronomiska Observatorium*, Series 2, No. 14, 1916.

1. The most distant globular cluster now known, N.G.C. 7006, appears to differ from much nearer systems only in the matter of distance. In general appearance and in the relation of size to brightness it conforms with typical globular clusters; and in phenomena of color and the frequency of red giant stars it is very similar to Messier 3 and 13 (Table III).

2. The similarity in the frequency of colors for near and distant clusters shows that the selective scattering of light in space, if acting uniformly, affects the color-indices of stars by less than two-millionths of a magnitude for each parsec of distance. With any reasonable assumption as to the dependence of light-scattering on stellar concentration, interstellar media appear unimportant in their effect on the colors of stars brighter than the fifteenth magnitude.

3. The parallax of N.G.C. 7006 is 0.″000015 on the basis of diameter measures, 0.″000014 according to measures of photographic magnitude, and 0.″000016 if the mean absolute photo-visual magnitude of its brightest stars be assumed equivalent to the corresponding means for Messier 3 and 13.

4. Counts of stars on photographs of the edges of globular clusters show that faint stars as well as the typical variables are widely dispersed. Probably the diameter of every typical globular cluster exceeds twenty million astronomical units.

5. As far from the center as 15′, corresponding to a linear distance of about 65 parsecs, the elongation of Messier 15 is found to be in general agreement with the results for the central regions, both as to direction and amount.

6. The cluster N.G.C. 4147, which, of all known, is by far the most distant from the galactic plane, may differ considerably from typical globular systems in frequency of giant stars and perhaps in linear extent. A revision of the photographic magnitudes in this cluster alters the previously adopted mean of the "25 brightest" stars by only five-hundredths of a magnitude.

7. A comparison of the direct and hypothetical parallaxes of seven eclipsing binaries show an average difference of ±0."01; the systematic difference is inappreciable.

8. Most if not all of the variable stars in the Orion nebula appear to be irregular in period and range; the color-indices of thirty situated in the denser nebulosity are large and positive. That the variables are actually associated with the nebula and therefore of faint absolute magnitude is not yet definitely proved.

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STUDIES BASED ON THE COLORS AND MAGNITUDES IN STELLAR CLUSTERS

TWELFTH PAPER: REMARKS ON THE ARRANGEMENT OF THE SIDEREAL UNIVERSE

By HARLOW SHAPLEY

I. THE GENERAL GALACTIC SYSTEM

1. *Introduction.*—A fairly definite conception of the arrangement of the sidereal system evolves naturally from the observational work discussed in the preceding *Contributions*. We find, in short, that globular clusters, though extensive and massive structures, are but subordinate items in the immensely greater organization which is dimly outlined by their positions. From the new point of view our galactic universe appears as a single, enormous, all-comprehending unit, the extent and form of which seems to be indicated through the dimensions of the widely extended assemblage of globular clusters. The fundamental nature of the galactic plane, in the dynamical structure of all that we now recognize as the sidereal universe, is manifested by the distribution of clusters in space. Near this plane lie the celestial objects that we customarily study. The open clusters, the diffused and planetary nebulae, the naked-eye stars, most variables, the objects that define and compose the star streams—all of these appear to be far within a relatively narrow equatorial region of the greater galactic system, a region in which such forces are at play that compact clusters of great mass apparently cannot form or exist. The Orion nebula and even the Magellanic clouds are miniature organizations in this general scheme, and undoubtedly are dependents of the Galaxy.

The adoption of such an arrangement of sidereal objects leaves us with no evidence of a plurality of stellar "universes." Even the remotest of recorded globular clusters do not seem to be independent organizations. The hypothesis that spiral nebulae are separate galactic systems now meets with further difficulties. So long as the high velocities of nebulae were unapproached by the motions of other objects and the maximum luminosity attainable by stars was beyond estimate, and so long as the diameter of the galactic system was thought to be only a thousand light-years or so, we had a fairly plausible case for the "island universe" hypothesis. But now we must consider radial velocities of several hundred kilometers a second as quite possible for objects in our own system; we must assume a moderate upper limit of luminosity, perhaps even for the most massive of novae; and any external "universe" must now be compared with a galactic system probably more than three hundred thousand light-years in diameter. As seen from the center of the galactic system, globular clusters would be distributed in the sky much as the spirals are when observed from the earth.

It is probable that the further accumulation of observations will modify to some extent the views outlined above and discussed more fully in the following pages. The present data may in some cases be susceptible of alternative interpretation, or possibly the conclusions may be questioned in the belief that the material is insufficient. But the greater part of the hypothesis proposed is merely the most direct and simple reading of recent observations.

2. *Outline of interpretation.*—The suggested plan of the galactic system may be concretely formulated through the following series of propositions; some of them are later amplified in so far as seems necessary; for others the discussion of preceding contributions will suffice. A few of the statements are obvious corollaries, while those designated B and F, in some of their details, may be less easy to maintain. Taken altogether they attempt to establish a general idea of the arrangement, extent, and constituency of the system of stars and nebulae.

A. The globular clusters are a part of the galactic system and knowledge of their distances seems at present to afford the best way to fathom the system.

B. The system of globular clusters, which is coincident in general, if not in detail, with the sidereal arrangement as a whole, appears to be somewhat ellipsoidal. The longest axis of the ellipsoid lies in the galactic plane and passes the sun at a distance of approximately three thousand parsecs. Its nearest point is in galactic longitude 240°, nearly coincident with the direction of the center assigned to the local system of stars. See F and Fig. 1.

C. The center of the sidereal system is distant from the earth some twenty thousand parsecs in the direction of the constellation Sagittarius; it lies in the galactic plane, which dynamically and statistically appears to be the symmetrical plane of the entire sidereal universe as now known. As seen from the sun the thinnest part of the Milky Way lies in Gemini, Taurus, and Auriga—a region rich in bright open clusters close to the galactic plane.

D. The axes of the system in the galactic plane and perpendicular to it may not differ greatly; but the gravitationally important equatorial segment, which apparently contains most of the stars, is at least thirty times as extended in the plane as at right angles thereto.

E. The equatorial region appears to be uninhabitable by compact systems, such as globular clusters, notwithstanding the greater abundance there of stellar material.

F. The stars in the neighborhood of the sun (practically all that go into our catalogues of spectrum, position, and motion) appear to compose (1) a large, open, moving subordinate group, and (2) a part of the surrounding and interpenetrating star fields of the equatorial segment of the greater galactic system. The center of the local system is in the direction of the constellation Carina, nearly at right angles to the direction of the center of the general galactic system, but less than one two-hundredths as far away. The plane of symmetry and condensation of the local cluster is inclined to the galactic plane about 12°; the center of the cluster is north of the galactic plane, and the sun is north of both planes.

G. The volume of space occupied by stars brighter than the sixth apparent magnitude, some of which, being absolutely very bright, are extremely distant as compared with the majority of naked-eye stars, is at most only a hundred-thousandth of the volume occupied by the other parts of the galactic system.

3. *Relation of present interpretation to earlier hypotheses.*—In order to show where the earlier working hypotheses stand with

respect to the interpretation now offered, it may be of interest to note the development, during the course of this work on clusters and variable stars, of the ideas concerning the relation of globular clusters to the galactic organization. Until the last year or so most students of stellar problems believed rather vaguely that the sun was not far from the center of the universe, and that the radius of the galactic system was of the order of 1000 parsecs. From the earlier observational data Seeliger and Newcomb derived a fairly central position for the sun. Hertzsprung[1] in 1906 estimated the "Dimensionen" of the visible Milky Way system to be of the order of 2000 parsecs, and some years later Walkey,[2] from consideration of extensive distributional data, estimated a distance of about seventeen hundred parsecs for the galactic main stream. In 1914, referring to the apparently lens-shaped sidereal system, Eddington[3] wrote, "There is little evidence as to the sun's position with respect to the perimeter of the lens; all that we can say is that it is not markedly eccentric"; and the diameter of the whole system (possibly excluding the peripheral ring of galactic clouds) was placed at some two or three thousand parsecs, with emphasis on the uncertainty. For a later computation Eddington[4] assumed the distance of the Milky Way to be 2000 parsecs.

The work on the hypothetical parallaxes of Cepheids and O-type stars by Hertzsprung, and of eclipsing binaries and Cepheids by Professor Russell and the writer, began to give concrete numerical expression to the distances of remote galactic objects, and in 1914 we have the statement:[5] "Our 'universe' of stars must be some thousands of light-years in diameter," but the computed radius of 2500 parsecs was reduced to 1200 by allowing for a presumably reasonable and necessary scattering of light in space. The necessity for such a correction seems now definitely to have vanished, but the general conception of the size of the stellar system has not materially changed. In his valuable memoir[1] on the B-type stars Charlier states:

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[1] *Zeitschrift für wissenschaftliche Photographie*, **5**, 106, 1907.

[2] *Monthly Notices*, **74**, 655, 1914.

[3] *Stellar Movements and the Structure of the Universe* (London, 1914), p. 32.

[4] *Ibid.*, p. 261.

[5] Russell and Shapley, *Astrophysical Journal*, **40**, 434, 1914.

> The luminosity of these stars is, indeed, so great that a star of this type, situated—as far as can be concluded—at the limits of our stellar universe, is scarcely fainter than the eighth magnitude. We are thus in the position to get, with the help of the B stars, what might appropriately be called a skeleton image of the Milky Way.

Only one star out of the 800 in Charlier's list is more distant from the sun than 800 parsecs, and a heliocentric sphere of 500 parsecs radius contains 95 per cent of the total number. Again, Charlier states that "the center of this cluster [of B-type stars], which may be assumed to coincide with the center of our stellar universe, is situated . . . . in the Constellation Carina" at a distance of 88 parsecs.[2]

In view of these prevailing beliefs, the working hypothesis that a globular cluster is a wholly distinct stellar system was quite appropriate when in 1915 it was found that the Hercules cluster (Messier 13) is possibly several hundred parsecs in diameter. It appeared then that the galactic system might be a large but subordinate unit, eccentrically situated with respect to the greater aggregation of globular clusters. The great distances of the clusters, the similarity of their stars, in many properties, to those of the Galaxy, and finally the discovery of galactic planes in a number of them, tended to emphasize further the comparability of clusters and our own surrounding galactic system of stars.

Two difficulties stood in the way of a definite hypothesis—the comparatively high condensation at the centers of globular clusters and the uncertainty relative to the constituency and distances of open clusters in the galactic clouds. The study of Messier 11 and of the dense star clouds in its neighborhood revealed the presence of faint blue stars—objects which, unless abnormal in size, must be quite similar in absolute magnitude to members of Charlier's cluster of B-type stars, but at least twenty times as distant as the limits of his group.

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[1] *Meddelanden från Lunds Astronomiska Observatorium*, Series 2, No. 14, p. 103, 1916.

[2] *Ibid.*, p. 104

Evidence was also found that the Milky Way clouds near Messier 11 were greatly extended in the line of sight. For a number of other parts of the Milky Way the study of stars in clusters and clouds indicated analogous conditions, and the conviction grew that the galactic system had an extent of at least 15,000 parsecs along its plane. This left little occasion for the direct comparison with globular clusters, the diameters of which were found by further study to be of the order of 150 parsecs. As a consequence, their relation to the general system was quite uncertain until the present determination of parallaxes and the discussion of the distribution in space indicated the position of globular clusters in the arrangement of sidereal objects and suggested that the actual diameter of the galactic system is of the order of 100,000 parsecs.

4. *The plane of symmetry and the equatorial segment.*—In the figures and discussion of the seventh paper of this series the dependence of globular clusters upon a larger symmetrical organization is definitely shown. Apparently there is no occasion to doubt the identity of the plane of symmetry in this system of globular clusters with the galactic plane defined by stellar condensation and the Milky Way. An agreement within a degree or two of the poles of two distinct and unrelated planes is far too unlikely for consideration. Further, we now know that in distance along the plane many of the globular clusters are nearer to us than stars in the open galactic clusters; and also we note that the distribution of Cepheid variables, as shown in Fig. 3 of the eighth paper, bridges the gap between the distant Milky Way clouds and the local system of stars.

That the equatorial segment is populated by stars throughout its whole extent seems very probable; both the arrangement of the clusters and the appearance of the Milky Way agglomerations support this view. But, owing to the evidence of rifts and divisions in the galactic clouds, it is impossible to suppose uniform stellar distribution or even to assume that the stars everywhere are concentrated progressively toward the medial plane. Stars of the galactic branch in Ophiuchus do not necessarily lie farther from the plane than the semi-width of the equatorial segment, for to be outside this region devoid of globular clusters the parallax of the stars in the cloud must be less than 0.″0001. Globular clusters that appear involved in the edges of the star clouds are almost without exception small and faint, and because of their greater radial distances are therefore clear of the equatorial region that contains the majority of known sidereal objects.

5. *The Milky Way and its asymmetry; regions of maximum star density.*—According to the present view of the galactic system, the phenomenon of the Milky Way is largely an optical one. Although the existence of local and occasionally very extensive condensations of Milky Way stars is not denied, the conception of a narrow encircling ring is abandoned. The Milky Way girdle is chiefly a matter of star depth, and its long recognized weakness between longitudes 90° and 180° is now taken to be a reflection of the eccentric position of the sun.

On the basis of the third and fourth diagrams of the seventh paper we estimate provisionally that the limit of the Galaxy is three times greater in longitude 325° than in the opposite direction. This does not require an impossible difference of stellar density in the two directions, even if there is a considerable condensation toward the center. A star of a given absolute luminosity situated in the galactic plane would appear less than two and a half magnitudes fainter at the boundary of the system beyond the center than at the opposite point, which is nearest the sun. The remarkable one-sidedness of the Milky Way has been little considered heretofore in works on stellar distribution. Nort,[1] in studying the Harvard map, has made an important beginning by showing that the star density is four or five times greater in the direction of the southern star clouds than in some of the shallower galactic regions of the north.

The surpassing stellar density in the direction now assigned to the center of the galactic system is particularly remarked by Chapman and Melotte[2] in their study of the Franklin-Adams plates. They state that one plate with center in $\alpha = 18^h$, $\delta = -30°$

> covers the Sagittarius region of the Southern Milky Way, and the star clouds on limited portions of it are so thick that in the case of twelve out of the

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[1] *Recherches Astronomique de l'Observatoire d'Utrecht*, **8**, 113, 1917.

[2] *Memoirs of the Royal Astronomical Society*, **60**, Part IV, p. 168, 1915.

twenty-five areas counted on it, it was found impossible to count every star shown; the images of the faintest stars in these regions merged into one another forming a continuous gray background. On every other plate of the Franklin-Adams series even the faintest star images shown were separate and distinct, and the counts included all stars visible. The extreme richness of the Sagittarius region may be judged of, then, when it is noticed that the incomplete counts on it show far more stars than are found in any other part of the Milky Way.

The fathoming of the sidereal universe need not long depend on globular clusters alone. If the nearest part of its boundary in the general direction of Auriga and Gemini is not more distant than 30,000 parsecs, no stars in that locality with absolute magnitude of zero or brighter will be fainter than the apparent magnitude 17.5. B-type stars will therefore contribute in future measurement of the extent of the system; and the Cepheid variables fainter than the fourteenth magnitude will in time be fully as valuable as the globular clusters in outlining the diameter and contour of the equatorial segment. As a ready qualitative check of the direction and distance of the center, the blue stars in the Milky Way should persist to a fainter magnitude in the southern sky than in the direction of the anti-center.

The possibly ellipsoidal form of the system of globular clusters is indicated in Fig. 1, which gives a projection on the galactic plane of the 60 clusters for which $R \sin\beta < 15{,}000$ parsecs. If the elongation be accepted as a real characteristic of the stars also, it is evident that the apparently densest star regions, depending on the faintness of the stars involved in the estimate, may lie in a longitude differing considerably from that of the center. The general direction of the galactic center is clearly toward the dense star clouds of Sagittarius and Scorpio; but the adopted galactic longitude, 325°, and the corresponding equatorial co-ordinates of the center, $\alpha = 17^{h}.5$, $\delta = -30°$, are necessarily approximate.

The statistical center derived by Charlier[1] from B-type stars is in Carina, in longitude 236°, a result referring entirely to the local group (within 500 parsecs of the sun) and not influenced by the arrangement of the general system. Strömberg,[2] from bright stars

[1] *Meddelanden från Lunds Astronomiska Observatorium*, Series 2, No. 14, 1916.

[2] *Astrophysical Journal*, **47**, 33, 1918.

of the redder spectral types, finds the dynamical center in longitude 257°. Nort,[1] using stars to the eleventh magnitude on the Harvard map of the sky, gets farther outside the bounds of the local cluster and obtains a maximum stellar density in the Milky Way between longitudes 280° and 290°; he finds a density but one-fifth as great in longitude 120°, the direction of the anti-center. Chapman and Melotte,[2] working to the still fainter limit of the Franklin-Adams plates, find in the clouds of Sagittarius the only region too dense for counting.

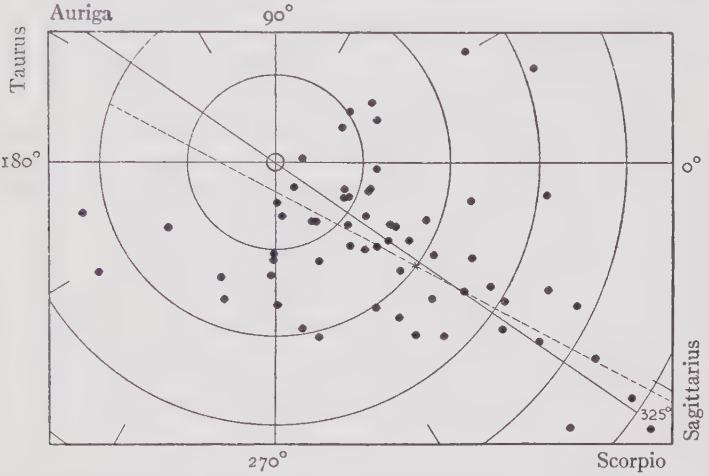

FIG. 1.—The system of globular clusters projected on the plane of the Galaxy. The galactic longitude is indicated for every 30°. The "local system" is completely within the smallest circle, which has a radius of 1000 parsecs. The larger circles, which are also heliocentric, have radii increasing by intervals of 10,000 parsecs. The dotted line indicates the suggested major axis of the system, and the cross the adopted center. The dots are about five times the actual diameters of the clusters on this scale. Nine clusters more distant from the plane than 15,000 parsecs are not included in the diagram.

This progressive increase of the longitude of maximum star density from 236° to 325° (with the increasing predominance of the general system over the local group), and the appearance to be expected of the star clouds in the directions of the two centers, are both in striking agreement with Gould's observations of the brightness of the Milky Way:[1]

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[1] *Op. cit.*

[2] *Ibid.*

> Its brightest portion is unquestionably in Sagittarius [the galactic center]; that in Carina [the local center] being slightly inferior to this as regards intrinsic brilliancy, although far more magnificent and impressive on account of the great number of bright stars with which it is there spangled.

6. *Absence of clusters from the equatorial segment and its dynamical significance.*—In an earlier paper we have remarked that the absence of globular clusters from the mid-galactic region is revealed by the galactic latitudes as well as by the distances from the plane. For the latitudes, however, the wide avoidance of that region is not so evident, because of the apparent nearness to the plane of some very remote systems. The relative frequency of galactic latitudes, plotted in Fig. 2, shows an equatorial belt 10° in width, which is as yet completely empty of known globular clusters.

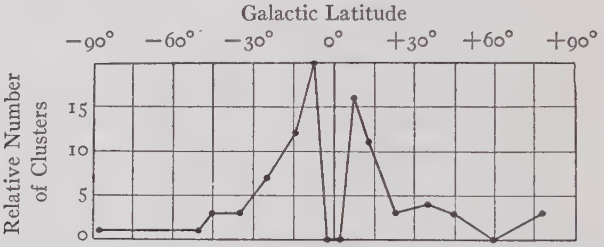

FIG. 2.—Frequency of galactic latitudes, illustrating the mid-galactic region devoid of globular clusters.

It is clear from the figure that any reasonable change in the adopted position of the galactic pole would neither eliminate the segment nor modify its coincidence with the mid-galactic stellar region. It is equally obvious that the region cannot be explained away by supposing the data incomplete through either the failure to record visible globular clusters or the obstruction of light in the equatorial segment. In support of this statement it will suffice to point out that already at a distance of 12,500 parsecs from the sun the absence of clusters is more than chance. Then, first, the suggestion that clusters have been overlooked is absurd, for some of the stars in globular systems at that distance would be photographically brighter than the fourteenth magnitude. That is, the overlooked clusters would be more conspicuous than Messier 3. In the second place, if general obstruction of light is to account for the absence of clusters, typical stars with negative color-indices should not be found in the Milky Way when fainter than the fifteenth magnitude, for at the distance of 12,500 parsecs $m-M=+15.5$. But such objects have been observed; and, moreover, it is highly improbable that the millions of Milky Way stars fainter than the eighteenth magnitude are all dwarfs.

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[1] *Uranometria Argentina*, p. 370, 1879.

We have referred to the equatorial segment as a region of avoidance. Slipher's observations of radial velocities suggest that the term is scarcely appropriate, since seven out of ten systems are approaching the sun (and probably the galactic region) with conspicuous velocity.[1] The segment seems rather to be a region of attraction and demolition, and if further work on radial velocities confirms the present indication of systematically high speeds of approach, a factor of very great importance in the evolution of the galactic system will be established.

Table I contains information relative to the ten clusters of measured radial velocity. The observed values in the third column and the computed quantities based upon them in the last column are admittedly provisional. The observed photographic magnitudes in the fourth column, taken from Table II of the seventh paper, show that two of the largest velocities pertain to the faintest and probably most difficult objects. The last column gives the observed radial velocity, V_p (expressed in the unit of 100 parsecs per million years), divided into the radial distance.

The present evidence suggests that globular clusters as a class may be falling rapidly into the regions rich in stars. None of the clusters in Table I is necessarily receding from the plane, for the positive velocity of Messier 5 is far within the uncertainty of the observation, and Messier 9 and 28 are in such low galactic latitudes that the radial component is no test of the true direction of motion perpendicular to the Milky Way. With the possible exception of Messier 5, therefore, all these clusters, with latitudes high enough

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[1] *Popular Astronomy*, **26**, 8, 1918.

to make it probable that the radial and perpendicular components have the same sign, are approaching the galactic plane with a speed that unless rapidly diminished will bring most of them to the dense stellar regions in less than 150 million years (cf. last column of Table I). Such an interval of time seems relatively short in the history of a stellar system, and hence these negative velocities make still more remarkable the absence of globular clusters from the equatorial segment.

TABLE I

RADIAL VELOCITIES OF GLOBULAR CLUSTERS

CLUSTER		RADIAL VELOCITY	MEAN PG. MAG. 25 BRIGHTEST STARS	GALACTIC		DISTANCE (UNIT IS 100 PARSECS) R	DISTANCE FROM GALACTIC PLANE $R \sin \beta$	$\frac{R}{V_p}$ (UNIT IS A MILLION YEARS)
N.G.C.	Messier			Long. λ	Lat. β			
		km						
5024...	53	−170	15.07	307°	+79°	189	+186	110
5272...	3	−125	14.23	8	+77	139	+135	110
5904...	5	+ 10	13.97	333	+45	125	+ 88	....
6205...	13	−300	13.75	26	+40	111	+ 71	40
6333...	9	+225	15.61	334	+ 9	250	+ 39	....
6341...	92	−160	13.86	35	+34	123	+ 69	80
6626...	28	0	14.87	336	− 7	185	− 23	....
6934...	....	−350	15.78	20	−20	333	−114	90
7078...	15	− 95	14.31	33	−29	147	− 71	150
7089...	2	− 10	14.61	22	−37	156	− 94	1600

7. *Possible explanation—a provisional hypothesis.*—There appears to be no very obvious or reasonable escape from the conclusion that some clusters at least are destined to enter the mid-galactic region. Revolution around the whole system is dynamically improbable; moreover, the present distances from the plane are small as compared with the mean equatorial diameter of the cluster system. With such great velocities and masses it seems incredible that clusters would not easily pass through the stratum of stars. Further observation may reveal survivors receding from the galactic plane, and we may be able to examine them for the effects of the passage.[1]

[1] The galactic latitudes of the large and small Magellanic clouds are 32° and 44°; their velocities of recession, according to observations by Wilson at the D. O. Mills Observatory, probably are 260 km/sec. and 150 km/sec., respectively. Reasonable estimates of the distance (cf. eleventh paper) give 70 and 130 as corresponding values of R/V_p in millions of years.

There is, however, suggestive though incomplete evidence that globular clusters are disrupted upon approaching the mid-galactic regions. As a general but not infallible rule, the clusters at moderate or high distances from the Galaxy are compact; the nearer ones are open. There is, for example, the well-known contrast between ω Centauri and 47 Tucanae, the two brightest and nearest systems, the former 1800 and the latter 4700 parsecs from the plane. Of the five systems within 2000 parsecs of the galactic plane four are decidedly open for globular clusters (Table II). The fifth, Messier 62, shows a structural irregularity that may be an indication of rupture; its parallax, depending only on its diameter, is perhaps inaccurate because of asymmetry.

TABLE II

Globular Clusters Nearest the Galactic Plane

N.G.C.	Galactic Latitude	$R \sin \beta$ (Parsecs)	Remarks
4372.......	− 9°	−1800	Large, faint, and little condensed. "Rather faint at center."—Melotte.
5139.......	+16	+1800	ω Centauri. Not strongly condensed.
6266.......	+ 7	+1900	M. 62. Well condensed but probably the most asymmetrical of globular clusters. See Bailey, *Harvard Annals*, **76**, 74, 1916.
6397.......	−12	−1700	Large cluster and one of the nearest. "Stars rather scattered."—Melotte.
6656.......	− 9	−1300	M. 22. The most open of large globular clusters.

The distribution of stars in open galactic clusters is possibly a second indication of the dissipation of globular clusters in the Milky Way. The diameter of some open clusters appears to be quite comparable with that of globular clusters. Several of them contain mainly stars that are highly luminous and probably of great mass. Counts in the surrounding star fields have shown in certain cases (Messier 67, Messier 11, h and χ Persei,[1] the Pleiades[2]) that the cluster stars are sparingly scattered over an area several times greater than that of the nucleus.

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[1] Inferred from radial velocity results by Adams and van Maanen, *Astronomical Journal*, **27**, 187, 1913.

[2] Trümpler, *Popular Astronomy*, **26**, 9, 1918.

As a provisional working hypothesis, which is supported by numerous observed conditions and may be further tested without difficulty, it is suggested that the obvious dynamical equilibrium of a globular cluster has been acquired originally at a great distance from external perturbing forces and is an extremely delicate adjustment that quickly breaks down under stresses such as those prevailing in the galactic region; further, that faint stars in globular clusters are of small mass and of more than average velocity (analogy with galactic stars), and in their orbital motions frequently attain great distances from the center. When the globular cluster approaches a disturbing body as massive as the general galactic system, such stars are of course most readily lost and intermingled with galactic stars.[1] On the other hand, the massive cluster stars, which are mostly of high luminosity, having low peculiar velocities and maintaining in their subsystem a high degree of stability, retain their organization longer in a disrupting field and, except for rare and accidental encounters with galactic stars, undergo as a single unit the general perturbations of the galactic system. It appears necessary to suppose that the gravitational or electrical field near the plane of the Milky Way is by itself more potent in the acceleration of stellar velocities than are near approaches; otherwise it seems impossible that through chance dynamical encounters a massive globular cluster, highly organized and presumably in a steady state, should be so speedily dissipated and transferred into an open galactic group.

8. *Remarks on the contraction theory of stellar evolution.*—In a preceding paragraph we have alluded to the scale of time in the development of the galactic system. An observational contribution to this problem, and more directly to the problem of the speed of evolution of spectral type, is afforded by the distant clusters. Recent discussion by Lindemann, Joly, Véronnet, Eddington, and Jeans, relative to the probable age of the earth and the sun from the standpoint of the contraction theory, emphasizes the persistent disagreement between this theory and the data of geology and physics in estimating the interval of time since the deposition of

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[1] A somewhat similar problem has been examined analytically by Jeans; in fact, he may have had this identical hypothesis in mind (*Monthly Notices*, **76**, 560, 1916).

the earliest sedimentary rocks. Lindemann and Jeans have argued that allowance for known radioactive or even for all electrical properties of matter cannot have prolonged the apparent constancy of solar radiation sufficiently to meet geological requirements. Specifically, Eddington computes that the duration of the total giant stage of a star, if contraction is its source of energy, is not likely to exceed one hundred thousand years; the development from a giant M to a giant G should be less than twenty-five thousand years. To obtain an appreciably slower development we must call upon sources of energy now unrecognized.

Giant stars of various spectral types are numerous in globular clusters and are easily studied in relation to the foregoing problem. We recall that, owing to the finite velocity of light, our investigations of clusters do not relate to contemporaneous conditions, but rather to events of the remote past. Stars in the nearest globular cluster are actually two hundred centuries older at present than they appear in our records, and the light we now receive from the most distant systems has been en route about two hundred thousand years. A significant consequence of these unlike distances, thanks to the structural uniformity of stars and of globular clusters, is that we are able to compare phenomena separated by enormous intervals of time. Instead of having to test stellar development observationally through the dissimilar records of one century or possibly two, with globular clusters we may make a test under identical conditions of climate and instrumental equipment at intervals up to more than a thousand centuries.

The clusters range in distance from twenty thousand to more than two hundred thousand light-years. The farther away the younger they are in our study and the less developed are their stars. (We assume, of course, that distance from the earth is totally irrelevant in the birth time of a globular cluster, though distance from the galactic center may not be.) The results at hand, though preliminary, are apparently decisive; and, as regards definite indications of evolution, are uniformly negative. We note, in particular, the conclusions reached in the eleventh paper, that no measurable change has occurred even in the 180,000 years separating the phenomena recorded for the nearby Hercules cluster and for the most distant system known.

It would have been impressive to find with decreasing distance and increasing age a progressive change in stellar conditions, thus enabling us to use this time scale to its greatest advantage. Perhaps, however, our negative result may be quite as important—our evidence, that is, of essential contemporaneousness from cluster to cluster. The result suggests the insufficiency of the contraction theory, thus supporting the geological time scale on the earth; it contributes observationally to the growing belief that stellar radiation involves sources of energy that are as yet wholly unknown; and, by indicating that an interval of 100,000 years is insufficient to show appreciable stellar development, it tends to alter our conceptions of the speed of evolutionary change.

II. THE LOCAL SYSTEM

9. *Discrepancy of centers—its interpretation.*—As has been stated above, the galactic center deduced from the arrangement of clusters, and roughly corroborated by the density of the Milky Way clouds, is about 90° from the center recently assigned by Walkey and Charlier on the basis of statistical discussions of the bright stars near the sun. The new position is not far distant, however, from the direction toward and from which catalogued stars show a preferential drift. To investigate this discrepancy of centers and the possible meaning of the star drifts in the mechanics of the general galactic system we are led to inquire what part the local stars play in the larger scheme. Let us consider the stellar phenomena manifested in our neighborhood from the standpoint of the greater galactic organization and think of the local stars as forming a group which is imbedded in an extensive stellar stratum and subordinate to the gravitational influences of the major system. Such a procedure brings us to the following hypothesis, which promises fairly satisfactory interpretations of a number of observational results, and appears to harmonize some of the earlier theories as to the systematic motions of the stars:

In the neighborhood of the sun (within 1000 parsecs) the stars fall into two categories: (1) Members of the general galactic system, having motions, distribution, frequencies of spectral type, and absolute magnitude, much the same as are believed to prevail throughout the greater part of the equatorial segment; these we shall call field stars. (2) Members of a moving local system of limited extent; these we shall call cluster stars. The cluster (Stream I) is receding in a nearly radial direction from the center of the galactic system and is surrounded by and intermingled with the field stars (Stream II). It is concentrated toward a plane of symmetry, which is inclined about 12° to the galactic plane as defined by remote non-cluster stars and the clouds of the Milky Way. Its motion of translation is parallel to the Galaxy rather than to its own equator; and there is, in addition, an internal motion in the cluster with respect to its center which appears to be about fifty parsecs north of the true galactic plane. The sun, evidently a field star, is at some distance laterally from the center of the cluster and a few parsecs above its central plane of symmetry. Cluster stars comprise nearly all of the B-type and a decided majority of the A-type stars brighter than the seventh magnitude and a large percentage of the redder stars within three or four hundred parsecs of the sun. The field stars within this limit include very few B's, a small percentage of A's, and possibly a majority of the redder spectral types. Cepheids, N-type stars, most of the O-type stars, the Ursa Major group, planetary nebulae, and stars of peculiarly high random velocity are probably all members of the field, which is, in general, a heterogeneous mixture of many groups and tendencies.

10. *The local cluster.*—In discussing the foregoing hypothesis the most important object will be to demonstrate the existence of a large, local, definitely organized cluster; for when we have satisfied ourselves that there is such a group moving in the neighborhood of the sun, and have admitted the reality of an extensive and populous galactic system, the present hypothesis appears to afford the most simple and natural explanation of the observed motion and distribution of local stars. As an analogy suppose we consider

the compactly organized galactic cluster Messier 11, which is deeply immersed in the rich star clouds of the Milky Way. Unless it be completely at rest with respect to its surroundings, its motion, as seen from a random-moving field star within its bounds, would give rise to exactly such phenomena as the systematic stellar motions we observe in our neighborhood.

We start from the well-known observational results that stars of spectral type B are much alike in actual luminosity, that they are much brighter than the average star, and that with decreasing apparent magnitude they do not increase in number as rapidly as other types of stars. A consideration of the star ratio shows that beyond the distance corresponding to apparent magnitude 7.5 the B stars must be very infrequent.[1] Charlier, computing the average luminosity of the early subclasses of this type, has derived the distance and distribution in space of all such stars contained in the Harvard catalogues and has given striking evidence that they form a large system consisting of about eight hundred stars, among which the sun is eccentrically situated. These two results by themselves seem to show the existence of an important local aggregation. But there is much additional evidence.

11. *Central plane of the B stars and its relation to the Galaxy.*— The first problem to be investigated is whether the system of bright B stars is other than a normal arrangement of galactic stars in the extensive equatorial segment. To determine the position of the central plane of the B-type group Charlier used 751 stars (without differentiation as regards distance from the sun) and obtained for the co-ordinates of the pole: $\alpha = 184\overset{\circ}{.}3$, $\delta = +28\overset{\circ}{.}7$. This position he adopts as the pole of the Milky Way,[2] considering it in satisfactory agreement with the pole of the plane defined by the Milky Way clouds: $\alpha = 191\overset{\circ}{.}2$, $\delta = +27\overset{\circ}{.}4$. Apparently he identifies the "Galaxy of B stars" with the entire stellar universe—a procedure that cluster studies obviously do not support, for only a few of the stars in Charlier's group are more distant from the sun than five hundred parsecs.

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[1] Cf. discussion by Seares, *Publications of the Astronomical Society of the Pacific*, 30, 114, 1918.

[2] *Op. cit.*, pp. 40 and 43.

If, however, the difference in the positions of the two poles be real, an inclination of the central plane of the B-type system to the Galaxy is indicated, and we have an additional argument for its existence as a distinct cluster. The diagrams in Charlier's memoir show, moreover, that the adopted plane is far from satisfying the distribution of the nearer B stars. In Fig. 3, which is based on data (Table III) derived from Charlier's tabulated results, the median values of *Z*, distance from the adopted central plane, are plotted for successive intervals of *Y*, that component of the distance in the plane which is perpendicular to the direction of the center from the sun. The area of each of the plotted points is proportional to the number of stars entering the median value and illustrates the rapid falling off of star density beyond a certain distance. The unit used by Charlier, the siriometer, is nearly equivalent to five parsecs or sixteen light-years. A cross indicates the position of the sun. The broken horizontal line represents the projection of the galactic plane adopted by him. The majority of B stars obviously lie below it, proving the sun's position to be above the cluster's central plane. The full horizontal line indicates the central plane adopted by Charlier; but considering only the inner part of the cluster we observe that the sun is much nearer to the central plane. The inclined broken line shows the projection of the galactic plane as derived from the stars of the Milky Way clouds; its inclination and its distance below (south of) the B

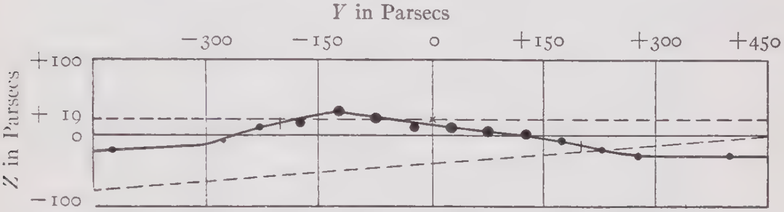

FIG. 3.—Projection of the local cluster of B stars on a plane perpendicular to the Galaxy and to the projection, on Charlier's adopted plane, of the line joining the sun and the center of the cluster. The center of the system of B stars, as derived by Charlier, is at the origin of co-ordinates. The cross shows the position of the sun. The true galactic plane is indicated by the inclined broken line. Vertical lines across the curve show the limits of distance adopted for the solution represented by Fig. 4.

cluster and the sun are derived from the results mentioned on a following page. The figure suggests in general that a few B stars of the Galaxy are sparingly intermixed with a definite and limited cluster.

TABLE III

Intervals of V in Siriometers	Number of Stars	Median Z in Siriometers
≦−60	41	−4
−59 to −50.........	16	−2
−49 " −40.........	52	+2
−39 " −30.........	59	+3
−29 " −20.........	72	+6
−19 " −10.........	80	+4
− 9 " 0.........	83	+2
0 " + 9.........	88	+2
+10 " +19.........	76	+1
+20 " +29.........	70	0
+30 " +39.........	60	−2
+40 " +49.........	37	−4
+50 " +59.........	36	−6
≧+60	35	−6

12. *Summarized discussion of the properties of the local cluster.*—The procedure from this point involves much computational work and statistical discussion, and for the present the most relevant results can best be given in a summarized form.

a) Charlier's determination of the mean absolute magnitudes of B stars is compared in Table IV with averages derived from the parallaxes given by Kapteyn in *Mount Wilson Contributions*, Nos. 82 and 147. While systematically fainter, Kapteyn's mean values

TABLE IV

Spectral Class	Kapteyn		Charlier		Difference
	Number of Stars	Absolute Magnitude	Number of Stars	Absolute Magnitude	
B0*...........	36	−0.34±0.23	27	−0.56	+0$^{\mathrm{M}}$22
B1...........	23	−2.00±0.16	26	−2.76	+0.76
B2...........	36	−1.21±0.16	40	−2.76	+1.55
B3...........	148	−0.38±0.06	157	−1.10	+0.72
B5...........	67	−0.21±0.09	104	−1.29	+1.08

* In Kapteyn's lists B stands for B0. The lack of homogeneity in this subtype, noted by both investigators, may be due to the inclusion of some (fainter stars) of the general class B with the true B0 stars.

verify the similarity of the subclasses B0, B3, and B5, and the much greater average luminosity of B1 and B2.

b) If we desire to limit our investigation of the B stars to the denser part of the group (say, to approximately two hundred parsecs from the sun, as indicated in Fig. 3), we should include only those brighter than apparent magnitude 4.0 for types B1 and B2, and brighter than 5.5 for types B0, B3, and B5. Such data permit a solution for the equatorial plane of the B cluster that is not seriously influenced by outside stars.

c) From the several lists in *Harvard Annals*, **56**, the galactic longitudes and latitudes have been discussed for 356 B-type stars falling within the foregoing limits of spectrum and brightness. The results, collected in Table V, give for the pole of the central plane of the flattened B-type cluster: $\alpha = 178^\circ$, $\delta = +31^\circ.2$.

TABLE V

Intervals of Galactic Longitude*	No. of Stars	Median Galactic Latitude	Mean Galactic Latitude	Residual from Curve
0°– 9°	15	+10°	+12°	+ 2°
10 – 19	13	+ 6	+ 8	− 1
20 – 29	19	+ 8	+11	+ 3
30 – 39	26	+ 8	+ 5	− 1
40 – 49	16	+ 2	− 5	−10
50 – 59	22	+ 4	+ 7	+ 4
60 – 69	22	+ 2	+ 3	+ 2
70 – 79	15	+ 4	− 1	0
80 – 89	26	− 1	− 3	− 0
90 – 99	13	− 3	− 2	+ 2
100 –109	14	−19	−17	−11
110 –119	24	− 8	− 4	+ 4
120 –129	20	−10	− 9	+ 0
130 –139	39	−16	−10	+ 1
140 –149	18	− 2	−11	+ 0
150 –159	14	− 6	−10	+ 2
160 –169	20	−16	−10	+ 2
170 –179	20	−16	−14	− 3

* Diametrically opposite regions are combined for this table.

d) The definiteness with which this secondary Galaxy is determined is shown in Fig. 4, where mean galactic latitudes are plotted against galactic longitude. The semi-amplitude of the resulting sine-curve yields 12° for the inclination of the two planes, and the nodes are in galactic longitudes 70° and 250°. The dip of the central plane due to the sun's position to the north is 5°. If stars in the constellation of Orion were deleted from the discussion, the result would not be materially affected. The average deviation of a mean value from the curve is +2°7.

e) A discussion of all A-type stars brighter than the sixth apparent magnitude confirms the foregoing results, showing that other spectral classes are involved in the cluster. The inclination derived from A stars, however, is only about 4°, probably an indication that many general galactic stars are included and also that the concentration to the plane of the cluster may not be so pronounced as for B-type stars.

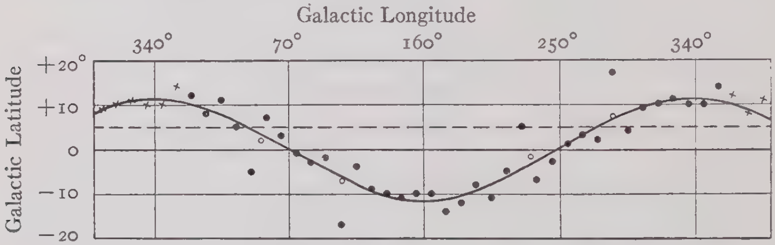

FIG. 4.—Solution for the inclination of the local cluster to the galactic plane

f) We have further suggestions that a majority of A stars probably belong to this limited cluster, first in Plummer's discussion of the preferential motion shown by their radial velocities and second in the observations of Boss and Eddington that the mean parallactic motion of these stars is the same in low and high galactic latitude although the density is twice as great near the Galaxy.

g) In Kapteyn's result that the B stars are members of Stream I we also have an indication that some stars of all the later types belong to the cluster.

> The work thus far done proves the continuity of the series of the B and A and the second-type stars. . . . . some B stars at least must be regarded as members of the Second Stream. It is now found that the remainder—the overwhelming majority—belong to the First Stream.[1]

Apparently, then, the constituency of Stream I shows the content of the cluster.

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[1] Kapteyn, *Mount Wilson Annual Report*, 1916, p. 256.

h) Gould's belt of bright stars, as revised by Newcomb,[1] is nearly coincident with this secondary Galaxy. A definite trace of it is given by all naked-eye stars when considered together, irrespective of type (lucid red and yellow stars contribute little because of their uniform distribution in the sky); stars of the ninth magnitude, however, give the same pole for the Milky Way plane as is given by galactic clouds (Newcomb).

i) Cepheid variables extend far outside the limits of the local cluster, and the pole of their plane of concentration is identical with that for the galactic clouds (Fig. 5). They afford the best determination yet available of the sun's distance north of the galactic plane—about sixty parsecs. A systematic error in the absolute magnitude of Cepheids would affect this distance proportionately, but would not alter the general conclusions. The N-type and most O-type stars are sufficiently remote and concentrated to the plane to afford a check of the sun's displacement, if reliable individual parallaxes become available. The poles of their planes of concentration are apparently identical with the pole derived from the galactic clouds.[2]

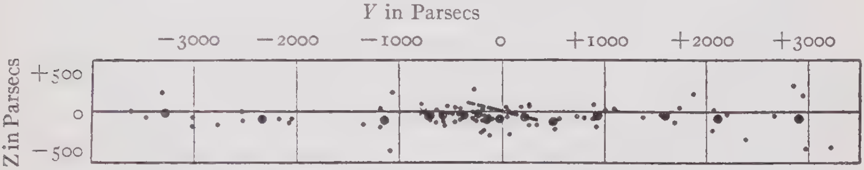

FIG. 5.—Projection of the positions of Cepheid variables on the same plane as in Fig. 3, the sun being at the origin. The small dots represent individual stars, the cross a special group in Carina, and the large dots mean values. The inclined line in the center represents the projected plane and the probable extent of the local cluster of B stars.

j) The center of the B-type cluster, adopting Charlier's values of absolute magnitude, is more than fifty parsecs north of the galactic plane, making the sun less than ten parsecs north of the central plane of the B stars (Fig. 3). For this last quantity Charlier derived 19.2 parsecs, but his work involves extra-cluster stars that apparently are concentrated to the true galactic plane. With Kapteyn's absolute magnitudes of the B's the distance of the cluster's central plane below the sun is still less, and the sun's distance from the local center is less than fifty parsecs.

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[1] *Carnegie Institution of Washington, Publication* No. 10, 1904.

[2] Hertzsprung, *Astronomische Nachrichten*, **192**, 261, 1912.

k) The evidence that the local cluster is a dynamical unit and has internal motion superposed upon a fairly high translational velocity includes: (1) the behavior of the star ratio for spectral types B and A; (2) the greatly flattened form of the system, at least for the B stars; (3) Kapteyn's announcement of the acceleration of Stream I;[1] (4) Strömberg's evidence from second-type stars of a rotation around a center approximately coincident in direction with the center of the cluster as derived from B stars;[2] and (5) the deviations of the vertices of the various groups of B stars, depending upon position in the sky. A study of the last three points, if not too much constrained by the magnitude of the observational errors, may show whether the internal motion is approximately radial or is a rotation in one general direction or in more than one.

12. *Consequences of motion—star-streaming.*—The foregoing abbreviated account seems to show conclusively that an open cluster surrounds the sun, and that it is comprehensive enough to include stars as numerous and varied as the components of a star stream. The continuity of the galactic star fields beyond the cluster's limits is strongly indicated by the arrangement in space of Cepheid variables (Fig. 5) and other types of distant stars, but especially by the distribution of the faint stars of the Bonner Durchmusterung (Seeliger, Newcomb) and by the increasing galactic concentration with decreasing apparent brightness of still fainter stars (Kapteyn, Seares, Van Rhijn). As seen from the solar system, therefore, the local cluster, if in motion, must inevitably produce star-streaming analogous to that observed; hence it is a natural assumption that the recorded systematic motions of the local stars are due wholly to such a cause. That the translation through the star field is approximately radial is of some dynamical significance, and that it is in the plane of the Galaxy was to be expected. The internal motion, on the other hand, probably

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[1] *Mount Wilson Annual Report*, 1916, p. 255.

[2] *Mt. Wilson Contr.*, No. 144; *Astrophysical Journal*, **47**, 7, 1918.

has a maximum component in the plane of the cluster. Unfortunately the small inclination may make difficult the separation of the two motions.

The present viewpoint of star-streaming seems to be greatly strengthened by the way in which important observed facts agree with its requirements. We have mentioned the form and limits of the cluster, its stellar content, its motion parallel to the galactic plane, the probable acceleration of Stream I, and the distribution of A-type stars. Two or three other points may be referred to here.

a) Since the plane of concentration of field stars lies south of the sun (Fig. 5), the theory requires that stars of Stream II should be relatively less numerous in northern than in southern galactic latitudes. In agreement with this Hough and Halm have found, from both radial velocities and proper motions, that the stars of Stream II are more than twice as frequent near the southern galactic pole as near the northern.[1] This point appears to be of high importance.

b) As the cluster is of limited extent the proportion of field stars should increase with increasing distance from the center. The ratio of the number in the first stream to that in the second is $\frac{3}{2}$ for naked-eye stars (excluding type B); for fainter stars Dyson and Thackeray[2] find the ratio $\frac{4}{3}$. Stars as distant as 1000 parsecs presumably would show only accidental traces of Stream I.

c) The declination of the apex of solar motion should be sensitive, on this cluster hypothesis, to the character and average distance of the stars involved in its determination. Thus the low declinations derived from stars of early types, of large proper motions, of faint absolute magnitude, of bright apparent magnitude, all indicate that the apex with respect to the galactic field lies considerably north of the value commonly used. Dyson and Thackeray's[3] work on Groombridge stars and Comstock's[4] results from the very faint "optical" companions of double stars perhaps most nearly indicate the "true" direction of solar motion.

[1] *Monthly Notices*, **70**, 85, 568, 1910; Halm, *ibid.*, **71**, 610, 1911.

[2] *Ibid.*, **77**, 590, 1917.

[3] *Ibid.*, **65**, 428, 1905.

[4] *Astronomical Journal*, **28**, 54, 1913.

In many of its features the present hypothesis makes no claim to novelty. It includes, indeed, a patchwork of many preceding suggestions, but is of course entirely independent in that the problem has been approached from the point of view of the recently acquired ideas on clusters and on the arrangement and constituency of the larger galactic system. Among earlier deductions and speculations related to this subject we note that Gould, forty years ago, thought his "Belt" of bright stars (inclination 20°, longitude of node 90°) represented a local cluster of possibly four hundred members; but Newcomb's later analysis attributed the whole phenomenon to a few chance aggregations. Hertzsprung and Charlier have found the deviation of the galactic pole for B stars, the former suggesting that the result might be significant. Perrine has noted that one of the vertices lies in the general direction of regions rich in globular clusters. An early hypothesis by Hough and Halm tentatively identified Stream II with the Milky Way stars. Turner's gravitational hypothesis of the streams assigned the center of the universe to the line of star-streaming. Jeans has considered theoretically several different possibilities, some of which resemble the present suggestions. But in developing this interpretation the obligation is by far the greatest to the various discussions of theories and observational data by Eddington and Kapteyn.

If a theory similar to the present one survives the test of future observation and analysis, it will be of interest to note how closely it approaches Kapteyn's original statement of star-streaming, so far as local phenomena are concerned. The "two cloud" hypothesis of the streams—the "dualistic" conception of the universe, as Charlier calls it—would be a more literal view of the conditions near the sun than that given by the ellipsoidal theory of streaming—the "monistic" conception. But so far as we now see the latter may apply better to the galactic system as a whole, and we also have a case for the ellipsoidal theory in the internal motions of the local cluster.

MOUNT WILSON SOLAR OBSERVATORY
April 1918

Contributions from the Mount Wilson Solar Observatory, No. 158

THE PHOTOGRAPHIC DETERMINATION OF STELLAR PARALLAXES WITH THE 60-INCH REFLECTOR

THIRD SERIES

By ADRIAAN VAN MAANEN

The work discussed in the present publication differs so little in general outline from that of the two previous series of parallaxes[1] that it will suffice to mention only a few details in regard to the taking and measuring of the photographs. As before, the two exposures to be compared have always been made at nearly the same hour-angle; the largest difference is 11°2, while in only 24 of the 204 cases is the difference greater than 5°. Since November, 1917, many of the plates have been developed with kodelon, the others with rodinal. The exposures as a rule were 15 minutes; for N.G.C. 224, 5194, and 6720, however, they were 20, 30, and 25 minutes, respectively.

In the fields of N.G.C. 224, Boss 660, Boss 3030, and N.G.C. 5194 it was necessary to use some comparison stars whose distances from the central star are greater than 8′; they did not, however, exceed 10′.

In this place we may point out once more the reasons why it seemed best to use faint comparison stars near the central star. This restatement seems the more necessary since Schlesinger has recently made the following remark about the use of the Cassegrain reflector for parallax work:

> The photographic refractor has a similar advantage over the Cassegrain reflector, a form of instrument that will doubtless play an important part in the astrometric work of the future. Here the field of good definition is so small that it is necessary to restrict the measures to within a few minutes of arc of the optical axis. Thus with the Mount Wilson reflector the available field has an angular area that is less than 2 per cent of that of one of the Thaw plates. As a consequence it is necessary to use very faint comparison stars (since brighter stars will be found in sufficient number within this small region

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[1] *Mt. Wilson Contr.*, Nos. 111 and 136, 1915 and 1917.

only as the result of a rare chance), and the length of exposure that this makes necessary is several times as long as with the visual refractor and many times as long as with the photographic refractor.[1]

In Schlesinger's paper I find no evidence that the available field of the Thaw refractor is 50 times that of the Mount Wilson Cassegrain reflector. In both previous series of parallaxes I have shown that in using fields of 10′ radius there is no vitiating influence of quadratic terms; that this holds for this third series also is shown on page 34. The Thaw refractor accordingly should have a field of 70′ radius available for accurate measures. But it is not clear that this is the case, as the comparison stars used in the paper cited are all within 32′ distance of the center; on the other hand, from the several discussions of plates taken with other photographic refractors, viz., those of the type used for the *Carte du Ciel*, we know that for fields of 60′ radius and even smaller the quadratic terms become very considerable and in accurate work cannot be neglected.[2] In an unpublished discussion of the proper motions derived from plates taken with the Yerkes 40-inch refractor it was also found that even there, although the field has a radius of only 22′, the quadratic terms cannot be neglected if the highest accuracy is required. On the other hand, it is not impossible that fields of considerably greater radius than 10′ could be used successfully with the Cassegrain combination of the 60-inch reflector; the matter has not been investigated simply because larger fields have not been required. The point is that reasons quite other than those of instrumental limitation have determined the use of faint comparison stars for these investigations. There are considerable advantages connected with such a procedure: the larger number of stars at our disposal makes it easier to fulfil the requirements for the most accurate work, viz., that the mean magnitude of the comparison stars should be about the same as that of the reduced parallax star; that there should be only a small range in the magnitudes of these stars; and that their centroid should coincide closely with the parallax star. An additional advantage is that we can use more comparison stars. This last fact is of importance, not only for a discussion of internal systematic errors, but also because it undoubtedly increases the accuracy of the results. There are several reasons for this: the greater the number of comparison stars the smaller will be the influence of an accidental error in any one of them on the final parallax; the fainter the stars the smaller the chance that any of them have considerable parallaxes or proper motions, which would be vitiating to the results; the larger the number that can be used the smaller the influence of any abnormal parallax or proper motion, while at the same time it is easier to identify stars having these peculiarities and, if necessary, reject them. Finally there is less uncertainty in the absolute value of the mean parallax of the fainter stars than in the case of the brighter stars, and the reduction from relative to absolute parallax will be therefore more accurate.

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[1] *Publications of the Allegheny Observatory*, **4**, 1, 1917.

[2] See, for instance, Kapteyn, *Publications of the Astronomical Laboratory at Groningen*, No. 1, 1900; van Maanen, *Recherches de l'Observatoire d'Utrecht*, v. **5**, 1911.

Since the parallax program of this observatory includes principally stars and nebulae of presumably small parallaxes, it is necessary to take every precaution to attain the highest possible accuracy; for this reason it was decided to give relatively long exposures in order to obtain the advantages gained by the use of faint comparison stars.

Although the focal length of the instrument used at Greenwich is relatively short and the number of exposures less than that of most of the other observatories, the probable errors found for the series of parallaxes published by the Greenwich Observatory[1] are relatively small; and this circumstance is undoubtedly due to the fact that the observers have also used more, fainter, and nearer comparison stars than is usually the case.

The detailed results for each field of the present series are collected in the tables, which are arranged in the same way as in the previous papers.

SUMMARY OF RESULTS

The results of these twenty-five fields are collected in Table I, which corresponds to the analogous tables in the previous papers. At the end of the table are the results of six objects not contained in Boss's *Preliminary General Catalogue*.

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[1] *Monthly Notices*, **75**, 592, 1915.

The magnitude for Nova Persei was derived from the Mount Wilson plates and is on the scale of *Lick Observatory Bulletin*, No. 8; the magnitude for B.D. +36°3956 is that given in the *Bonner Durchmusterung*, while that for the central star in N.G.C. 6720 is the value given by Barnard.[1]

TABLE I

Number in Boss *P.G.C.*	Mag. Harvard	Spectrum	P.M. Boss	Relative Parallax	P.E.	No. of Exp.	μ_α Boss	μ_α vM	Boss−vM
161	5.58	Mb(G7)	0″064	+0″012	0″005	16	−0″038	−0″050	+0″012
582	5.62	Ma(G7)	0.062	+0.013	0.005	16	+0.030	+0.015	+0.015
660	5.94	Mc(G8)	0.018	+0.009	0.006	16	−0.009	−0.007	−0.002
712	6.48	K1	0.042	+0.029	0.005	16	+0.038	+0.048	−0.010
1154	5.67	Ma*	0.034	+0.007	0.002	14	+0.028	+0.041	−0.013
1643	5.88	K1	0.019	+0.015	0.005	20	+0.019	+0.015	+0.004
1822	5.60	G5	0.087	+0.034	0.004	16	−0.054	−0.054	0.000
2245	5.75	Mb(G4)	0.116	+0.027	0.005	16	−0.025	−0.030	+0.005
2378	6.31	Mb(G7)	0.022	−0.001	0.009	16	−0.016	−0.036	+0.020
2442_1	7.26	F5*	0.161	+0.018	0.007	14	−0.161	−0.134	−0.027
2442_2	7.74			+0.016	0.006	16		−0.117	−0.044
2660	5.99	K*	0.068	+0.004	0.006	16	+0.027	+0.035	−0.008
3030	5.74	G9	0.078	+0.022	0.007	14	−0.078	−0.067	−0.011
3169	6.13	K1	0.189	+0.026	0.005	18	+0.045	+0.029	+0.016
3658	6.22	K5*	0.122	+0.023	0.005	16	−0.004	−0.006	+0.002
4343	6.75	Mb(G7)	0.058	+0.007	0.005	20	+0.039	+0.027	+0.012
5083	5.51	B2	0.015	+0.007	0.005	16	+0.006	+0.003	+0.003
5409	5.96	Ma*	0.075	+0.007	0.004	16	−0.034	−0.043	+0.009
5602	6.48	K2	0.148	+0.003	0.005	14	+0.147	+0.143	+0.004
6129	6.55	K3p	0.332	+0.097	0.004	18	+0.329	+0.312	+0.017
Nova Persei No. 2	13.0	O	....	+0.007	0.004	14	....	−0.017	....
B.D. +36°3956	8.0	O	....	−0.005	0.006	20	....	+0.007	....
N.G.C. 224	......	Sp. Neb.	....	+0.004	0.005	16	....	−0.013	....
N.G.C. 2392	9.42	Pl. Neb.	....	+0.020	0.003	18	....	−0.006	....
N.G.C. 5194	......	Sp. Neb.	....	+0.005	0.008	16	....	−0.005	....
N.G.C. 6720	13.6	Pl. Neb.	....	+0.002	0.005	14	....	+0.010	....

The mean number of exposures is 16; the mean probable error is 0″0052. The correction necessary to convert the relative into absolute parallaxes is +0″002.

Some of the fields give occasion for the following remarks:

a) The field of Boss 2378 also contains Boss 2380 = 55 Cancri. A faint star 1′1 east and 0′9 south of 55 Cancri was included among the comparison stars; during the measurement it soon became evident that this star has a considerable proper motion; accordingly it was rejected for comparison purposes, but the parallax and proper motion were derived with respect to four stars in the neighborhood; the results are:

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[1] *Monthly Notices*, **66**, 109, 1906.

$$\pi_{\text{rel.}}=+0''.042\pm0''.018$$
$$\mu=0''.541 \text{ in } p=242^\circ.5$$[1]

The star is probably a companion of 55 Cancri, which has $\mu=0''.55$ in $p=243^\circ.9$.

b) B.D. +36°3956 is the third of the Wolf-Rayet stars whose parallaxes were first derived by Kapteyn;[2] the parallaxes of B.D. +35°4001 and B.D. +35°4013 were published in a previous paper.[3] On account of the proximity of these three stars of this type we may suppose that in reality they are close together and have the same parallax; the mean absolute parallax as derived from the Mount Wilson plates is $+0''.0047$. The absolute magnitudes would accordingly be +1.9, +1.4, and +1.4, or, in the mean, +1.6. If on the other hand we accept the hypothesis that the absolute magnitudes are the same (we know that the dispersion is very small for stars of this type), we may use a formula, derived by Mr. Strömberg for computing mean absolute magnitude, viz.:

$$M=5\log(\Sigma\ \pi_0 10^{-0.2m'})-5\log(\Sigma\ 10^{-0.4m'}),$$

where π_0 is the observed parallax and $m'=m+5$. The result for M is then +1.2.

c) The absolute parallax of Nova Persei No. 2, $+0''.009$, is in agreement with the parallaxes found by other observers, viz., $+0''.028\pm0''.020$ by Hasenstein, $-0''.003\pm0''.023$ by Chase, $+0''.036\pm0''.010$ by Bergstrand, and $+0''.007\pm0''.004$ by Küstner. The weighted means of these five values is $+0''.013\pm0''.004$. This agrees well with the values derived from the motion of the nebulosity in the early days of the famous star; from these Bell found $\pi\geqq0''.020$, Kapteyn $+0''.011$, Seeliger $+0''.010$, and Very $>0''.025$.

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[1] *Publications of the Astronomical Society of the Pacific*, **30**, 191, 1918.

[2] *Publications of the Astronomical Laboratory at Groningen*, **1**, 1900.

[3] *Mt. Wilson Contr.*, No. 136; *Publications of the Astronomical Society of the Pacific*, **28**, 279, 1916.

For only two other novae are parallaxes available, viz., for Nova Geminorum No. 2: $\pi_{\text{abs.}}=+0''.012\pm0''.007$ by Slocum and $-0''.013\pm0''.013$ by Miller; and for Nova Lacertae: $\pi_{\text{abs.}}=+0''.013\pm0''.012$ by Slocum and Mitchell and $0''.000\pm0''.020$ by Balanowsky. The weighted mean parallaxes are accordingly $+0''.003\pm0''.008$ and $+0''.008\pm0''.004$, respectively. Using these we find the absolute magnitudes at maximum and minimum as given in Table II. The mean absolute magnitudes at maximum and minimum are therefore -2.9 and $+6.9$; this is of interest in connection with the appearance of new stars in spiral nebulae. As to their minima, we can only say that they are below the twentieth or twenty-first apparent magnitude; the mean maximum of the 15 novae in spirals, which are now known, is $+14.7$ mag. Supposing that they reached the same absolute magnitude at maximum as the three novae for which parallaxes are known, we derive $+0''.00003$ for the mean parallax of the spirals in which they appeared.[1]

TABLE II

Object	$\pi_{\text{abs.}}$	m max.	m min.	M max.	M min.
Nova Persei No. 2......	$+0''.013$	0.0	+13.0	−4.4	+8.6
Nova Gem. No. 2.......	+0.003	+3.7	+12.3	−3.9	+4.7
Nova Lacertae.........	+0.008	+5.0	+13.0	−0.5	+7.5

The three novae for which the parallaxes are given in Table II, are, however, among the brightest found since the first discovery of a nova in a spiral nebula in 1885. The exclusion of the novae found before this time may seem arbitrary, yet when we keep in mind the fact that the new stars in spirals appear to be at least five magnitudes fainter on the average than those found in the Milky Way, it is clear that the chances for their discovery in the early days were extremely small; it thus seems allowable to use only the more recent material in a comparison of the two kinds of novae. Comparing therefore the two bright novae found in the spirals, which attained an apparent magnitude of 7, with the brighter ones in the Milky Way, we find a parallax of 0″.0010 for those spirals. The two parallaxes, 0″.00003 and 0″.0010, based on different suppositions, would give velocities of 3400 and 100 km per sec., respectively, corresponding to the internal motions found by Kostinsky and the writer in Messier 51, 101, and 81. Neither of these values seems to be impossible, while the parallaxes required by the island-universe theory lead to highly improbable velocities for the internal motions, to corroborate which we can only regret that no more material is available.

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[1] This value is of the same order as the parallaxes derived by Shapley for the globular clusters, which appear unquestionably to be members of our galactic system.

A discussion based on the magnitudes of novae has, however, certain weak points: we know from the objects discovered in the Andromeda Nebula that the dispersion at maximum is considerable—at least 10 magnitudes; moreover, unless we accept the island-universe theory on other grounds, there is reason to expect quite different absolute magnitudes for novae within and outside of the spirals.

d) The results for the parallaxes of N.G.C. 224 and 5194 given in Table I are larger than most astronomers will like to accept; but, although the exposures show images of the central parts of these two spirals which are fairly measurable, it seems that the diffuseness of their images as compared with those of the comparison stars may easily have introduced a systematic error of the order of the parallaxes themselves; the only conclusion indicated by these values seems to be that the parallaxes of the two spirals can hardly surpass 0″.010 or 0″.015.

e) This difficulty does not hold for the central stars of the three planetaries for which the measures have now been finished; here the images are quite sharp and as good as those of the surrounding stars. Incidentally it may be mentioned that in the case of N.G.C. 2392 a sector of 36° opening was required in order to cut down the magnitude of the central star.

There are available determinations of parallaxes of planetaries by other observers: for N.G.C. 2392, −0″.036 by Bohlin; for 6543, +0″.066 by Brünnow and −0″.046 by Bredichin; for 7027, about −0″.100 by Wilsing; for 7662, −0″.063 by Wilsing-Huss; and for 6720, +0″.067 and +0″.015 by Newkirk. Only the last of these values seems to deserve much weight.

Including Newkirk's second determination for N.G.C. 6720, we may use the parallaxes as given in Table III to derive data on the size of these planetaries and the absolute magnitudes of their central stars.

The mean absolute magnitude is accordingly +7.9, a result strikingly close to the absolute magnitude +6.9 found above for the novae at minimum; the fact demands the more consideration, as the spectra of the two kinds of bodies show much resemblance. Caution should be used, however, in drawing any far-reaching conclusion, both on account of the scarcity of the material as yet available and because considerable difficulty is encountered in deriving any conclusion which involves absolute magnitudes of bright-line stars. It is noteworthy that in both cases we find evidence that these bright-line stars are of faint absolute magnitude, while the regular stars of type O have very high luminosities; thus for the three Wolf-Rayet stars, whose parallaxes were measured at Mount Wilson, we have found $M=+1.2$. For 10 other O-type stars, parallaxes by other observers are available; the mean absolute parallax is negative, viz., −0″.007, while the mean apparent magnitude is 6.55; these results also point to very high luminosities. From parallactic motions Gyllenberg[1] derives an absolute magnitude of −1.2 for the O-type stars, while Hertzsprung[2] finds −2.8.

TABLE III

Object	$\pi_{abs.}$	Mag.	Diameter	M	Diameter in Light Years
N.G.C. 2392	+0″.022	9.4	46″	+6.1	0.03
6720	+0.008	13.6	80	+8.1	0.16
7662	+0.023	12.6	31	+9.4	0.02

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[1] *Arkiv för Matematik, Astronomi och Fysik*, **11**, No. 28, 1917.

[2] *Astronomische Nachrichten*, **196**, 201, 1914

N.G.C. 224 $\alpha = 0^h37^m17^s$ $\delta = +40°43'$

Pl. No.	Date			t	Obs.	Qy	p	m	Remarks
1166 ..	1916	Nov.	17	− 2°5	vM	g	−0.644	+1.88	
1167 ..			17	+ 3.0	H	g	−0.644	+1.88	Thick
1174 ..			18	− 0.2	vM	fg	−0.655	+1.89	Hazy 25^m exp.
1175 ..			18	+ 7.2	vM	fg	−0.655	+1.89	Hazy 25^m exp.
1485_1 ..	1917	July	29	− 9.0	vM	fg	+0.837	+2.58	
1485_2 ..			29	− 4.0	H	fg	+0.837	+2.58	
1493 ..		Aug.	10	− 8.5	vM	g	+0.737	+2.61	Trail of Boss 141
1494 ..			10	− 1.7	H	g	+0.737	+2.61	Clouds 25^m exp.
1509 ..			11	+ 1.5	vM	fg	+0.729	+2.61	
1510 ..			11	+ 7.2	H	fg	+0.727	+2.61	
1511 ..			11	+12.7	vM	fg	+0.727	+2.61	
1520 ..			12	− 0.2	H	g	+0.719	+2.62	
1606 ..		Nov.	8	− 3.0	vM	fg	−0.534	+2.86	Hazy 28^m exp.
1607 ..			8	+ 5.7	H	fg	−0.534	+2.86	Hazy 30^m exp.
1620 ..		Dec.	6	− 2.5	H	pf	−0.812	+2.93	35^m exp.
1622 ..			7	− 1.5	H	pf	−0.821	+2.94	35^m exp.

Mor.-Ev.	Δp	Δm	M	v	Δt
1485_1–1606...	+1.37	−0.28	− 1	− 6	−6°0
1493 –1620...	+1.55	−0 32	+20	+13	−6.0
1485_2–1622...	+1.66	−0.36	−14	−21	−2.5
1494 –1166...	+1.38	+0.73	− 9	− 6	+0.8
1520 –1174...	+1.37	+0.73	+ 9	+12	0.0
1509 –1167...	+1.37	+0.73	+ 7	+10	−1.5
1510 –1175...	+1.38	+0.72	−16	−13	0.0
1511 –1607...	+1.26	−0.25	+23	+18	+7.0

$$+2.48\,\mu_a + 2.20\,\pi = -14 \qquad \pi = +2.5 = +0''.004 \pm 0''.005$$
$$+2.20\,\mu_a + 16.19\,\pi = +24 \qquad \mu_a = -7.9 = -0''.013 \pm 0''.011$$

Comp. Star	Br.	x	y	π	P.E.
1.........	bf	−6'.0	+2'.5	+0".017	±0".006
2.........	bf	−1.8	+4.5	−0.011	0.007
3.........	f	+3.2	+3.2	−0.001	0.012
4.........	bf	+6.0	+0.4	+0.013	0.006
5.........	f	+5.5	−1.9	−0.012	0.006
6.........	bf	+1.2	−3.6	+0.011	0.011
7.........	f	+0.1	−2.6	+0.001	0.011
8.........	b	−2.1	−0.3	−0.010	0.011
9.........	bf	−7.8	−2.3	−0.005	±0.006

Boss 161 $\alpha=0^h41^m19^s$ $\delta=+14^\circ56'$

Pl. No.	Date	t	Obs.	Qy	p	m	Remarks
1083 ..	1916 Aug. 5	+3°0	H	fg	+0.790	+1.60	
1173_1..	Nov. 18	−16.7	vM	fg	−0.643	+1.89	Hazy 20^m exp.
1173_2..	18	−10.5	vM	fg	−0.643	+1.89	Hazy 20^m exp.
1188_1..	Dec. 16	+2.5	vM	g	−0.867	+1.96	
1188_2..	16	+6.5	H	g	−0.867	+1.96	
1206 ..	17	+2.5	vM	g	−0.872	+1.96	
1508_1..	1917 Aug. 11	−10.5	vM	g	+0.740	+2.61	
1508_2..	11	−6.2	H	g	+0.740	+2.61	
1518_1..	12	−17.7	vM	g	+0.730	+2.62	
1518_2..	12	−13.5	H	g	+0.730	+2.62	
1519 ..	12	−9.0	vM	fg	+0.730	+2.62	
1533_1..	Sept. 12	+10.5	vM	fg	+0.338	+2.70	
1533_2..	12	+15.0	H	fg	+0.338	+2.70	
1608 ..	Nov. 8	+13.2	vM	g	−0.520	+2.86	Hazy 20^m exp. Trail
1637_1..	Dec. 8	−9.0	vM	g	−0.821	+2.94	
1637_2..	8	−4.5	H	fg	−0.821	+2.94	

Mor.-Ev.	Δp	Δm	M	v	Δt
1518_1–1173_1 ..	+1.37	+0.73	+2	+14	−1°0
1518_2–1173_2 ..	+1.37	+0.73	−29	−17	−3.0
1508_1–1637_1 ..	+1.56	−0.33	+27	+6	−1.5
1519 –1637_2 ..	+1.55	−0.32	+40	+19	−4.5
1508_2–1206 ..	+1.61	+0.65	+8	+16	−8.7
1083 –1188_1 ..	+1.66	−0.36	0	−23	+0.5
1533_1–1188_2 ..	+1.21	+0.74	−28	−14	+4.0
1533_2–1608 ..	+0.86	−0.16	+2	−9	+1.8

$$+2.40\,\mu_\alpha + 2.20\,\pi = -58 \qquad \pi = +7.2 = +0''.012 \pm 0''.005$$
$$+2.20\,\mu_\alpha + 16.14\,\pi = +48 \qquad \mu_\alpha = -30.6 = -0''.050 \pm 0''.014$$

Comp. Star	Br.	x	y	π	P.E.
1	bf	+3'.2	+6'.5	−0''.013	±0''.004
2	f	+4.1	+2.6	+0.008	0.009
3	f	+2.9	−0.8	−0.004	0.006
4	bf	+0.4	−7.1	+0.011	0.003
5	f	−1.0	−6.5	−0.012	0.007
6	ff	−2.7	−3.4	−0.010	0.011
7	f	−1.6	+2.1	+0.021	0.008
8	b	−4.4	+5.6	−0.003	±0.006

Boss 582 $\alpha = 2^h29^m43^s$ $\delta = +34^\circ15'$

Pl. No.	Date		t	Obs.	Qy	p	m	Remarks
1112_1..	1916	Sept. 20	− 6°.2	vM	fg	+0.636	+1.72	
1112_2..		20	− 1.7	H	g	+0.636	+1.72	
1190_1..		Dec. 16	− 4.0	vM	g	−0.658	+1.96	Trail
1190_2..		16	+ 0.5	H	fg	−0.658	+1.96	
1208_1..		17	− 6.5	H	g	−0.670	+1.96	Trail
1208_2..		17	− 2.0	vM	vg	−0.670	+1.96	
1229_1..		18	− 9.0	H	fg	−0.681	+1.97	
1229_2..		18	− 5.0	vM	fg	−0.681	+1.97	
1534_1..	1917	Sept. 12	− 6.0	vM	fg	+0.732	+2.70	
1534_2..		12	− 1.7	H	f	+0.732	+2.70	
1548_1..		13	− 4.0	vM	g	+0.719	+2.70	
1548_2..		13	0.0	H	g	+0.719	+2.70	
1559_1..		14	0.0	vM	g	+0.709	+2.71	
1559_2..		14	+ 4.2	H	g	+0.709	+2.71	
1625_1..		Dec. 7	− 3.5	vM	fg	−0.542	+2.94	20^m exp.
1625_2..		7	+ 2.2	H	fg	−0.542	+2.94	20^m exp.

Mor.-Ev.	Δp	Δm	M	v	Δt	Remarks
1112_1–1229_1...	+1.32	−0.25	+12	+ 4	+2°.8	Star 7 not measured
1534_1–1208_1...	+1.40	+0.74	+10	− 8	+0.5	
1548_1–1229_2...	+1.40	+0.73	+31	+13	+1.0	
1112_2–1190_1...	+1.29	−0.24	+20	+12	+2.3	
1534_2–1625_1...	+1.27	−0.24	− 7	−15	+1.8	
1548_2–1208_2...	+1.39	+0.74	+33	+15	+2.0	
1559_1–1190_2...	+1.37	+0.75	− 2	−20	−0.5	
1559_2–1625_2...	+1.25	−0.23	+ 3	− 5	+2.0	

$$+2.42\,\mu_\alpha + 2.89\,\pi = +45 \qquad \pi = +7.7 = +0''.013 \pm 0''.005$$

$$+2.89\,\mu_\alpha + 14.30\,\pi = +137 \qquad \mu_\alpha = +9.4 = +0''.015 \pm 0''.012$$

Comp. Star	Br.	x	y	π	P.E.
1........	bf	+0′.3	+1′.1	0″.000	±0″.004
2........	f	+1.1	+2.1	−0.001	0.009
3........	bf	+3.2	+4.8	+0.002	0.008
4........	f	+6.2	0.0	+0.006	0.007
5........	bf	+2.1	−3.6	−0.015	0.009
6........	f	+1.4	−6.9	+0.009	0.007
7........	f	−2.6	−3.3	−0.009	0.011
8........	bf	−5.4	−3.2	+0.009	0.008
9........	f	−3.8	+4.0	+0.005	0.007
10........	bf	−2.2	+4.3	−0.008	±0.012

Boss 660 $\alpha = 2^h 50^m 11^s$ $\delta = +17^\circ 56'$

Pl. No.	Date	t	Obs.	Qy	p	m	Remarks
1094 . .	1916 Sept. 19	$-5^\circ.7$	H	fg	$+0.715$	$+1.72$	
1095 . .	19	-1.0	vM	fg	$+0.715$	$+1.72$	Trail
1191$_1$ . .	Dec. 16	$+2.0$	vM	g	-0.601	$+1.96$	
1191$_2$ . .	16	$+6.2$	H	g	-0.601	$+1.96$	
1209$_1$ . .	17	0.0	H	g	-0.615	$+1.96$	
1209$_2$ . .	17	$+4.5$	vM	g	-0.615	$+1.96$	
1230$_1$ . .	18	-4.0	H	g	-0.627	$+1.97$	
1230$_2$ . .	18	0.0	vM	g	-0.627	$+1.97$	
1535$_1$ . .	1917 Sept. 12	-1.0	vM	g	$+0.792$	$+2.70$	
1535$_2$ . .	12	$+3.5$	H	f, as	$+0.790$	$+2.70$	
1549$_1$ . .	13	$+1.0$	vM	fg	$+0.780$	$+2.70$	
1549$_2$ . .	13	$+5.5$	H	fg	$+0.780$	$+2.70$	
1560$_1$ . .	14	$+5.7$	vM	g	$+0.770$	$+2.71$	
1560$_2$ . .	14	$+9.7$	H	g	$+0.770$	$+2.71$	
1641$_1$ . .	Dec. 8	$+4.2$	vM	fg	-0.489	$+2.94$	
1641$_2$ . .	8	$+8.5$	H	fg	-0.489	$+2.94$	

Mor.-Ev.	Δp	Δm	M	v	Δt
1094 –1230$_1$ . .	$+1.34$	-0.25	-1	-10	$-1^\circ.7$
1095 –1209$_1$ . .	$+1.33$	-0.24	$+21$	$+12$	-1.0
1535$_1$–1230$_2$ . .	$+1.42$	$+0.73$	$+2$	-3	-1.0
1549$_1$–1191$_1$ . .	$+1.38$	$+0.74$	-17	-22	-1.0
1535$_2$–1641$_1$ . .	$+1.28$	-0.24	$+21$	$+13$	-0.7
1549$_2$–1209$_2$ . .	$+1.40$	$+0.74$	$+27$	$+22$	$+1.0$
1560$_1$–1191$_2$ . .	$+1.37$	$+0.75$	$+8$	$+3$	-0.5
1560$_2$–1641$_2$ . .	$+1.26$	-0.23	-10	-18	$+1.2$

$$+2\ 42\,\mu_a + 2.87\,\pi = +6 \qquad \pi = +5.7 = +0''.009 \pm 0''.006$$
$$+2.87\,\mu_a + 14.56\,\pi = +70 \qquad \mu_a = -4.3 = -0''.007 \pm 0''.014$$

Comp. Star	Br.	x	y	π	P.E.
1	ff	$+9'.6$	$0'.0$	$-0''.007$	$\pm 0''.007$
2	f	$+0.5$	-2.7	$+0.011$	0.009
3	f	$+1.2$	-8.2	-0.007	0.005
4	f	-8.9	$+1.0$	-0.009	0.007
5	f	-1.1	$+2.1$	$+0.019$	0.013
6	bf	-0.3	$+3.4$	$+0.002$	0.008
7	ff	-1.1	$+6.0$	-0.010	± 0.011

Boss 712 $\alpha=3^h2^m41^s$ $\delta=+18°25'$

Pl. No.	Date	t	Obs.	Qy	p	m	Remarks
745_1	1915 Oct. 1	+5°.0	vM	g	+0.620	+0.75	
745_2	1	+10.0	H	g	+0.620	+0.75	Trail
1192_1	1916 Dec. 16	+9.0	vM	g	−0.563	+1.96	
1192_2	16	+13.0	H	fg	−0.563	+1.96	
1210_1	17	+7.7	H	fg	−0.576	+1.96	
1210_2	17	+12.0	vM	g	−0.576	+1.96	
1231_1	18	+3.0	H	fg	−0.589	+1.97	
1231_2	18	+7.0	vM	g	−0.589	+1.97	
1536_1	1917 Sept. 12	+6.5	vM	fg	+0.822	+2.70	
1536_2	12	+10.7	H	f, as	+0.822	+2.70	
1537	12	+15.0	vM	fg	+0.822	+2.70	
1550_1	13	+8.5	vM	fg	+0.811	+2.70	
1550_2	13	+13.0	H	fg	+0.811	+2.70	
1573	Oct. 8	+9.5	vM	g, as	+0.521	+2.77	
1642_1	Dec. 8	+11.7	vM	fg	−0.447	+2.94	
1642_2	8	+15.7	H	fg	−0.447	+2.94	

Mor.-Ev.	Δp	Δm	M	v	Δt
745_1–1231_1	+1.21	−1.22	−36	−22	+2°.0
1536_1–1231_2	+1.41	+0.73	+59	+12	−0.5
1550_1–1210_1	+1.39	+0.74	+54	+7	+0.8
1573–1192_1	+1.08	+0.81	+23	−20	+0.5
745_2–1210_2	+1.20	−1.21	−7	+8	−2.0
1536_2–1642_1	+1.27	−0.24	+21	+5	−1.0
1550_2–1192_2	+1.37	+0.74	+34	−12	0.0
1537–1642_2	+1.27	−0.24	+32	+16	−0.7

$$+5.36\,\mu_\alpha + 0.41\,\pi = +166 \qquad \pi = +17.8 = +0''.029 \pm 0''.005$$
$$+0.41\,\mu_\alpha + 13.09\,\pi = +246 \qquad \mu_\alpha = +29.6 = +0''.048 \pm 0''.008$$

Comp. Star	Br.	x	y	π	P.E.
1	f	+1′.5	+2′.1	−0″.020	±0″.012
2	f	+4.1	+2.1	−0.005	0.004
3	bf	+7.0	−2.3	+0.020	0.014
4	f	+0.3	−4.4	−0.005	0.009
5	f	+1.5	−5.6	−0.013	0.017
6	b	−3.7	−2.1	+0.025	0.010
7	f	−3.8	−0.4	−0.002	0.010
8	f	−3.0	0.0	−0.008	0.008
9	f	−1.6	+5.8	−0.010	0.006
10	bf	−1.1	+5.8	+0.019	±0.005

Nova Persei $\alpha = 3^h24^m24^s$ $\delta = +43°34'$

Pl. No.	Date	t	Obs.	Qy	p	m	Remarks
670 . .	1915 Aug. 31	−14°0	H	fg	+0.984	+0.67	
698 . .	Sept. 2	− 2.2	H	fg	+0.939	+0.67	
728_1 . .	30	+ 2.5	vM	g	+0.704	+0.75	
728_2 . .	30	+ 6.7	C	fg	+0.704	+0.75	
792 . .	1916 Jan. 11	+ 5.5	vM	f	−0.796	+1.03	
793 . .	11	+11.0	vM	fg	−0.798	+1.03	
804 . .	12	+ 5.2	vM	fg	−0.806	+1.04	Clouds
1232_1 . .	Dec. 18	+ 9.0	H	g	−0.522	+1.97	
1232_2 . .	18	+13.0	vM	g	−0.522	+1.97	
1561_1 . .	1917 Sept. 14	+ 8.5	vM	fg	+0.856	+2.71	
1561_2 . .	14	+12.5	H	fg	+0.856	+2.71	
1574 . .	Oct. 8	+13.2	Pt	fg	+0.597	+2.77	
1653 . .	1918 Jan. 5	−10.0	vM	g	−0.743	+3.02	
1671 . .	6	−12.7	H&vM	fg	−0.755	+3.02	Clouds 50^m exp. Trail of Boss 805

Mor.-Ev.	Δp	Δm	M	v	Δt
728_1− 804 . .	+1.51	−0.29	+15	+ 6	−2°7
1561_1− 792 . .	+1.65	+1.68	−10	+ 1	+3.0
728_2−1232_1 . .	+1.23	−1.22	+43	+25	−2.3
1561_2− 793 . .	+1.65	+1.68	−19	− 8	+1.5
1574 −1232_2 . .	+1.12	+0.80	− 2	+ 1	+0.2
670 −1671 . .	+1.70	−2.35	+19	−13	−1.3
698 −1653 . .	+1.68	−2.35	+27	− 5	+7.8

$$+18.89\,\mu_\alpha - 3.45\,\pi = -215 \qquad \pi = +\ 4.1 = +0''.007 \pm 0''.004$$
$$-\ 3.45\,\mu_\alpha + 16.19\,\pi = +103 \qquad \mu_\alpha = -10.6 = -0''.017 \pm 0''.003$$

Comp. Star	Br.	x	y	π	P.E.
1	f	+2'.3	+1'.2	+0".018	±0".012
2	f	+3.6	+1.6	+0.009	0.011
3	f	+3.7	+2.2	−0.008	0.011
4	f	+1.5	−0.8	+0.007	0.006
5	f	+2.6	−1.1	−0.023	0.007
6	bf	−3.7	−3.8	−0.006	0.004
7	bf	−4.4	−1.5	+0.015	0.006
8	f	−2.7	+1.2	−0.001	0.007
9	bf	−3.4	+2.1	−0.015	±0.006

Boss 1154 $\alpha = 4^h48^m10^s$ $\delta = +2^\circ21'$

Pl. No.	Date	t	Obs.	Qy	p	m	Remarks
1116 ..	1916 Sept. 20	− 2°.5	H	g	+0.965	+1.72	
1133_1..	Oct. 21	− 6.5	vM	fg	+0.697	+1.81	Hazy 25^m exp.
1133_2..	21	+ 1.5	H	fg	+0.697	+1.81	Hazy 25^m exp.
1254_1..	1917 Feb. 12	− 6.5	vM	g	−0.923	+2.12	
1254_2..	12	− 2.5	H	fg	−0.923	+2.12	
1255_1..	12	+ 2.5	vM	fg	−0.923	+2.12	
1255_2..	12	+ 6.7	H	fg	−0.923	+2.12	Trail
1273 ..	13	− 6.5	vM	fg	−0.930	+2.12	20^m exp.
1274_1..	13	− 1.0	H	fg	−0.930	+2.12	20^m exp.
1274_2..	13	+ 4.5	vM	fg	−0.930	+2.12	20^m exp.
1575_1..	Oct. 8	− 1.0	vM	fg	+0.841	+2.77	
1575_2..	8	+ 3.0	vM	g, as	+0.841	+2.77	
1591_1..	24	+ 2.0	vM	fg	+0.663	+2.82	
1591_2..	24	+ 6.5	H	fg	+0.663	+2.82	

Mor.-Ev.	Δp	Δm	M	v	Δt
1133_1–1254_1..	+1.62	−0.31	−10	− 9	0°.0
1116 –1273 ..	+1.90	−0.40	− 3	− 1	+4.0
1575_1–1254_2..	+1.76	+0.65	+19	− 4	+1.5
1133_2–1274_1..	+1.63	−0.31	+ 9	+10	+2.5
1591_1–1255_1..	+1.59	+0.70	+30	+ 5	−0.5
1575_2–1274_2..	+1.77	+0.65	+26	+ 3	−1.5
1591_2–1255_2..	+1.59	+0.70	+21	− 4	−0.2

$$+2.17\,\mu_\alpha + 2.74\,\pi = +66 \qquad \pi = +4.1 = +0''.007 \pm 0''.002$$
$$+2.74\,\mu_\alpha + 20.18\,\pi = +153 \qquad \mu_\alpha = +25.3 = +0''.041 \pm 0''.006$$

Comp. Star	Br.	x	y	π	P.E.
1.........	bb	+0'.5	+4'.0	−0''.014	±0''.009
2.........	f	+3.0	+3.7	+0.008	0.005
3.........	f	+5.4	−1.8	−0.001	0.007
4.........	bf	+4.4	−4.7	+0.011	0.007
5.........	f	−1.0	−3.0	−0.024	0.006
6.........	f	−2.5	−0.7	+0.005	0.008
7.........	b	−5.9	+0.3	+0.012	0.006
8.........	f	−2.1	+1.9	+0.002	±0.003

Boss 1643 $\alpha = 6^h 24^m 57^s$ $\delta = +78° 5'$

Pl. No.	Date			t	Obs.	Qy	p	m	Remarks
299 ..	1914	Oct.	11	−11.°5	vM	fg	+0.971	−0.22	
300 ..			11	− 6.2	H	fg	+0.971	−0.22	
349_1 ..			14	−11.0	vM	g	+0.960	−0.21	
349_2 ..			14	− 7.0	vM	fg	+0.960	−0.21	Trail
475 ..	1915	Feb.	7	− 0.2	vM&H	f	−0.668	+0.11	2 exp. superposed
476_1 ..			7	+ 7.5	vM	fg	−0.668	+0.11	
476_2 ..			7	+12.5	H	fg	−0.668	+0.11	
490_1 ..		Mar.	8	− 8.0	vM	g	−0.942	+0.19	
490_2 ..			8	− 3.0	vM	g	−0.942	+0.19	
1153 ..	1916	Nov.	16	− 8.5	vM	g	+0.648	+1.88	
1182_1 ..			18	− 9.5	vM	fg	+0.622	+1.89	20^m exp.
1182_2 ..			18	− 3.2	vM	f	+0.622	+1.89	20^m exp.
1247 ..	1917	Jan.	29	+ 2.7	vM	fg	−0.553	+2.08	20^m exp.
1248 ..			29	+ 9.0	vM	fg	−0.553	+2.08	20^m exp.
1256_1 ..		Feb.	12	− 9.0	vM	fg	−0.734	+2.12	
1256_2 ..			12	− 4.7	H	fg	−0.734	+2.12	
1289 ..		Mar.	15	+ 2.0	vM	g	−0.976	+2.21	
1577 ..		Oct.	8	− 3.7	vM	g	+0.981	+2.77	13^m exp.
1593_1 ..			24	+ 1.7	vM	fg	+0.895	+2.82	20^m exp.
1593_2 ..			24	+ 7.2	H	fg	+0.894	+2.82	20^m exp.

Mor.-Ev.	Δp	Δm	M	v	Δt
299 −1256_1 ..	+1.71	−2.34	−23	−17	− 2.°5
1182_1− 490_1 ..	+1.56	+1.70	+65	+35	− 1.5
349_1−1256_2 ..	+1.69	−2.33	− 2	+ 4	− 6.3
1153 − 490_2 ..	+1.59	+1.69	+54	+23	− 5.5
349_2− 475 ..	+1.63	−0.32	+ 6	− 6	− 6.8
300 −1289 ..	+1.95	−2.43	−10	− 5	− 8.2
1182_2−1247 ..	+1.18	−0.19	+34	+25	− 5.9
1577 − 476_1 ..	+1.65	+2.66	+16	−24	−11.2
1593_1−1248 ..	+1.45	+0.74	+22	+ 2	− 7.3
1593_2− 476_2 ..	+1.56	+2.71	+13	−27	− 5.3

$$+37.67\,\mu_a + 1.61\,\pi = +371 \qquad \pi = +9.2 = +0.''015 \pm 0.''005$$
$$+1.61\,\mu_a + 25.84\,\pi = +253 \qquad \mu_a = +9.5 = +0.''015 \pm 0.''004$$

Comp. Star	Br.	x	y	π	P.E.
1	f	+3.′4	+4.′8	−0.″011	±0.″002
2	ff	+3.3	+1.6	+0.002	0.007
3	f	+6.9	0.0	+0.007	0.005
4	bb	+0.8	−6.5	−0.006	0.004
5	bf	−0.4	−2.6	+0.004	0.006
6	f	−6.5	−1.9	−0.002	0.006
7	f	−5.9	+3.6	+0.004	±0.005

Boss 1822 $\alpha=6^h59^m36^s$ $\delta=+34°38'$

Pl. No.	Date	t	Obs.	Qy	p	m	Remarks
1183_1..	1916 Nov. 18	− 4°2	vM	fg	+0.719	+1.89	20^m exp.
1183_2..	18	+ 2.5	vM	fg	+0.719	+1.89	20^m exp.
1257_1..	1917 Feb. 12	− 6.0	vM	fg	−0.631	+2.12	
1257_2..	12	− 1.7	H	fg	−0.631	+2.12	
1258_1..	12	+ 3.5	vM	fg	−0.631	+2.12	
1258_2..	12	+ 7.7	H	fg	−0.631	+2.12	
1281_1..	Mar. 13	− 6.0	vM	g	−0.926	+2.20	
1281_2..	13	− 1.7	H	g	−0.926	+2.20	Trail
1599_1..	Nov. 7	−10.2	vM	fg	+0.840	+2.85	
1599_2..	7	− 6.5	H	g	+0.840	+2.85	
1616_1..	8	− 8.0	H	fg	+0.832	+2.86	
1616_2..	8	− 3.7	vM	g	+0.832	+2.86	
1630_1..	Dec. 7	+ 1.2	H	fg	+0.462	+2.94	
1630_2..	7	+ 7.5	vM	fg	+0.462	+2.94	
1727_1..	1918 Mar. 3	− 5.5	vM	g	−0.844	+3.17	
1727_2..	3	− 1.5	vM	fg	−0.844	+3.17	

Mor.-Ev.	Δp	Δm	M	v	Δt
1599_1–1257_1..	+1.47	+0.73	− 4	−10	−4°2
1616_1–1281_1..	+1.76	+0.66	+28	+14	−2.0
1599_2–1727_1..	+1.68	−0.32	+44	− 2	−1.0
1183_1–1281_2..	+1.65	−0.31	+33	−11	−2.5
1616_2–1257_2..	+1.46	+0.74	+20	+15	−2.0
1630_1–1727_2..	+1.31	−0.23	+36	+ 1	+2.7
1183_2–1258_1..	+1.35	−0.23	+43	+ 7	−1.0
1630_2–1258_2..	+1.09	+0.82	−23	−18	−0.2

$$+2.49\,\mu_a+\ \ 2.53\,\pi=-\ \ 31 \qquad \pi\ =+20.6=+0''.034\pm0''.004$$
$$+2.53\,\mu_a+17.66\,\pi=+280 \qquad \mu_a=-33.4=-0''.054\pm0''.010$$

Comp. Star	Br.	y	y	π	P.E.
1..........	f	+3'.8	+1'.4	−0''.009	±0''.010
2..........	ff	+2.9	+0.1	+0.001	0.009
3..........	bf	+4.5	−1.4	+0.026	0.007
4..........	f	0.0	−2.5	−0.030	0.009
5..........	f	−1.4	−0.4	−0.001	0.011
6..........	f	−5.2	−0.7	+0.007	0.004
7..........	f	−4.4	+0.3	+0.010	0.005
8..........	f	−0.3	+3.7	−0.006	±0.007

N.G.C. 2392 $\alpha = 7^h 23^m 15^s$ $\delta = +21^{\circ} 7'$

Pl. No.	Date	t	Obs.	Qy	p	m	Remarks
1259 . .	1917 Feb. 12	$+9\overset{\circ}{.}0$	vM	fg	-0.555	$+2.12$	
1282_1 . .	Mar. 13	$+1.5$	vM	g	-0.883	$+2.20$	20^m exp.
1282_2 . .	13	$+6.7$	H	g	-0.883	$+2.20$	20^m exp.
1305_1 . .	16	$+1.5$	vM	g	-0.905	$+2.21$	
1305_2 . .	16	$+5.7$	H	fg	-0.905	$+2.21$	
1600_1 . .	Nov. 7	-5.2	vM	g	$+0.881$	$+2.85$	
1600_2 . .	7	-1.0	H	fg	$+0.881$	$+2.85$	
1601 . .	7	$+7.0$	vM	fg	$+0.881$	$+2.85$	
1617_1 . .	8	-3.0	H	g, as	$+0.871$	$+2.86$	Hazy 20^m exp.
1617_2 . .	8	$+2.5$	vM	g, as	$+0.871$	$+2.86$	Hazy 20^m exp.
1631 . .	Dec. 7	$+14.5$	vM	fg	$+0.540$	$+2.94$	20^m exp.
1647_1 . .	8	$+2.0$	vM	fg	$+0.529$	$+2.94$	
1647_2 . .	8	$+6.0$	H	g	$+0.529$	$+2.94$	
1648 . .	8	$+10.5$	vM	g	$+0.529$	$+2.94$	
1728_1 . .	1918 Mar. 3	$+2.5$	Hum	fg	-0.786	$+3.17$	Hazy 20^m exp.
1728_2 . .	3	$+8.5$	vM	g	-0.786	$+3.17$	Hazy 20^m exp.
1738 . .	4	-5.2	Hum	f	-0.798	$+3.18$	Hazy 25^m exp. Trail of β Gem.
1745 . .	April 4	$+11.5$	vM	g	-0.986	$+3.26$	Clouds 25^m exp.

Mor.-Ev.	Δp	Δm	M	v	Δt
1600_1–1738 . .	$+1.68$	-0.33	$+14$	-7	$0\overset{\circ}{.}0$
1617_1–1282_1 . .	$+1.75$	$+0.66$	$+12$	-7	-4.5
1600_2–1305_1 . .	$+1.79$	$+0.64$	$+33$	$+13$	-2.5
1647_1–1728_1 . .	$+1.32$	-0.23	$+18$	$+1$	-0.5
1617_2–1305_2 . .	$+1.78$	$+0.65$	$+2$	-18	-3.2
1601 –1282_2 . .	$+1.76$	$+0.65$	$+28$	$+9$	$+0.3$
1647_2–1728_2 . .	$+1.32$	-0.23	$+19$	$+2$	-2.5
1648 –1259 . .	$+1.08$	$+0.82$	$+14$	$+4$	$+1.5$
1631 –1745 . .	$+1.53$	-0.32	$+25$	$+5$	$+3.0$

$$+2.67\,\mu_a + 3.86\,\pi = +38 \qquad \pi = +12.2 = +0\overset{''}{.}020 \pm 0\overset{''}{.}003$$
$$+3.86\,\mu_a + 22.34\,\pi = +259 \qquad \mu_a = -3.4 = -0\overset{''}{.}006 \pm 0\overset{''}{.}008$$

Comp. Star	Br.	x	y	π	P.E.
1.	f	$+2\overset{'}{.}2$	$+2\overset{'}{.}1$	$-0\overset{''}{.}011$	$\pm 0\overset{''}{.}006$
2.	bf	$+6.9$	-0.5	-0.001	0.005
3.	f	$+1.9$	-0.8	$+0.022$	0.009
4.	bf	-0.5	-3.3	-0.009	0.004
5.	bf	-5.2	-1.6	-0.006	0.004
6.	bf	-3.7	$+1.1$	$+0.018$	0.009
7.	bf	-1.1	$+1.0$	-0.015	0.006
8.	ff	-0.3	$+3.2$	-0.001	± 0.005

Boss 2245 $\alpha = 8^h 21^m 12^s$ $\delta = +12^{\circ} 59'$

Pl. No.	Date	t	Obs.	Qy	p	m	Remarks
815 ..	1916 Feb. 24	$-2^{\circ}.7$	H	fg	-0.513	$+1.15$	Clouds 20^m exp.
1198_1..	Dec. 16	-7.7	H	g	$+0.590$	$+1.96$	
1198_2..	16	-3.0	vM	g	$+0.590$	$+1.96$	
1199_1..	16	$+2.5$	H	g	$+0.590$	$+1.96$	
1199_2..	16	$+6.5$	vM	g	$+0.590$	$+1.96$	
1283_1..	1917 Mar. 13	$+0.2$	vM	fg	-0.741	$+2.20$	19^m exp.
1283_2..	13	$+5.0$	H	g	-0.741	$+2.20$	
1292_1..	15	0.0	H	fg	-0.761	$+2.21$	
1292_2..	15	$+5.5$	vM	fg	-0.761	$+2.21$	20^m exp.
1306_1..	16	-2.0	vM	g	-0.772	$+2.21$	
1306_2..	16	$+2.2$	H	g	-0.772	$+2.21$	
1318 ..	April 12	$+6.0$	vM	fg	-0.959	$+2.28$	
1602_1..	Nov. 7	-2.0	H	fg	$+0.941$	$+2.85$	
1602_2..	7	$+2.2$	vM	g	$+0.941$	$+2.85$	
1618_1..	8	-4.7	H	fg	$+0.938$	$+2.86$	Hazy 20^m exp.
1618_2..	8	$+1.0$	vM	fg	$+0.938$	$+2.86$	Hazy 20^m exp. Trail

Mor.-Ev.	Δp	Δm	M	v	Δt
1618_1– 815 ..	$+1.45$	$+1.71$	-12	-5	$-2^{\circ}.0$
1198_1–1306_1..	$+1.36$	-0.25	$+8$	-19	-5.7
1198_2–1292_1..	$+1.35$	-0.25	$+29$	$+2$	-3.0
1602_1–1283_1..	$+1.68$	$+0.65$	$+34$	$+18$	-2.2
1618_2–1306_2..	$+1.71$	$+0.65$	$+30$	$+14$	-1.2
1602_2–1283_2..	$+1.68$	$+0.65$	-6	-22	-2.8
1199_1–1292_2..	$+1.35$	-0.25	$+16$	-11	-3.0
1199_2–1318 ..	$+1.55$	-0.32	$+49$	$+17$	$+0.5$

$$+4.48\,\mu_a + 4.25\,\pi = -12 \qquad \pi = +16.5 = +0''.027 \pm 0''.005$$
$$+4.25\,\mu_a + 18.55\,\pi = +229 \qquad \mu_a = -18.3 = -0''.030 \pm 0''.010$$

Comp. Star	Br.	x	y	π	P.E.
1........	f	$+2'.7$	$+3'.6$	$-0''.008$	$\pm 0''.008$
2........	f	$+1.8$	-0.3	-0.004	0.009
3........	f	$+2.3$	-2.6	$+0.003$	0.017
4........	bf	$+4.8$	-5.8	$+0.022$	0.007
5........	bf	$+2.5$	-6.0	-0.016	0.006
6........	bf	-4.9	-1.1	$+0.008$	0.006
7........	f	-3.0	$+1.5$	-0.007	0.004
8........	bf	-4.7	$+3.6$	$+0.001$	0.008
9........	bf	-1.2	$+6.0$	$+0.017$	0.008
10........	bf	-0.5	$+1.8$	-0.015	± 0.008

Boss 2378 = Bu 4823 $\alpha = 8^h46^m28^s$ $\delta = +28°38'$

Pl. No.	Date	t	Obs.	Qy	p	m	Remarks
1219_1 . .	1916 Dec. 17	−7°.0	vM	g	+0.648	+1.96	
1219_2 . .	17	−2.2	H	g	+0.648	+1.96	
1284_1 . .	1917 Mar. 13	+5.0	vM	fg	−0.662	+2.20	
1284_2 . .	13	+9.5	H	g	−0.662	+2.20	
1307_1 . .	16	+1.5	vM	g	−0.699	+2.21	
1307_2 . .	16	+5.7	H	g	−0.699	+2.21	
1332_1 . .	April 13	+2.0	vM	fg	−0.931	+2.28	
1332_2 . .	13	+6.2	H	fg	−0.931	+2.28	
1632_1 . .	Dec. 7	+1.5	H	fg	+0.762	+2.94	20^m exp.
1632_2 . .	7	+6.0	vM	fg	+0.762	+2.94	
1633 . .	7	+10.5	H	f	+0.762	+2.94	
1649_1 . .	8	−3.7	H	fg	+0.751	+2.94	
1649_2 . .	8	+0.5	vM	g	+0.751	+2.94	
1665 . .	1918 Jan. 5	+7.0	H	fg	+0.387	+3.02	
1729 . .	Mar. 3	−2.5	Hum	g	−0.524	+3.17	Clouds
1739 . .	4	−3.7	vM	fg	−0.542	+3.18	Clouds 30^m exp. Trail of Boss 2348

Mor.-Ev.	Δp	Δm	M	v	Δt
1219_1–1307_1 . .	+1.35	−0.25	+14	+9	−8°.5
1649_1–1739 . .	+1.29	−0.24	+34	+30	0.0
1219_2–1332_1 . .	+1.58	−0.32	−21	−27	−4.2
1649_2–1729 . .	+1.28	−0.23	+1	−3	+3.0
1632_1–1284_1 . .	+1.42	+0.74	+27	+45	−3.5
1632_2–1307_2 . .	+1.46	+0.73	−47	−30	+0.3
1665 –1332_2 . .	+1.32	+0.74	−33	−15	+0.8
1633 –1284_2 . .	+1.42	+0.74	−17	+1	+1.0

$$+2.45\,\mu_\alpha + 2.70\,\pi = -56 \qquad \pi = -0.5 = -0''.001 \pm 0''.009$$

$$+2.70\,\mu_\alpha + 15.53\,\pi = -68 \qquad \mu_\alpha = -22.3 = -0''.036 \pm 0''.022$$

The faint component is comparison star 6; it is not known if the two stars form a physical pair.

Comp. Star	Br.	x	y	π	P.E.
1	bf	+3′.7	−0′.8	0″.000	±0″.005
2	ff	+1.5	−4.5	−0.006	0.004
3	f	−1.4	−2.5	+0.013	0.006
4	f	−3.8	−0.5	−0.004	0.005
5	bf	−0.4	+4.0	0.000	0.005
6	bf	−0.4	+0.7	−0.005	±0.008

Boss 2442 = Bu 4929 $\alpha = 9^h 1^m 41^s$ $\delta = +23°23'$

Pl. No.	Date			t	Obs.	Qy	p	m	Remarks
1220_1..	1916	Dec.	17	+ 0°.2	vM	fg	+0.688	+1.96	
1220_2..			17	+ 4.5	H	g	+0.688	+1.96	
1285 ..	1917	Mar.	13	+12.0	vM	g	−0.612	+2.20	
1293_1..			15	+ 3.0	H	f	−0.637	+2.21	20^m exp.
1293_2..			15	+ 9.2	vM	fg	−0.637	+2.21	20^m exp.
1308_1..			16	+ 7.5	vM	g	−0.650	+2.21	
1308_2..			16	+11.7	H	fg	−0.650	+2.21	Trail; 2442_1 not behind sector
1347 ..		April	14	+ 3.0	vM	pf	−0.904	+2.29	25^m exp.
1650_1..		Dec.	8	+ 4.5	H	fg	+0.784	+2.94	
1650_2..			8	+ 9.0	vM	g	+0.784	+2.94	
1666_1..	1918	Jan.	5	+ 9.0	vM	g	+0.441	+3.02	
1666_2..			5	+13.2	H	g	+0.441	+3.02	
1678_1..			6	+ 1.7	vM	g	+0.426	+3.02	Clouds 25^m exp.
1678_2..			6	+ 8.5	H	fg	+0.426	+3.02	Clouds 25^m exp.
1746_1..		April	4	− 4.2	H	fg	−0.842	+3.26	Clouds 20^m exp.
1746_2..			4	+ 2.0	vM	g	−0.842	+3.26	Clouds 20^m exp.

Mor.-Ev.	Δp	Δm	M (2442_1)	M (2442_2)	v (2442_1)	v (2442_2)	Δt
1220_1–1347	+1.59	−0.33	+65	+45	+20	+ 5	−2°.8
1678_1–1746_1.....	+1.27	−0.24	+45	+26	+11	− 4	+5.9
1220_2–1293_1.....	+1.33	−0.25	+27	+47	− 9	+16	+1.5
1650_1–1746_2.....	+1.63	−0.32	+25	+26	−19	−13	+2.5
1678_2–1308_1.....	+1.08	+0.81	−73	−73	−18	−26	+1.0
1650_2–1293_2.....	+1.42	+0.73	−56	−50	−12	−12	−0.2
1666_1–1308_2.....	+1.09	+0.81	....	−46	....	+ 1	−2.7
1666_2–1285	+1.05	+0.82	−26	−13	+29	+36	+1.2

$$\left.\begin{array}{l} +2.19\,\mu_a + 1.10\,\pi = -168 \\ +1.10\,\mu_a + 12.86\,\pi = +53 \end{array}\right\}(2442_1) \qquad \left.\begin{array}{l} +2.85\,\mu_a + 1.98\,\pi = -185 \\ +1.98\,\mu_a + 14.05\,\pi = -4 \end{array}\right\}(2442_2)$$

$$(2442_1)\left\{\begin{array}{l} \pi = +11.1 = +0''.018 \pm 0''.007 \\ \mu_a = -82.3 = -0''.134 \pm 0''.016 \end{array}\right. \qquad (2442_2)\left\{\begin{array}{l} \pi = +9.9 = +0''.016 \pm 0''.006 \\ \mu_a = -71.8 = -0''.117 \pm 0''.014 \end{array}\right.$$

The 12^M companion, being covered by the sector, does not show any images on the plates.

Comp. Star	Br.	x	y	π	P.E.
1.........	f	+2′.9	+2′.2	−0″.012	±0″.011
2.........	f	+4.7	+5.5	−0.008	0.005
3.........	ff	+5.6	+3.8	+0.008	0.012
4.........	f	+3.8	−6.9	+0.011	0.003
5.........	f	−5.5	−4.4	−0.016	0.006
6.........	ff	−7.7	−1.0	+0.007	0.008
7.........	f	−4.1	+4.3	+0.011	±0.008

Boss 2660 $\alpha = 9^h 50^m 15^s$ $\delta = +57^\circ 54'$

Pl. No.	Date	t	Obs.	Qy	p	m	Remarks
76_1..	1914 Jan. 19	+1°.0	vM	g	+0.406	−0.95	
76_2..	19	+6.0	vM	fg	+0.406	−0.95	
84_1..	20	−1.5	vM	fg	+0.391	−0.94	
84_2..	20	+2.0	vM	g	+0.391	−0.94	
105 ..	April 17	+3.0	vM	f	−0.833	−0.70	
114_1..	18	+4.5	vM	g, as	−0.840	−0.70	
114_2..	18	+8.7	vM	fg	−0.840	−0.70	
881_1..	1916 April 22	+8.7	H	fg	−0.875	+1.31	
881_2..	22	+13.0	vM	g	−0.875	+1.31	
1320_1..	1917 April 12	+1.7	H	g	−0.791	+2.28	
1320_2..	12	+5.7	vM	g	−0.793	+2.28	
1334 ..	13	+8.0	vM	g	−0.801	+2.28	
1634_1..	Dec. 7	+1.2	vM	fg	+0.868	+2.94	
1634_2..	7	+5.2	H	fg	+0.868	+2.94	
1667 ..	1918 Jan. 5	+12.7	H	g	+0.597	+3.02	
1679 ..	6	+5.2	vM	fg	+0.584	+3.02	Clouds 26^m exp. Trail

Mor.-Ev.	Δp	Δm	M	v	Δt
84_1–1320_1...	+1.18	−3.22	−47	+18	−3°.2
76_1– 105 ...	+1.24	−0.25	−45	−43	−2.0
1634_1– 114_1...	+1.71	+3.64	+84	+3	−3.3
84_2–1320_2...	+1.18	−3.22	−65	0	−3.7
1634_2–1334 ...	+1.67	+0.66	+28	+10	−2.8
76_2– 114_2...	+1.25	−0.25	−17	−15	−2.7
1679 – 881_1...	+1.46	+1.71	+59	+20	−3.5
1667 – 881_2...	+1.47	+1.71	+38	−1	−0.3

$$+40.39\,\mu_\alpha + 4.11\,\pi = +865 \qquad \pi = +2.3 = +0''.004 \pm 0''.006$$

$$+4.11\,\mu_\alpha + 15.88\,\pi = +124 \qquad \mu_\alpha = +21.2 = +0''.035 \pm 0''.004$$

Comp. Star	Br.	x	y	π	P.E.
1.........	f	+1′.6	+0′.8	+0″.004	±0″.011
2.........	bf	+3.6	+4.8	−0.015	0.006
3.........	b	+6.0	−2.1	+0.005	0.005
4.........	bf	+1.0	−4.5	+0.001	0.007
5.........	f	−3.4	−6.7	−0.011	0.006
6.........	bf	−2.5	−2.5	+0.006	0.005
7.........	bf	−4.9	+5.2	−0.003	0.002
8.........	bf	−0.8	+5.1	+0.013	±0.006

Boss 3030 $\alpha = 11^h25^m16^s$ $\delta = +18°58'$

Pl. No.	Date	t	Obs.	Qy	p	m	Remarks
799_1 . .	1916 Jan. 11	− 4°.0	vM	f	+0.781	+1.03	
799_2 . .	11	0.0	vM	fg	+0.781	+1.03	
882_1 . .	April 22	− 3.2	H	g	−0.620	+1.31	
882_2 . .	22	+ 2.0	vM	fg	−0.620	+1.31	
883 . .	22	+ 7.2	H	fg	−0.620	+1.31	
898_1 . .	23	− 9.5	H	g, as	−0.631	+1.31	Some astigmatism of 60-in. mirror
898_2 . .	23	− 5.0	vM	g, as	−0.631	+1.31	
1224 . .	Dec. 17	− 7.0	H	fg	+0.899	+1.96	
1225 . .	17	− 3.5	vM	fg	+0.899	+1.96	
1253_1 . .	1917 Jan. 29	− 7.5	vM	fg	+0.593	+2.08	
1253_2 . .	29	− 3.5	vM	f	+0.593	+2.08	
1266 . .	Feb. 12	+ 6.5	H	f	+0.406	+2.12	
1322_1 . .	April 12	− 0.7	vM	g	−0.489	+2.28	
1322_2 . .	12	+ 3.2	H	g	−0.489	+2.28	Trail

Mor.-Ev.	Δp	Δm	M	v	Δt
1253_1 − 898_1 . . .	+1.22	+0.77	− 4	+11	+2°.0
1224 − 898_2 . . .	+1.53	+0.65	+18	+24	−2.0
799_1 − 882_1 . . .	+1.40	−0.28	+ 5	−25	−0.8
1225 − 1322_1 . . .	+1.39	−0.32	+47	+15	−2.8
1253_2 − 882_2 . . .	+1.21	+0.77	−40	−25	−5.5
799_2 − 1322_2 . . .	+1.27	−1.25	+69	+ 1	−3.2
1266 − 883 . . .	+1.03	+0.81	−27	− 8	−0.7

$$+4.00\,\mu_a + 1.27\,\pi = -146 \qquad \pi = +13.4 = +0''.022 \pm 0''.007$$

$$+1.27\,\mu_a + 11.85\,\pi = +107 \qquad \mu_a = -40.8 = -0''.067 \pm 0''.012$$

Comp. Star	Br.	x	y	π	P.E.
1.	f	+1′.9	+0′.3	+0″.008	±0″.010
2.	f	+5.6	+0.8	+0.013	0.010
3.	f	+8.2	−4.5	−0.010	0.012
4.	f	−2.3	−0.7	−0.003	0.010
5.	ff	−8.5	+2.6	+0.003	0.006
6.	f	−1.2	+6.0	−0.012	±0.007

Boss 3169 $\alpha = 12^h 4^m 34^s$ $\delta = +2^\circ 28'$

Pl. No.	Date	t	Obs.	Qy	p	m	Remarks
448 . .	1915 Jan. 10	$-8.^\circ 0$	H	fg	+0.853	+0.03	Clouds 20^m exp.
449 . .	10	−1.7	vM	fg	+0.853	+0.03	Thick 20^m exp.
450 . .	10	+5.7	H	fg	+0.853	+0.03	Thick 20^m exp. Trail
481 . .	Feb. 7	−9.0	vM	g, as	+0.616	+0.11	
915_1 . .	1916 April 24	+1.0	vM	g, as	−0.506	+1.32	
915_2 . .	24	+5.0	vM	g	−0.506	+1.32	
1351_1 . .	1917 May 12	+6.5	H	fg	−0.717	+2.36	
1351_2 . .	12	+10.5	vM	fg	−0.717	+2.36	
1364_1 . .	13	+2.0	vM	g, as	−0.725	+2.37	
1364_2 . .	13	+6.0	vM	g, as	−0.725	+2.37	
1365 . .	13	+13.0	vM	fg	−0.725	+2.37	
1377_1 . .	14	−8.0	vM	g	−0.735	+2.37	
1377_2 . .	14	−3.7	vM	fg	−0.735	+2.37	
1695 . .	1918 Feb. 2	0.0	vM	fg	+0.669	+3.09	
1696 . .	2	+5.0	H	g	+0.669	+3.09	
1697 . .	2	+9.5	vM	fg	+0.667	+3.09	
1716 . .	3	+5.5	vM	g	+0.656	+3.10	Hazy 20^m exp.
1717 . .	3	+11.7	H	g	+0.656	+3.10	Hazy 20^m exp.

Mor.-Ev.	Δp	Δm	M	v	Δt
481 − 1377_1 . . .	+1.35	−2.26	−5	+13	$-1.^\circ 0$
448 − 1377_2 . . .	+1.59	−2.34	−9	+7	−4.3
449 − 915_1 . . .	+1.36	−1.29	+3	+4	−2.7
1695 − 1364_1 . . .	+1.39	+0.72	+12	−24	−2.0
1696 − 915_2 . . .	+1.18	+1.77	+45	−6	0.0
1716 − 1364_2 . . .	+1.38	+0.73	+53	+18	−0.5
450 − 1351_1 . . .	+1.57	−2.33	−40	−24	−0.8
1697 − 1351_2 . . .	+1.38	+0.73	+57	+22	−1.0
1717 − 1365 . . .	+1.38	+0.73	+25	−10	−1.3

$$+22.92\,\mu_\alpha - 6.06\,\pi = +309 \qquad \pi = +16.2 = +0.''026 \pm 0.''005$$
$$-6.06\,\mu_\alpha + 17.68\,\pi = +179 \qquad \mu_\alpha = +17.8 = +0.''029 \pm 0.''004$$

Comp. Star	Br.	x	y	π	P.E.
1	f	$+5.'2$	$+0.'7$	$+0.''011$	$\pm 0.''008$
2	f	+5.4	+1.0	−0.004	0.006
3	f	+4.0	−3.6	−0.008	0.004
4	f	−5.9	−3.0	+0.004	0.002
5	f	−3.3	+4.5	−0.003	±0.002

N.G.C. 5194 $\alpha = 13^h 25^m 40^s$ $\delta = +47^\circ 43'$

Pl. No.	Date	t	Obs.	Qy	p	m	Remarks
832 ..	1916 Mar. 11	+ 1.°2	vM	g	+0.488	+1.20	
932 ..	May 22	− 5.0	vM	f	−0.586	+1.39	40^m exp.
943 ..	24	+ 2.5	vM	f	−0.611	+1.40	45^m exp.
953 ..	25	− 3.5	vM	g	−0.623	+1.40	35^m exp.
954 ..	25	+ 7.5	H	fg	−0.623	+1.40	35^m exp.
1298 ..	1917 Mar. 15	− 5.2	vM	fg	+0.436	+2.20	
1299 ..	15	+ 3.2	H	fg	+0.436	+2.20	
1313 ..	16	− 4.7	H	fg	+0.423	+2.21	
1314 ..	16	+ 3.2	vM	fg	+0.422	+2.21	
1352 ..	May 12	+ 0.7	H	g, as	−0.448	+2.36	
1366 ..	13	+ 2.5	Pt	fg	−0.463	+2.37	
1380 ..	14	+ 9.0	vM	fg	−0.475	+2.37	40^m exp.
1388 ..	June 11	+ 6.2	vM	fg	−0.795	+2.45	Trail
1698 ..	1918 Feb. 2	− 0.7	H	g	+0.856	+3.09	
1699 ..	2	+ 7.2	vM	fg	+0.856	+3.09	
1719 ..	3	+11.2	H&vM	fg	+0.851	+3.10	Hazy 45^m exp.

Mor.-Ev.	Δp	Δm	M	v	Δt
1298 − 932 ...	+1.02	+0.81	− 9	− 9	−0.°2
1313 −1352 ...	+0.87	−0.15	− 7	−10	−5.4
1698 − 953 ...	+1.48	+1.69	+26	+26	+2.8
832 − 943 ...	+1.10	−0.20	+24	+19	−1.3
1299 −1366 ...	+0.90	−0.17	+20	+16	+0.7
1314 −1388 ...	+1.22	−0.24	−14	−19	−3.0
1699 − 954 ...	+1.48	+1.69	−21	−21	−0.3
1719 −1380 ...	+1.33	+0.73	0	− 2	+2.2

$$+7.06\,\mu_\alpha + 6.01\,\pi = -2 \qquad \pi = +3.3 = +0.''005 \pm 0.''008$$

$$+6.01\,\mu_\alpha + 11.46\,\pi = +19 \qquad \mu_\alpha = -3.1 = -0.''005 \pm 0.''011$$

Comp. Star	Br.	x	y	π	P.E.
1.........	bf	+7.′0	+4.′9	−0.″006	±0.″004
2.........	bf	+3.8	−1.4	+0.025	0.014
3.........	f	+2.6	−6.5	+0.022	0.010
4.........	f	−1.5	−2.5	−0.036	0.013
5.........	bf	−1.1	−1.0	−0.019	0.017
6.........	f	−1.9	+0.1	−0.032	0.015
7.........	f	−2.3	+1.9	+0.015	0.012
8.........	bf	−6.6	+2.6	+0.028	±0.013

Boss 3658 $\alpha = 14^h 10^m 22^s$ $\delta = +41°59'$

Pl. No.	Date	t	Obs.	Qy	p	m	Remarks
955_1..	1916 May 25	+6°.0	vM	g	−0.473	+1.40	
955_2..	25	+10.5	H	g	−0.473	+1.40	
1269 ..	1917 Feb. 12	−16.0	vM	fg	+0.876	+2.12	20^m exp.
1270 ..	12	− 9.2	H	fg	+0.876	+2.12	20^m exp.
1271 ..	12	− 2.5	vM	fg	+0.876	+2.12	20^m exp.
1272 ..	12	+ 4.5	H	fg	+0.876	+2.12	20^m exp.
1300_1..	Mar. 15	+ 1.7	vM	g	+0.599	+2.21	20^m exp.
1300_2..	15	+ 7.2	H	g	+0.599	+2.21	20^m exp.
1389_1..	June 11	+ 5.5	H	fg	−0.684	+2.45	17^m exp.
1389_2..	11	+ 9.5	vM	fg	−0.684	+2.45	
1400 ..	12	− 6.0	vM	fg	−0.695	+2.45	Some astigmatism of 60-in. mirror
1401_1..	12	− 0.7	H	fg	−0.697	+2.45	
1401_2..	12	+ 3.0	vM	g, as	−0.697	+2.45	
1415 ..	13	− 5.0	vM	g, as	−0.707	+2.45	
1700 ..	1918 Feb. 2	+ 4.0	H	fg	+0.923	+3.09	
1736 ..	Mar. 3	+10.0	vM	fg	+0.734	+3.17	Clouds 25^m exp. Trail

Mor.-Ev.	Δp	Δm	M	v	Δt
1269 −1400 ...	+1.57	−0.33	+42	+18	−10°.0
1270 −1415 ...	+1.58	−0.33	+12	−12	− 4.2
1271 −1401_1...	+1.57	−0.33	+15	− 9	− 1.8
1300_1−1401_2...	+1.30	−0.24	+16	− 4	− 1.3
1700 −1389_1...	+1.61	+0.64	+49	+28	− 1.5
1272 − 955_1...	+1.35	+0.72	− 5	−21	− 1.5
1300_2− 955_2...	+1.07	+0.81	+20	+ 8	− 3.3
1736 −1389_2...	+1.42	+0.72	+ 6	−11	+ 0.5

$$+2.50\,\mu_\alpha + 2.02\,\pi = +20 \qquad \pi = +14.4 = +0''.023 \pm 0''.005$$

$$+2.02\,\mu_\alpha + 16.68\,\pi = +232 \qquad \mu_\alpha = -3.6 = -0''.006 \pm 0''.013$$

Comp. Star	Br.	x	y	π	P.E.
1.........	f	+2'.5	+2'.3	−0".029	±0".010
2.........	bf	+3.4	−3.7	−0.012	0.010
3.........	f	+5.2	−3.8	+0.028	0.015
4.........	bf	−1.8	−5.5	−0.001	0.002
5.........	f	−3.2	+1.4	−0.016	0.004
6.........	b	−2.1	+4.3	+0.011	0.012
7.........	bf	−3.8	+6.5	+0.016	±0.011

Boss 4343 $\alpha = 16^h 59^m 55^s$ $\delta = +35^\circ 33'$

Pl. No.	Date	t	Obs.	Qy	p	m	Remarks
921_1..	1916 April 24	−13.7	vM	vg	+0.662	+1.32	
921_2..	24	− 8.7	H	g	+0.662	+1.32	
1026_1..	July 22	+ 6.0	H	g	−0.697	+1.56	Some astigmatism of 60-in. mirror
1026_2..	22	+10.0	vM	g	−0.697	+1.56	
1040_1..	23	0.0	vM	g	−0.710	+1.56	
1040_2..	23	+ 4.0	vM	fg	−0.710	+1.56	
1328 ..	1917 April 12	−11.5	vM	fg	+0.801	+2.28	
1329 ..	12	− 6.5	H	fg	+0.801	+2.28	Trail
1345_1..	13	+ 0.5	vM	g	+0.791	+2.28	
1345_2..	13	+ 4.7	H	fg	+0.789	+2.28	
1346 ..	13	+ 9.2	vM	g	+0.789	+2.28	
1384_1..	May 14	+ 3.0	Pt	fg	+0.383	+2.37	20^m exp.
1384_2..	14	+ 8.5	vM	fg	+0.383	+2.37	20^m exp.
1385 ..	14	+14.7	Pt	f	+0.383	+2.37	20^m exp.
1456_1..	July 11	+ 6.5	H	g	−0.550	+2.53	
1456_2..	11	+10.5	vM	g	−0.550	+2.53	
1461 ..	13	+ 2.5	vM	g	−0.578	+2.53	Hazy 20^m exp.
1467 ..	28	− 7.7	vM	fg	−0.764	+2.58	
1468 ..	28	− 3.5	H	fg	−0.764	+2.58	
1475 ..	29	− 6.0	vM	fg	−0.774	+2.58	

Mor.-Ev.	Δp	Δm	M	v	Δt
921_1–1467 ...	+1.43	−1.26	−17	− 2	−6°0
1328 –1475 ...	+1.58	−0.30	+ 2	0	−5.5
921_2–1468 ...	+1.43	−1.26	−13	+ 2	−5.2
1329 –1040_1...	+1.51	+0.72	+26	+ 8	−6.5
1345_1–1461 ...	+1.37	−0.25	− 9	−11	−2.0
1384_1–1040_2...	+1.09	+0.81	+14	− 4	−1.0
1345_2–1026_1...	+1.49	+0.72	+21	+ 3	−1.3
1346 –1456_1...	+1.34	−0.25	+40	+38	+2.7
1384_2–1026_2...	+1.08	+0.81	+13	− 4	−1.5
1385 –1456_2...	+0.93	−0.16	−43	−44	+4.2

$$+5.78\,\mu_\alpha - 0.99\,\pi = +91 \qquad \pi = +4.3 = +0''.007 \pm 0''.005$$
$$-0.99\,\mu_\alpha + 17.98\,\pi = +61 \qquad \mu_\alpha = +16.5 = +0''.027 \pm 0''.010$$

Comp. Star	Br.	x	y	π	P.E.
1.........	f	+0'.5	+2'.6	+0''.016	±0''.009
2.........	ff	+4.7	+1.5	−0.013	0.011
3.........	f	+3.3	+1.1	−0.002	0.006
4.........	ff	+2.9	−0.7	−0.006	0.007
5.........	f	−0.1	−2.1	+0.027	0.007
6.........	bf	−3.3	−2.9	−0.012	0.006
7.........	f	−4.9	−1.0	−0.004	0.007
8.........	f	−4.0	+1.6	−0.004	±0.007

N.G.C. 6720 $\alpha = 18^h49^m53^s$ $\delta = +32°54'$

Pl. No.	Date	t	Obs.	Qy	p	m	Remarks
1056 ..	1916 Aug. 3	+ 6.5	H	g	−0.496	+1.59	Hazy
1071 ..	5	− 7.0	vM	fg	−0.532	+1.60	Hazy
1072 ..	5	0.0	H	fg	−0.532	+1.60	Hazy
1073 ..	5	+ 7.0	vM	g	−0.532	+1.60	Hazy
1360 ..	1917 May 12	−10.0	H	fg	+0.768	+2.36	
1361 ..	12	− 3.0	vM	g	+0.768	+2.36	
1362 ..	12	+ 6.2	H	fg	+0.766	+2.36	
1374 ..	13	−12.0	Pt	fg	+0.757	+2.37	30^m exp.
1375 ..	13	− 4.2	vM	fg	+0.755	+2.37	30^m exp.
1376 ..	13	+ 6.0	vM	fg	+0.755	+2.37	
1386 ..	14	− 3.7	vM	fg	+0.744	+2.37	30^m exp.
1488 ..	Aug. 10	− 4.5	H	g	−0.598	+2.61	Trail of Boss 4775
1497 ..	11	− 9.7	vM	fg	−0.612	+2.61	30^m exp.
1498 ..	11	+ 0.2	H	g	−0.612	+2.61	30^m exp.

Mor.-Ev.	Δp	Δm	M	v	Δt
1374–1497....	+1.37	−0.24	+ 1	0	−2°3
1360–1071....	+1.30	+0.76	+12	+ 6	−3.0
1375–1488....	+1.35	−0.24	+10	+10	+0.3
1386–1072....	+1.28	+0.77	+16	+10	−3.7
1361–1498....	+1.38	−0.25	−11	−11	−3.2
1376–1056....	+1.25	+0.78	−16	−22	−0.5
1362–1073....	+1.30	+0.76	+12	+ 6	−0.8

$$+2.54\,\mu_a + 2.95\,\pi = +19 \qquad \pi = +1.1 = +0''.002 \pm 0''.005$$
$$+2.95\,\mu_a + 12.18\,\pi = +32 \qquad \mu_a = +6.2 = +0''.010 \pm 0''.011$$

Comp. Star	Br.	x	y	π	P.E.
1.........	f	+2′.9	+3′.3	+0″.017	±0″.008
2.........	f	+2.2	+2.2	−0.006	0.013
3.........	bf	+1.0	0.0	−0.003	0.011
4.........	f	+1.6	−1.0	−0.032	0.008
5.........	bf	+2.7	−2.7	+0.022	0.005
6.........	bf	−1.9	−3.8	−0.004	0.007
7.........	bf	−4.3	−1.6	+0.013	0.005
8.........	f	−1.0	+0.8	−0.007	0.007
9.........	bf	−2.7	+2.9	0.000	±0.008

Boss 5083 $\alpha=19^h48^m58^s$ $\delta=+46°46'$

Pl. No.	Date	t	Obs.	Qy	p	m	Remarks
251 ..	1914 Sept. 15	−14.0	vM	fg	−0.827	−0.29	
267_1..	16	−13.7	vM	fg	−0.839	−0.29	
267_2..	16	− 9.7	vM	fg	−0.839	−0.29	
268 ..	16	− 4.0	H	g	−0.839	−0.29	Trail
1396_1..	1917 June 11	−10.5	vM	fg	+0.567	+2.45	
1396_2..	11	− 6.5	H	fg	+0.567	+2.45	
1397_1..	11	− 1.0	vM	fg	+0.567	+2.45	
1397_2..	11	+ 3.0	H	fg	+0.566	+2.45	
1410_1..	12	−14.5	H	fg	+0.554	+2.45	
1410_2..	12	−10.5	vM	fg	+0.554	+2.45	
1424_1..	13	−10.5	H	fg	+0.539	+2.45	
1424_2..	13	− 6.2	vM	fg	+0.539	+2.45	
1522 ..	Sept. 12	−16.0	vM	fg	−0.804	+2.70	
1523_1..	12	−10.7	H	fg	−0.804	+2.70	
1523_2..	12	− 6.5	vM	fg	−0.804	+2.70	
1540 ..	13	+ 6.2	vM	fg	−0.813	+2.70	Hazy 20^m exp.

Mor.-Ev.	Δp	Δm	M	v	Δt
1410_1–1522 ...	+1.36	−0.25	+13	+ 7	+1°5
1410_2– 251 ...	+1.38	+2.74	− 3	−14	+3.5
1424_1– 267_1...	+1.38	+2.74	+37	+26	+3.2
1396_1–1523_1...	+1.37	−0.25	+11	+ 5	+0.2
1396_2– 267_2...	+1.41	+2.74	+13	+ 2	+3.2
1424_2–1523_2...	+1.34	−0.25	− 5	−11	+0.3
1397_1– 268 ...	+1.41	+2.74	− 3	−14	+3.0
1397_2–1540 ...	+1.38	−0.25	+ 3	− 3	−3.2

$$+30.28\,\mu_\alpha+13.92\,\pi=+115 \qquad \pi=+4.3=+0''.007\pm0''.005$$
$$+13.92\,\mu_\alpha+15.21\,\pi=+\ 91 \qquad \mu_\alpha=+1.8=+0''.003\pm0''.004$$

Comp. Star	Br.	x	y	π	P.E.
1........	bf	+3′.8	+4′.9	+0″.013	±0″.013
2........	bf	+2.1	+3.7	−0.006	0.007
3........	f	+1.1	+1.4	+0.015	0.007
4........	f	+1.5	+0.1	−0.006	0.011
5........	bf	+4.8	−0.7	−0.010	0.006
6........	bf	+1.5	−6.3	−0.008	0.007
7........	f	−2.3	−4.9	+0.017	0.006
8........	bf	−5.4	−3.6	−0.002	0.008
9........	b	−6.0	+2.7	−0.006	0.008
10........	bf	−1.4	+3.8	−0.008	±0.007

B.D. +36° 3956 $\alpha = 20^h 10^m 47^s$ $\delta = +36°21'$

Pl. No.	Date	t	Obs.	Qy	p	m	Remarks
717_1..	1915 Sept. 30	+ 1.5	vM	fg	−0.894	+0.75	
717_2..	30	+ 5.7	vM	fg	−0.894	+0.75	
733_1..	Oct. 1	−10.5	vM	fg	−0.900	+0.75	
733_2..	1	− 6.5	vM	fg	−0.900	+0.75	
734 ..	1	− 0.5	H	fg	−0.900	+0.75	
1398 ..	1917 June 11	+ 6.0	vM	fg	+0.635	+2.45	
1399 ..	11	+10.7	H	fg	+0.635	+2.45	
1411_1..	12	−10.2	H	fg	+0.623	+2.45	
1411_2..	12	− 6.2	vM	fg	+0.623	+2.45	
1412_1..	12	− 1.2	H	fg	+0.623	+2.45	
1412_2..	12	+ 2.5	vM	fg	+0.623	+2.45	
1425_1..	13	− 5.0	H	fg	+0.611	+2.45	
1425_2..	13	− 0.2	vM	fg	+0.609	+2.45	
1438_1..	26	+ 4.5	vM	fg	+0.428	+2.49	
1438_2..	26	+ 8.7	H	g	+0.428	+2.49	
1524_1..	Sept. 12	− 5.2	H	fg	−0.743	+2.70	
1524_2..	12	− 1.2	vM	fg	−0.743	+2.70	
1525 ..	12	+ 3.0	H	fg	−0.743	+2.70	Trail of Boss 5188
1541_1..	13	+ 8.5	H	fg	−0.753	+2.70	
1541_2..	13	+13.0	vM	fg	−0.753	+2.70	

Mor.-Ev.	Δp	Δm	M	v	Δt
1411_1− 733_1...	+1.52	+1.70	+10	+ 8	+0°.3
1411_2− 733_2...	+1.52	+1.70	−11	−13	+0.3
1425_1−1524_1...	+1.35	−0.25	−26	−21	+0.2
1412_1−1524_2...	+1.37	−0.25	+ 5	+10	0.0
1425_2− 734 ...	+1.51	+1.70	+22	+20	+0.3
1412_2− 717_1...	+1.52	+1.70	−25	−27	+1.0
1438_1−1525 ...	+1.17	−0.21	−17	−12	+1.5
1398 − 717_2...	+1.53	+1.70	+19	+17	+0.3
1438_2−1541_1...	+1.18	−0.21	+ 3	+ 8	+0.2
1399 −1541_2...	+1.39	−0.25	+ 9	+14	−2.3

$$+14.71\,\mu_\alpha + 11.38\,\pi = +31 \qquad \pi = -3.0 = -0''.005 \pm 0''.006$$
$$+11.38\,\mu_\alpha + 19.94\,\pi = -\ 9 \qquad \mu_\alpha = +4.4 = +0''.007 \pm 0''.007$$

Comp. Star	Br.	x	y	π	P.E.
1.........	bf	+0'.3	+0'.7	−0''.007	±0''.008
2.........	f	+1.4	+1.9	+0.016	0.004
3.........	f	+1.5	+0.3	−0.016	0.009
4.........	f	+3.6	−0.7	+0.004	0.004
5.........	bf	+0.3	−3.8	+0.003	0.006
6.........	b	−0.5	−1.6	−0.009	0.011
7.........	bf	−1.1	−0.5	+0.015	0.012
8.........	b	−3.4	+1.6	+0.015	0.008
9.........	f	−2.1	+1.8	−0.021	±0.008

Boss 5409 = Bu 10663 $\alpha = 20^h55^m53^s$ $\delta = +18^\circ56'$

Pl. No.	Date			t	Obs.	Qy	p	m	Remarks
1103_1..	1916	Sept.	20	$+0^\circ.5$	vM	vg	-0.697	$+1.72$	
1103_2..			20	$+4.7$	H	vg	-0.697	$+1.72$	
1121_1..		Oct.	21	-6.5	vM	fg	-0.929	$+1.81$	
1121_2..			21	-2.5	vM	fg	-0.929	$+1.81$	
1413 ..	1917	June	12	-2.7	H	fg	$+0.747$	$+2.45$	
1414 ..			12	$+1.0$	vM	fg	$+0.747$	$+2.45$	
1426_1..			13	-4.5	H	f, as	$+0.737$	$+2.45$	
1426_2..			13	-0.5	vM	fg	$+0.737$	$+2.45$	Trail
1439_1..			26	$+3.5$	vM	g	$+0.581$	$+2.49$	
1439_2..			26	$+7.7$	H	fg	$+0.581$	$+2.49$	
1440 ..			26	$+12.0$	vM	fg	$+0.581$	$+2.49$	
1450 ..			27	-9.0	vM	fg	$+0.570$	$+2.49$	
1526_1..		Sept.	12	-0.7	vM	fg	-0.597	$+2.70$	
1526_2..			12	$+3.2$	H	fg	-0.597	$+2.70$	Trail
1542_1..			13	$+7.5$	H	fg	-0.609	$+2.70$	
1542_2..			13	$+12.7$	vM	fg	-0.609	$+2.70$	

Mor.-Ev.	Δp	Δm	M	v	Δt
$1450-1121_1$...	$+1.50$	$+0.68$	-5	$+7$	$-2^\circ.5$
1426_1-1121_2...	$+1.67$	$+0.64$	-19	-9	-2.0
$1413-1526_1$...	$+1.34$	-0.25	$+30$	$+17$	-2.0
1426_2-1103_1...	$+1.43$	$+0.73$	-21	-8	-1.0
$1414-1526_2$...	$+1.34$	-0.25	$+7$	-6	-2.2
1439_1-1103_2...	$+1.28$	$+0.77$	-4	$+11$	-1.2
1439_2-1542_1...	$+1.19$	-0.21	-9	-20	$+0.2$
$1440-1542_2$...	$+1.19$	-0.21	$+19$	$+8$	-0.7

$$+2.19\,\mu_\alpha + 2.95\,\pi = -45 \qquad \pi = +4.2 = +0''.007 \pm 0''.004$$

$$+2.95\,\mu_\alpha + 15.14\,\pi = -14 \qquad \mu_\alpha = -26.2 = -0''.043 \pm 0''.012$$

The $8^M.5$ star at $x = -0'.4$, $y = +0'.7$, which does not have the same p.m., is found to have a parallax $= -0''.004 \pm 0''.007$.

Comp. Star	Br.	x	y	π	P.E.
1.........	bf	$+0'.4$	$+2'.5$	$+0''.013$	$\pm 0''.008$
2.........	f	$+1.8$	$+2.9$	-0.001	0.005
3.........	bf	$+3.7$	-1.1	-0.002	0.006
4.........	f	$+1.0$	-2.3	-0.019	0.009
5.........	bf	-1.5	-4.0	$+0.010$	0.003
6.........	bf	-1.9	$+0.8$	$+0.018$	0.005
7.........	bf	-3.7	$+2.7$	-0.019	± 0.002

Boss 5602 $\alpha=21^h41^m51^s$ $\delta=+25^\circ6'$

Pl. No.	Date	t	Obs.	Qy	p	m	Remarks
701_1 . .	1915 Sept. 29	− 8.2	vM	fg	−0.642	+0.75	
701_2 . .	29	− 4.2	vM	g	−0.642	+0.75	
702 . .	29	+ 0.7	vM	fg	−0.642	+0.75	
736 . .	Oct. 1	− 7.0	vM	fg	−0.661	+0.75	Trail
1451_1 . .	1917 June 27	−14.0	H	f	+0.706	+2.49	
1451_2 . .	27	− 9.7	vM	f	+0.706	+2.49	
1452_1 . .	27	− 4.2	H	fg	+0.706	+2.49	
1452_2 . .	27	0.0	vM	fg	+0.706	+2.49	
1465 . .	July 13	+ 2.5	vM	fg	+0.510	+2.53	
1466_1 . .	13	+ 7.7	Pt	fg	+0.508	+2.53	
1466_2 . .	13	+12.7	vM	fg	+0.508	+2.53	Clouds 20^m exp.
1565 . .	Oct. 8	+12.7	vM	g	−0.742	+2.77	
1580 . .	9	+ 2.0	H	g	−0.754	+2.78	
1586 . .	10	+ 4.0	H	fg	−0.763	+2.78	Hazy 25^m exp.

Mor.-Ev.	Δp	Δm	M	v	Δt
1451_1− 701_1 . . .	+1.35	+1.74	+152	− 4	−5.°8
1451_2− 736 . . .	+1.37	+1.74	+155	− 1	−2.7
1452_1− 701_2 . . .	+1.35	+1.74	+150	− 6	0.0
1452_2− 702 . . .	+1.35	+1.74	+165	+ 9	−0.7
1465 −1580 . . .	+1.26	−0.25	− 39	−20	+0.5
1466_1−1586 . . .	+1.27	−0.25	− 5	+14	+3.7
1466_2−1565 . . .	+1.25	−0.24	− 14	+ 4	0.0

$$+12.30\,\mu_\alpha + 8.50\,\pi = +1096 \qquad \pi = +2.1 = +0''.003 \pm 0''.005$$
$$+8.50\,\mu_\alpha + 12.10\,\pi = +770 \qquad \mu_\alpha = +87.7 = +0''.143 \pm 0''.005$$

Comp. Star	Br.	x	y	π	P.E.
1.	f	+0.′8	+1.′4	−0.″034	±0.″021
2.	bf	+1.6	+0.3	−0.026	0.009
3.	f	+4.3	+0.5	+0.027	0.010
4.	bb	+1.8	−3.4	+0.009	0.010
5.	b	0.0	−5.4	−0.016	0.010
6.	f	−1.2	−1.1	+0.033	0.012
7.	bf	−4.1	+3.2	+0.002	0.009
8.	f	−2.7	+3.8	+0.004	±0.015

Boss 6129 = Bu 12608 $\alpha = 23^h 47^m 32^s$ $\delta = +74°59'$

Pl. No.	Date	t	Obs.	Qy	p	m	Remarks
1064_1..	1916 Aug. 3	+ 3.0	vM	fg	+0.658	+1.59	
1064_2..	3	+ 7.5	H	g	+0.658	+1.59	Trail
1172_1..	Nov. 18	−17.0	vM	fg	−0.783	+1.89	
1172_2..	18	−13.0	vM	fg	−0.783	+1.89	
1187_1..	Dec. 16	+ 5.0	vM	g	−0.902	+1.96	Some astigmatism of 60-in. mirror
1187_2..	16	+ 9.5	H	fg	−0.902	+1.96	
1205_1..	17	+ 6.0	vM	fg	−0.902	+1.96	
1205_2..	17	+10.0	H	g	−0.902	+1.96	
1474_1..	1917 July 28	+ 3.0	vM	fg	+0.728	+2.58	Hazy 20^m exp.
1474_2..	28	+ 7.5	H	f	+0.728	+2.58	Hazy 20^m exp.
1483_1..	29	−18.5	vM	fg	+0.716	+2.58	
1483_2..	29	−14.5	H	fg	+0.716	+2.58	
1492_1..	Aug. 10	− 9.0	vM	g	+0.582	+2.61	
1492_2..	10	− 4.2	H	g	+0.582	+2.61	
1507 ..	11	− 4.0	H	g	+0.568	+2.61	
1605 ..	Nov. 8	− 7.2	vM	g	−0.691	+2.86	Hazy 20^m exp.
1635 ..	Dec. 8	− 9.0	vM	g	−0.888	+2.94	
1636 ..	8	− 2.5	H	fg	−0.888	+2.94	

Mor.-Ev.	Δp	Δm	M	v	Δt
1483_1–1172_1...	+1.50	+0.69	+241	+19	−1°5
1483_2–1172_2...	+1.50	+0.69	+222	0	−1.5
1064_1–1187_1...	+1.56	−0.37	+ 22	− 1	−2.0
1474_1–1205_1...	+1.63	+0.62	+200	−15	−3.0
1474_2–1187_2...	+1.63	+0.62	+215	0	−2.0
1064_2–1205_2...	+1.56	−0.37	+ 35	+12	−2.5
1492_1–1605 ...	+1.27	−0.25	+ 18	−11	−1.8
1507 –1636 ...	+1.46	−0.33	− 1	−26	−1.5
1492_2–1635 ...	+1.47	−0.33	+ 44	+19	+4.8

$$+2.28\,\mu_\alpha + 1.65\,\pi = +535 \qquad \pi = +59.8 = +0''.097 \pm 0''.004$$

$$+1.65\,\mu_\alpha + 20.58\,\pi = +1547 \qquad \mu_\alpha = +191.4 = +0''.312 \pm 0''.012$$

The 11^M5 companion, being covered by the sector, does not show any images on the plates.

Comp. Star	Br.	x	y	π	P.E.
1.........	f	+0′.1	+4′.4	+0″.003	±0″.008
2.........	ff	+1.5	+2.5	−0.001	0.008
3.........	f	+7.0	−1.4	0.000	0.005
4.........	f	0.0	−3.4	+0.001	0.008
5.........	f	+0.8	−4.3	−0.002	0.005
6.........	bf	−4.7	−1.4	+0.003	0.004
7.........	b	−4.4	+4.5	−0.004	±0.006

SYSTEMATIC ERRORS

The discussion of systematic errors in the second series of parallaxes showed that there was no reason to suppose that these errors surpassed 0.″003. In a briefer form the same line of reasoning will be sufficient to show that the same conclusion holds for the material published in the present series.

Internal systematic errors.—*a*) The practical absence of a magnitude-error can be seen from Table IV, which gives the mean parallaxes, their probable errors, and the numbers of comparison stars of different brightness used in the present paper.

TABLE IV

	bb	b	bf	f	ff	All
Mean.......	−0.″003	+0.″002	+0.″002	−0.″001	−0.″003	0.″000
Mean P.E....	0.″008	0.″008	0.″007	0.″008	0.″009	0.″008
n..........	3	11	70	99	13	196

The parallax of only one physical double star has been measured since the publication of *Contribution* No. 136; this would add one more value to Table III of that paper, viz.,

$$\left.\begin{array}{r}\text{Boss } 2442_1\\ 2442_2\end{array}\right\}=\text{Bu } 4929\left\{\begin{array}{l}7.26,\ \pi=+0.''018\\ 7.74,\ \pi=+0.''016\end{array}\right\}\Delta m=-0.48\qquad \Delta\pi=+0.''002$$

b) For an investigation of a systematic error due to the neglect of the quadratic terms in x and y the comparison stars were divided into 49 groups according to their position on the plates, each group containing 1 to 11 stars. The weighted mean of each group was used as the second member of an equation of condition of the form:

$$+a+bx+cy+dx^2+exy+fy^2=\text{mean } \pi.$$

Solved by least squares these equations gave as a resulting correction to the parallaxes of the central stars +0.″0009 ± 0.″0008.

External systematic errors.—*a*) Although five additional fields with available parallax determinations by other observers are included in the present paper, the material is still too scanty for any definite conclusion, as many of these observations have little weight. For N.G.C. 224, for instance, there are five other observations, ranging from −0″.320 by Franz to +0″.171 by Bohlin; for N.G.C. 2392 one value is given, viz., −0″.036 by Bohlin, and for N.G.C. 6720 we have the old value by Newkirk, +0″.067, and his later determination, +0″.015; for B.D. +36°3956 Kapteyn gives +0″.087, which, corrected for a magnitude error, would become +0″.062.[1] For Nova Persei No. 2 there are available, however, four rather consistent values, which are mentioned on page 5; the weighted mean is +0″.015 ± 0″.008.

b) The mean difference μ_aBoss *minus* μ_avM in this series is 0″.000, and accordingly the indicated correction for the parallaxes is also 0″.000.

c) For twelve stars in this series we can derive parallaxes from van Rhijn's tables. The mean difference $\pi_{\mathrm{vR}}-\pi_{\mathrm{vM}}$ is −0″.0094; this large quantity is principally due to Boss 6129, for which the difference is −0″.062. From spectroscopic observations Adams derives for this star a parallax of +0″.132, and it is probable that in reality it is considerably nearer than would appear from its apparent magnitude and proper motion. Excluding this object, the difference $\pi_{\mathrm{vR}}-\pi_{\mathrm{vM}}$ becomes −0″.0046, which is in good agreement with the value −0″.0056 found before.

d) In the discussion accompanying the second series of parallaxes we compared the Mount Wilson results with those of other observers by means of van Rhijn's tables for the stars of types F, G, and K having magnitudes between 5.00 and 6.99 and proper motions less than 0″.500. There were only 68 stars available with 86 observations. This material has later been extended to all stars of spectra B to K whose proper motions are known, excluding only those stars whose proper motion exceeds 1″. From this comparison, which includes 776 observations, we find for the Mount Wilson parallaxes a correction of −0″.002.

e) A comparison of the values published here with the parallaxes determined by Mr. Adams by his spectroscopic method gives for 13 stars, which we have in common, a difference $\pi_{\mathrm{Adams}}-\pi_{\mathrm{vM}}$, −0″.0042, which also agrees well with the value −0″.0033 found before.

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[1] See *Publications of the Astronomical Society of the Pacific*, **28**, 280, 1916.

The conclusion that the systematic error in the Mount Wilson parallaxes can hardly surpass 0″003 seems therefore to find corroboration in the results of the present series.

Miss Coral Wolfe, of the Computing Division, has given much assistance in the computations, while Mr. W. P. Hoge has again rendered most efficient help in securing the exposures; to both I wish to express my acknowledgment.

Mount Wilson Solar Observatory
April 15, 1918

ERRATA TO CONTRIBUTION NO. 96

Page 9, No. 240 *for* $\phi = 35°0$ *read* 145°0
Page 11, No. 296 " $\phi = 296.3$ " 243.7
Page 11, No. 310 " $\phi = 242.4$ " 297.6
Page 12, No. 345 " $\phi = 211.1$ " 328.9
Page 16, No. 480 " $\phi = 263.6$ " 276.4

ERRATA TO CONTRIBUTION NO. 136

Page 35, Table III, *for* 1904 *read* 1014
Page 41, 11 lines from the bottom, *for* (log) *read* log

Contributions from the Mount Wilson Solar Observatory, No. 159
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THE VARIATION IN LIGHT AND COLOR OF RS BOÖTIS

By FREDERICK H. SEARES and HARLOW SHAPLEY

A variation in the light of RS Boötis[1] was first noted by Mrs. Fleming on a photograph made at Arequipa on May 24, 1906.[2] Other plates in the Harvard collection, some made as early as 1890, indicated a period of about 12 hours and an amplitude of one magnitude. Visual observations by Seares and Haynes at the Laws Observatory in June and July, 1908, gave the provisional elements:[3]

$$\text{Max.} = \text{J.D. } 2418115.626 + 0.37722\ E, \text{ G.M.T.} \tag{1}$$

and revealed a variation similar to that of the short-period variables so numerous in certain globular clusters.

Only a few such objects were known among the stars at large. Williams, during 1900–1903, had recognized Y Lyrae, UY Cygni, and RZ Lyrae; and observations at the Laws Observatory in 1906–1908 had identified RV Capricorni, SU Draconis, and SW Andromedae. RR Lyrae, RR Geminorum, and one or two others had previously been discovered; but the total number of isolated cluster-type variables, now about forty-five, did not then exceed a dozen. The discovery of RS Boötis therefore emphasized the interest of this peculiar type of variability, and the visual observations begun at the Laws Observatory were accordingly continued with some persistence until July, 1910. The results of these measures, 542 observations in all, are collected in Table I.[4]

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[1] B.D. $+32^\circ 2489$, 9.3, $14^h 27^m 21^s$, $+32^\circ 23'.4$ (1855). For a modern position cf. *Astronomische Abhandlungen der Hamburger Sternwarte*, **1**, No. 3, 1909.

[2] *Harvard Circular*, No. 124; *Astronomische Nachrichten*, **174**, 105, 1907; **176**, 188, 1907.

[3] *Laws Observatory Bulletins*, **1**, 240, 1908.

[4] These observations were made with the assistance of a grant from the Gould Fund of the National Academy of Sciences.

TABLE I

VISUAL OBSERVATIONS OF RS BOÖTIS

J.D.	Phase	Star, Obs.	Vis. Mag.	O−C	J.D.	Phase	Star, Obs.	Vis. Mag.	O−C
2417-					712.661..	59	*b*H	10.22	+16
660.611..	80	*a*H	10.17	−2	.672..	70	*b*H	10.30	+17
		*b*H	10.06	−13	.685..	83	*b*H	10.36	+15
.685..	154	*a*H	10.68	+20	.700..	98	*b*H	10.47	+20
		*b*H	10.50	+2	.728..	126	*b*H	10.61	+22
.752..	221	*a*H	10.94	+32	.748..	146	*b*H	10.65	+19
		*b*H	10.86	+24	715.618..	374	*b*H	10.33	−1
.786..	255	*a*H	10.94	+28	.635..	14	*b*H	10.36	+30
		*b*H	10.98	+32	.649..	28	*b*H	10.45	+59
667.608..	285	*a*H	10.85	+16	.667..	46	*b*H	10.33	+35
		*b*H	10.80	+11	.683..	62	*b*H	10.38	+30
.651..	328	*a*H	10.94	+24	.695..	74	*b*H	10.53	+38
		*b*H	10.83	+13	.727..	106	*b*H	10.43	+13
.697..	374	*a*H	10.48	+14	.744..	123	*b*H	10.55	+17
		*b*H	10.46	+12	.761..	140	*b*H	10.47	+3
.728..	28	*a*H	10.30	+44	.774..	153	*b*H	10.52	+4
		*b*H	10.36	+50	.790..	169	*b*H	10.55	+2
.764..	64	*a*H	10.25	+16	.812..	191	*b*H	10.54	−3
		*b*H	10.19	+10	.827..	206	*b*H	10.59	−1
.869..	169	*a*H	10.53	0	.845..	224	*b*H	10.46	−17
		*b*H	10.62	+9	.862..	241	*b*H	10.47	−18
.910..	210	*a*H	10.62	+1	.880..	259	*b*H	10.43	−24
		*b*H	10.63	+2	716.617..	241	*b*H	10.57	−8
675.909..	285	*a*H	10.96	+27	.640..	264	*b*H	10.76	+9
		*b*H	10.95	+26	.656..	280	*b*H	10.73	+4
679.751..	353	*a*H	10.24	−39	.671..	295	*b*H	10.71	+1
		*b*H	10.39	−24	742.623..	212	*a*H	10.35	−26
687.590..	268	*b*H	10.81	+13			*b*H	10.25	−36
.826..	127	*b*H	10.70	+30	.654..	243	*b*H	10.50	−15
.849..	150	*b*H	10.62	+15	.681..	270	*b*H	10.54	−14
.868..	169	*b*H	10.63	+10	.701..	290	*b*H	10.59	−11
.890..	191	*b*H	10.60	+3	.737..	326	*b*H	10.75	+5
689.703..	117	*b*H	10.52	+16	.777..	366	*b*H	10.41	−6
.735..	149	*b*H	10.53	+6	.806..	18	*b*H	9.94	−6
.765..	179	*b*H	10.63	+8	.832..	44	*b*H	10.30	+34
.810..	224	*b*H	10.82	+19	.865..	77	*b*H	10.32	+15
.840..	254	*b*H	10.77	+11	762.632..	224	*a*H	10.62	−1
.872..	286	*b*H	10.80	+10	.634..	226	*b*H	10.56	−7
690.585..	245	*b*H	11.05	+40	.660..	252	*a*H	10.62	−4
.601..	261	*b*H	10.96	+29	.663..	255	*b*H	10.57	−9
697.580..	70	*b*H	10.28	+15	.682..	274	*a*H	10.78	+10
.605..	95	*b*H	10.25	−1	.686..	278	*b*H	10.89	+20
.632..	122	*a*H	10.75	+37	.688..	280	*a*H	10.92	+23
		*b*H	10.46	+8	.692..	284	*b*H	10.54	−15
.658..	148	*b*H	10.76	+30	.714..	306	*a*H	10.89	+19
.690..	180	*b*H	10.81	+26	.716..	308	*b*H	10.68	−2
.733..	223	*b*H	10.87	+24	.749..	341	*a*H	10.74	+5
.778..	268	*b*H	10.89	+21	.751..	343	*b*H	10.84	+16
.793..	283	*b*H	10.99	+30	.795..	10	*a*H	10.23	+11
.811..	301	*b*H	10.80	+10	.798..	13	*b*H	10.42	+34
.873..	363	*b*H	10.59	+7	.838..	53	*a*H	10.02	−1
712.649..	47	*b*H	10.26	+27	.840..	55	*b*H	10.08	+4

TABLE I—*Continued*

J.D.	Phase	Star, Obs.	Vis. Mag.	O−C
763.643..	103	*a*H	10.59	+30
.645..	105	*b*H	10.34	+ 4
.677..	137	*a*H	10.44	+ 1
.678..	138	*b*H	10.32	−11
.702..	162	*a*H	10.57	+ 6
.704..	164	*b*H	10.54	+ 3
.707..	167	*a*H	10.62	+10
.709..	169	*b*H	10.46	− 7
777.629..	128	*b*H	10.40	0
.654..	153	*b*H	10.59	+11
.701..	200	*b*H	10.50	− 9
.750..	249	*b*H	10.82	+16
.798..	297	*b*H	10.63	− 7
787.612..	302	*a*H	10.77	+ 7
.614..	304	*b*H	10.41	−29
.632..	322	*a*H	10.80	+10
.634..	324	*b*H	10.98	+28
.706..	19	*a*H	9.73	−26
.708..	21	*b*H	10.14	+18
788.606..	163	*a*H	10.44	− 7
.608..	165	*b*H	10.56	+ 5
.624..	181	*a*H	10.61	+ 6
.626..	183	*b*H	10.45	−11
.642..	199	*a*H	10.61	+ 2
.645..	202	*b*H	10.60	+ 1
.661..	218	*a*H	10.44	−18
.664..	221	*b*H	10.78	+16
.685..	242	*a*H	10.73	+ 8
.687..	244	*b*H	10.65	0
.709..	266	*a*H	10.55	−13
.712..	269	*b*H	10.83	+15
.735..	292	*a*H	10.69	− 1
.737..	294	*b*H	10.63	− 7
.759..	316	*a*H	10.96	+26
.761..	318	*b*H	10.74	+ 4
994.683..	212	*a*H	10.77	+16
		*b*H	10.58	− 3
.709..	238	*a*H	10.38	+27
		*b*H	10.46	+19
.776..	305	*a*H	10.47	−23
		*b*H	10.47	−23
.874..	26	*a*H	9.81	− 7
		*b*H	9.87	− 1
.898..	50	*a*H	9.66	−35
		*b*H	9.79	−22
.913..	65	*a*H	9.89	−21
		*b*H	9.81	−29
2418-				
037.598..	109	*b*H	10.42	+10
.656..	167	*b*H	10.41	−11
.706..	217	*b*H	10.65	+ 3
.767..	278	*b*H	10.54	−15
.899..	33	*b*H	10.13	+27
037.936..	70	*a*H	10.11	− 2
		*b*H	10.23	+10
057.667..	180	*a*H	10.19	−36
		*b*H	10.33	−22
.683..	196	*b*H	10.34	−24
.701..	214	*b*H	10.34	−27
.725..	238	*b*H	10.30	−35
115.626..	32	*a*S	9.72	−13
.665..	71	*a*S	10.02	−12
.708..	114	*a*S	10.38	+ 4
.711..	117	*a*S	10.49	+13
.745..	151	*a*S	10.79	+32
.786..	192	*a*S	10.85	+28
.790..	196	*a*H	10.72	+14
		*b*H	10.61	+ 3
.836..	242	*a*S	11.00	+35
.840..	246	*a*H	10.68	+ 2
.843..	249	*a*S	10.80	+14
.883..	289	*a*S	11.05	+35
.887..	293	*a*H	10.79	+ 9
117.619..	138	*a*H	10.36	− 7
		*b*H	10.32	−11
.657..	176	*a*S	10.58	+ 4
		*b*S	10.61	+ 7
.704..	223	*a*S	10.56	− 7
		*b*S	10.58	− 5
.741..	260	*a*S	10.47	−20
		*b*S	10.63	− 4
.783..	302	*b*S	10.61	− 9
.787..	306	*b*H	10.67	− 3
.829..	348	*b*S	10.62	− 4
.832..	351	*b*H	10.70	+ 6
.865..	7	*b*S	10.21	+ 4
.868..	10	*b*H	10.15	+ 3
.872..	14	*b*S	10.00	− 6
.874..	16	*b*H	9.98	− 5
.878..	20	*b*S	9.80	−17
.880..	22	*a*S	9.74	−20
.883..	25	*a*H	9.74	−16
.886..	28	*a*S	9.85	− 1
		*b*S	9.86	0
.890..	32	*a*H	9.72	−13
.893..	35	*a*S	9.76	−11
.894..	36	*a*H	9.71	−17
.897..	39	*a*S	9.81	−10
118.604..	369	*a*S	10.31	−12
.607..	372	*a*S	10.37	0
.610..	375	*a*S	10.26	− 6
.611..	376	*a*S	10.17	−13
.614..	1	*a*S	10.08	−19
.617..	4	*a*S	10.01	−21
.620..	7	*a*S	9.89	−28
.622..	9	*a*S	9.83	−31

TABLE I—*Continued*

J.D.	Phase	Star, Obs.	Vis. Mag.	O−C	J.D.	Phase	Star, Obs.	Vis. Mag.	O−C
118.625..	12	*a*S	9.81	−28	131.785..	344	*b*S	10.80	+13
.627..	14	*a*S	9.84	−22	.788..	347	*b*H	10.62	− 4
.629..	16	*a*S	9.72	−31	.790..	349	*b*S	10.76	+11
.631..	18	*a*S	9.72	−28	.792..	351	*b*H	10.75	+11
.635..	22	*a*S	9.71	−23	.801..	360	*b*S	10.63	+ 6
.638..	25	*a*S	9.77	−13	.803..	362	*b*H	10.52	− 2
.639..	26	*a*S	9.69	−19	.805..	364	*b*S	10.50	− 1
.643..	30	*a*S	9.74	−11	.807..	366	*b*H	10.62	+15
.646..	33	*a*S	9.82	− 4	.810..	369	*b*S	10.40	− 3
.649..	36	*a*S	9.86	− 2	.812..	371	*b*H	10.47	+ 8
.651..	38	*a*S	9.82	− 8	.819..	1	*b*S	10.16	−11
.668..	55	*a*S	9.93	−11	.822..	4	*b*H	10.00	−22
.670..	57	*a*S	9.93	−12	.826..	8	*b*S	10.09	− 6
.685..	72	*a*S	10.15	+ 1	132.633..	60	*a*H	10.07	0
.687..	74	*a*S	10.06	− 9			*b*H	10.02	− 5
.708..	95	*a*S	10.26	0	.667..	94	*a*H	10.27	+ 1
.710..	97	*a*S	10.34	+ 7			*b*H	10.30	+ 4
.731..	118	*a*S	10.23	−13	.718..	145	*a*H	10.45	0
.733..	120	*a*H	10.44	+ 7			*b*H	10.40	− 5
.769..	156	*a*S	10.58	+ 9	.764..	191	*a*H	10.68	+11
.772..	159	*a*H	10.55	+ 5			*b*H	10.59	+ 2
.835..	222	*a*S	10.50	−12	139.615..	250	*a*S	10.39	−27
.838..	225	*a*S	10.74	+11			*b*S	10.54	−12
.840..	227	*a*H	10.66	+ 3	.618..	253	*a*H	10.76	+10
.844..	231	*a*H	10.62	− 2			*b*H	10.74	+ 8
		*b*H	10.64	0	.690..	325	*a*H	10.70	0
120.604..	105	*a*H	10.14	−16			*b*H	10.69	− 1
.606..	107	*a*H	10.20	−11	.714..	349	*a*S	10.53	−12
.610..	111	*a*S	10.17	−16			*b*S	10.62	− 3
.612..	113	*a*S	10.19	−15	.718..	353	*a*H	10.53	−10
.614..	115	*a*H	10.31	− 4			*b*H	10.57	− 6
.616..	117	*a*H	10.26	−10	.723..	358	*a*S	10.43	−16
.619..	120	*a*S	10.25	−12			*b*S	10.53	− 6
.622..	123	*a*S	10.12	−26	.728..	363	*a*H	10.49	− 3
.626..	127	*a*H	10.38	− 2			*b*H	10.48	− 4
		*b*H	10.35	− 5	.732..	367	*a*S	10.38	− 8
.631..	132	*a*S	10.29	−13			*b*S	10.33	−13
		*b*S	10.35	− 7	.736..	371	*a*H	10.31	− 8
122.615..	229	*a*S	10.44	−20			*b*H	10.42	+ 3
		*b*S	10.48	−16	.740..	375	*a*S	10.26	− 6
.671..	285	*a*S	10.54	−15			*b*S	10.43	+11
		*b*S	10.45	−24	.747..	5	*a*H	10.16	− 4
.716..	330	*a*S	10.72	+ 2			*b*H	10.23	+ 3
		*b*S	10.50	−20	.751..	9	*a*S	9.92	−22
131.654..	213	*a*H	10.58	− 3			*b*S	10.24	+10
		*b*H	10.61	0	.756..	14	*a*H	10.02	− 4
.697..	256	*a*S	10.58	− 9			*b*H	10.07	+ 1
		*b*S	10.62	− 5	.760..	18	*a*S	9.86	−14
.700..	259	*b*H	10.65	− 2			*b*S	9.91	− 9
.743..	302	*b*S	10.65	− 5	.765..	23	*a*H	9.89	− 4
.745..	304	*b*H	10.73	+ 3			*b*H	9.97	+ 4
.774..	333	*b*S	10.77	+ 7	.770..	28	*a*S	9.63	−23
.776..	335	*b*H	10.76	+ 6			*b*S	9.84	− 2

TABLE I—*Continued*

J.D.	Phase	Star, Obs.	Vis. Mag.	O−C	J.D.	Phase	Star, Obs.	Vis. Mag.	O−C
139.774..	32	*a*H	9.80	−5	178.624..	19	*a*H	10.02	+3
		*b*H	9.91	+6			*b*H	10.05	+6
.779..	37	*a*H	9.82	−7	.626..	21	*a*H	9.95	−1
.782..	40	*a*H	9.76	−16			*b*H	10.03	+7
.786..	44	*a*H	9.89	−7	.630..	25	*a*H	9.76	−14
		*b*H	9.97	+1			*b*H	9.89	−1
.799..	57	*a*H	9.97	−8	.633..	28	*a*H	9.89	+3
.801..	59	*a*H	9.96	−10			*b*H	9.94	+8
143.631..	115	*a*S	10.33	−2	.638..	33	*a*H	9.87	+1
		*b*S	10.29	−6			*b*H	9.90	+4
.649..	133	*a*S	10.17	−25	.642..	37	*a*H	9.93	+4
		*b*S	10.27	−15			*b*H	9.89	0
153.677..	351	*a*S	10.69	+5	.646..	41	*a*H	9.85	−8
		*b*S	10.57	−7			*b*H	9.97	+4
.683..	357	*a*H	10.75	+15	.656..	51	*a*H	9.97	−5
		*b*H	10.74	+14			*b*H	10.05	+3
.705..	2	*a*S	10.45	+20	.667..	62	*a*H	10.00	−8
		*b*S	10.24	−1			*b*H	10.12	+4
.712..	9	*b*H	10.16	+2	.695..	90	*a*H	10.21	−3
.713..	10	*b*H	10.07	−5			*b*H	10.18	−6
.716..	13	*b*S	10.06	−2	473.682..	373	*a*Sh	10.20	−16
.717..	14	*b*S	9.92	−14			*b*Sh	10.17	−19
.721..	18	*b*H	9.82	−18	475.673..	100	*a*H	10.08	−20
.725..	22	*b*H	9.71	−23			*b*H	10.24	−4
.727..	24	*b*S	9.79	−12	.711..	138	*a*H	10.18	−25
.729..	26	*b*S	9.67	−21			*b*H	10.32	−11
.733..	30	*a*H	9.84	−1	.740..	167	*a*H	10.22	−30
.735..	32	*a*H	9.86	+1			*b*H	10.39	−13
169.639..	89	*a*H	10.10	−14	477.761..	302	*a*H	10.69	−1
		*b*H	10.04	−20			*b*H	10.46	−24
.656..	106	*a*H	10.23	−7	.785..	326	*a*H	10.86	+16
		*b*H	10.18	−12			*b*H	10.49	−21
170.659..	354	*b*H	10.69	+6	.842..	6	*a*H	10.17	−1
.699..	17	*b*H	10.13	+11			*b*H	10.34	+16
.719..	37	*b*H	10.02	+13	.849..	13	*a*Sh	10.24	+16
176.612..	271	*a*H	10.88	+20			*b*Sh	10.18	+10
		*b*H	11.00	+32	.854..	18	*a*H	10.06	+6
.633..	292	*a*H	10.73	+3			*b*H	10.17	+17
		*b*H	10.91	+21	.867..	31	*a*Sh	9.71	−14
.653..	312	*a*H	10.73	+3	.869..	33	*a*Sh	9.82	−4
		*b*H	10.80	+10	.872..	36	*a*H	9.73	−15
.675..	334	*a*H	10.82	+12			*b*H	9.67	−21
		*b*H	11.05	+35	479.704..	359	*a*H	10.69	+11
.702..	361	*a*H	10.88	+33			*b*H	10.74	+16
		*b*H	10.85	+30	.709..	364	*a*H	10.57	+6
.716..	375	*a*H	10.76	+44			*b*H	10.54	+3
		*b*H	10.89	+57	.733..	11	*a*H	10.22	+11
178.612..	7	*a*H	10.11	−6			*b*H	10.15	+4
		*b*H	10.18	+1	.739..	17	*a*H	9.95	−7
.615..	10	*a*H	10.20	+8			*b*H	9.91	−11
		*b*H	10.10	−2	.760..	38	*a*H	9.87	−3
.618..	13	*a*H	10.08	0			*b*H	9.95	+5
		*b*H	10.22	+14	.765..	43	*a*H	9.78	−17

TABLE I—*Continued*

J.D.	Phase	Star, Obs.	Vis. Mag.	O−C	J.D.	Phase	Star, Obs.	Vis. Mag.	O−C
479.765..	43	*b*H	9.84	−11	504.658..	33	*a*H	9.86	0
.788..	66	*a*H	10.05	− 6			*b*H	9.85	− 1
		*b*H	9.91	−20	.674..	49	*a*H	9.99	− 1
501.609..	2	*a*H	10.40	+15			*b*H	9.96	− 4
		*b*H	10.29	+ 4	.681..	56	*a*H	10.01	− 4
.614..	7	*a*H	10.28	+11			*b*H	9.93	−12
		*b*H	10.26	+ 9	.698..	73	*a*H	10.20	+ 5
.620..	13	*a*H	10.04	− 4			*b*H	10.12	− 3
		*b*H	10.16	+ 8	.704..	79	*a*H	10.19	+ 1
.627..	20	*a*H	10.09	+12			*b*H	10.13	+ 5
		*b*H	10.02	+ 5	837.627..	190	*a*Sh	10.52	− 5
.635..	28	*a*H	9.92	+ 6			*b*Sh	10.69	+12
		*b*H	9.89	+ 3	.637..	200	*b*Sh	10.60	+ 1
.640..	33	*a*H	9.91	+ 5	.643..	206	*a*Sh	10.60	0
		*b*H	9.83	− 3			*b*Sh	10.76	+16
.646..	39	*a*H	9.91	0	.678..	241	*b*Sh	10.66	+ 1
		*b*H	9.75	−16	.683..	246	*b*Sh	10.61	− 5
.653..	46	*a*H	10.00	+ 2	.691..	254	*b*Sh	10.66	0
		*b*H	9.95	− 3	.721..	284	*b*Sh	10.58	−11
.675..	68	*a*H	10.09	− 3	.736..	299	*b*Sh	10.67	− 3
		*b*H	10.11	− 1	840.642..	186	*b*Sh	10.31	−25
.680..	73	*a*H	10.12	− 3	.716..	260	*a*Sh	10.59	− 8
		*b*H	10.09	− 6			*b*Sh	10.74	+ 7
.707..	100	*a*H	10.14	−14	863.721..	250	*a*Sh	10.53	−13
		*b*H	10.16	−12			*b*Sh	10.58	− 8
.714..	107	*a*H	10.32	+ 1	865.621..	263	*b*Sh	10.76	+ 9
		*b*H	10.21	−10	.624..	266	*b*Sh	10.67	− 1
.737..	130	*a*H	10.56	+15	.678..	320	*b*Sh	10.55	−15
		*b*H	10.38	− 3	.719..	361	*b*Sh	10.27	−28
.742..	135	*a*H	10.49	+ 7	.724..	366	*a*Sh	10.35	−12
		*b*H	10.45	+ 3			*b*Sh	10.24	−23
502.643..	281	*a*H	10.55	−14	.728..	370	*b*Sh	10.27	−14
		*b*H	10.58	−11	.735..	0	*b*Sh	10.18	−11
.649..	287	*a*H	10.60	−10	.741..	6	*b*Sh	10.07	−11
		*b*H	10.60	−10	.744..	9	*b*Sh	10.06	− 8
504.612..	364	*a*H	10.77	+26	.747..	12	*b*Sh	10.02	− 7
		*b*H	10.78	+27	.753..	18	*b*Sh	9.84	−16
.617..	369	*a*H	10.62	+19	.756..	21	*a*Sh	10.23	+27
		*b*H	10.73	+30	.760..	25	*a*Sh	10.06	+16
.623..	375	*a*H	10.45	+13			*b*Sh	9.91	+ 1
		*b*H	10.50	+18	.767..	32	*b*Sh	10.03	+18
.628..	3	*a*H	10.31	+ 7	.770..	35	*a*Sh	10.05	+18
		*b*H	10.28	+ 4	.775..	40	*a*Sh	10.12	+20
.633..	8	*a*H	10.27	+12			*b*Sh	10.14	+22
		*b*H	10.12	− 3	.786..	51	*b*Sh	10.04	+ 2
.638..	13	*a*H	10.19	+11	871.754..	359	*a*Sh	10.54	− 4
		*b*H	10.13	+ 5	.759..	364	*a*Sh	10.32	−19
.643..	18	*a*H	10.01	+ 1			*b*Sh	10.28	−23
		*b*H	9.99	− 1	.766..	371	*a*Sh	10.26	−13
.648..	23	*a*H	9.83	−10	.773..	1	*a*Sh	10.01	−26
		*b*H	10.02	+ 9	.778..	6	*a*Sh	10.06	−12
.653..	28	*a*H	9.89	+ 3	.784..	12	*a*Sh	10.02	− 7
		*b*H	9 87	+ 1	.789..	17	*a*Sh	10.13	+11

TABLE I—*Continued*

J.D.	Phase	Star, Obs.	Vis. Mag.	O−C	J.D.	Phase	Star, Obs.	Vis. Mag.	O−C
876.652..	353	*a*Sh	10.52	−11	879.705..	10	*a*Sh	10.09	− 3
.656..	357	*a*Sh	10.59	− 1	.708..	13	*a*Sh	10.09	+ 1
		*b*Sh	10.46	−14	.718..	23	*a*Sh	9.98	+ 5
879.635..	317	*a*Sh	10.50	−20	.720..	25	*a*Sh	9.92	+ 2
		*b*Sh	10.61	− 9	.732..	37	*a*Sh	10.12	+23
.662..	344	*a*Sh	10.76	+ 9	.736..	41	*a*Sh	10.10	+17
.694..	376	*a*Sh	10.23	− 7	.745..	50	*a*Sh	10.15	+14
.697..	2	*a*Sh	10.22	− 3			*b*Sh	10.01	0

An important question concerning cluster-type variables is that of a change in color paralleling their fluctuations in light. Their many analogies with the Cepheids, which were known to vary in color, had suggested such a possibility, although the cluster variables were not then classed with the Cepheids as they now usually are. The photometric methods developed for use with the 60-inch reflector afforded a ready means of measuring color; and photographic and photo-visual observations of RS Boötis on three successive nights in July, 1914, gave the data in Table XI, and established beyond question a variation in color similar to that previously detected in the case of the longer-period Cepheids.

These two series of measures, made with different instruments by wholly different methods and separated by an interval of several years, are discussed in the present paper for the purpose of revising the light-elements and determining mean light- and color-curves. The publication of the results has been long delayed, partly through occupation with other work, but more particularly by a series of mischances in securing the polar-comparison plates necessary for the determination of the zero-points, so that it was not until recently that the magnitudes could finally be reduced to the international system.[1]

THE VISUAL OBSERVATIONS

Most of the visual observations listed in Table I were made with an equalizing wedge photometer attached to the $7\frac{1}{2}$-inch equatorial of the Laws Observatory; those for J.D. 2417762–88, however, were obtained with a Zöllner photometer and a $4\frac{1}{2}$-inch equatorial. An account of the methods of observation and reduction will be found in *Laws Observatory Bulletins*, **1**, 94, 187, 1905, 1907.

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[1] Results of a preliminary reduction were given in *Publications of the Astronomical Society of the Pacific*, **26**, 202, 1914.

The decimals of the Julian Day in the first column of the table are expressed in Greenwich mean time. The unit for the phase in the second column is 0.001 day. The letters in the third column indicate the comparison star (a=B.D. +32°2481, mag. 9.2; b=B.D. +32°2483, mag. 9.4) and the observer (S=Seares, H=Haynes, Sh=Shapley). The visual magnitude has been referred to the international zero-point by methods described below; the residuals in the last column (unit=0.01 mag.) are referred to the mean visual light-curve given in Table V.

The measures with the wedge photometer were usually arranged so that a single group of settings gave values for the difference in brightness of the variable and each of the comparison stars a and b, and, at the same time, the magnitude difference of the comparison stars themselves. The first step in the reduction was the reference of the comparisons made with star a to star b, in order that all the observations might be expressed as differences of the form $v-b$. This required a knowledge of the difference $a-b$.

TABLE II

VALUES OF $a-b$ BY HAYNES

J.D.	Mean	Av. Dev.	No. Values	J.D.	Mean	Av. Dev.	No. Values
7660	−0.72	± 6	4	8118	−0.71	....	1
7667	.65	5	7	8120	.61	....	1
7675	.65	....	1	8131	.62	....	1
7679	.51	....	1	8132	.70	± 7	4
7697	.79	....	1	8139	.61	4	10
7742	.78	....	1	8169	.70	2	2
7762	.59	18	8	8176	.55	6	6
7763	.78	2	4	8178	.59	5	13
7787	.34	23	3	8475	.49	2	3
7788	.59	21	8	8477	.72	16	5
7994	.61	±10	6	8479	.65	4	7
8037	.61	....	1	8501	.69	7	14
8057	.53	....	1	8502	.60	1	2
8115	.71	....	1	8504	−0.66	± 6	14
8117	−0.61	....	1				

The most extensive series of values for $a-b$ is by Haynes. His results for each night are collected in Table II, which gives the

Julian Day, the mean for the night, the average deviation of a single observation from this mean, and the number of values. The accidental error of measures with the Zöllner photometer (J.D. 7762–88) is large; otherwise the agreement is satisfactory, and there is no evidence of variability in the comparison stars. The mean for the series is $a-b=-0.64$ mag. (108 obsns.). Similar, but less extensive, series by Seares and Shapley give -0.64 (22 obsns.) and -0.55 (13 obsns.), respectively. The adopted value is $a-b=-0.64$ mag. (143 obsns.).

After adding $a-b$ to the values of $v-a$, the entire series of results for $v-b$ was plotted with the G.M.T. of observation as abscissa. With the aid of an approximate light-curve drawn on tracing paper, a number of epochs were determined for which the variable, during increasing light, was equal to star b. The results and their estimated relative weights are in the second and third columns of Table III. With two similar epochs derived from the Mount Wilson photo-visual observations on J.D. 2420331–2, these are the data used for the revision of the elements of variation.

TABLE III

EPOCHS FOR $v=b$; REVISION OF PERIOD

E	J.D. Epoch $v=b$	Wt.	Red. to Sun	$O-C_1$	Cal. Epoch	$O-C_2$
− 988..........	7742.783	1	−2	− 1	7742.786	− 5
935..........	7762.796	1	−1	+14	7762.785	+10
869..........	7787.690	1	+1	+ 6	7787.689	+ 2
− 320..........	7794.853	1	−3	+10	7994.846	+ 4
0..........	8115.588	1	−1	0	8115.593	− 6
+ 6..........	8117.857	5	−1	+ 5	8117.857	− 1
8..........	8118.604	5	−1	− 3	8118.612	− 9
43..........	8131.816	4	0	+ 4	8131.818	− 2
45..........	8132.574	1	0	+ 7	8132.573	+ 1
64..........	8139.736	6	0	0	8139.742	− 6
101..........	8153.703	5	+1	+ 6	8153.704	0
146..........	8170.690	1	+2	+14	8170.684	+ 8
167..........	8178.610	4	+3	+11	8178.608	+ 5
960..........	8477.841	4	−2	+12	8477.835	+ 4
965..........	8479.724	5	−1	+10	8479.721	+ 2
1023..........	8501.611	5	0	+12	8501.607	+ 4
1031..........	8504.630	6	0	+13	8504.625	+ 5
2004..........	8871.762	2	0	0	8871.772	−10
2025..........	8879.693	2	+1	+ 8	8879.696	− 2
5873..........	0331.683	5	0	+19	0331.682	+ 1
+5876..........	0332.812	6	0	+16	0332.814	− 2

Heliocentric Time = Geocentric Time $-0^{\mathrm{d}}00416\cos(\odot-201^{\circ})$.
The unit for the reduction to the sun and for $O-C$ is 0.001 day.

For the derivation of final elements the following provisional values were used

$$m_{v=b}=\text{J.D. } 2418115.587+0.377333\,\text{E} \qquad (2)$$

The epoch is that of elements (1) corrected by −0.039 day, which is an estimate for the interval separating $v=\text{max.}$ from $v=b$. The period is a value by Pračka[1] based on an interval of 1185 periods.

The representation by elements (2) is shown by the differences $O-C_1$ in Table III. Discussed in the usual manner by least squares, these residuals give the corrections and probable errors,

$$\begin{aligned} E-E_0&=+0.0057\pm0.00085 \text{ days}\\ P-P_0&=+0.00000206\pm0.00000035 \text{ days,}\end{aligned}$$

whence

$$\left.\begin{aligned} E_{v=b}&=\text{J.D. } 2418115.593, \text{ G.M.T.}\\ P&=0.37733506 \text{ days}\end{aligned}\right\} \qquad (3)$$

The very small uncertainty in the period, $\pm0^s.03$, is to be attributed to the long interval of seven years, or nearly seven thousand complete cycles of variation, covered by the observations. The residuals $O-C_2$ corresponding to the revised constants are all small. The epoch numbers show that the rather conspicuous grouping of signs can scarcely be accepted as evidence of irregularity in the period. The residuals for Pračka's epochs[2] ($E=-1185$ and -1143), neither of which was used in revising the elements, are +0.002 and +0.084 days, respectively.

The mean visual light-curve has been derived from the 491 observations made with the wedge photometer, those with the Zöllner instrument being omitted because of large accidental error (see Table II). The values of $v-b$ were arranged in the order of heliocentric phase and divided into 29 groups of 17 each (16 observations in the first and last groups), whose means are in the first two columns of Table IV. The resulting light-curve, reduced to the international zero-point, is in Table V. The normal points and the curve are illustrated in the upper part of Fig. 1.

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[1] *Bulletin international de l'Académie des sciences de Bohême*, **15**, 1910.

[2] *Loc. cit.*

Table IV also gives the deviation of the normals from the curve and the probable error of a single observation based upon the

TABLE IV

NORMAL POINTS FOR MEAN VISUAL LIGHT-CURVE

Phase	Mean $v-b$	Dev.	P.E. One Obsn.	Rel. Wt.	Phase	Mean $v-b$	Dev.	P.E. One Obsn.	Rel. Wt.
0.003	−0.12	+4	± 9	12	0.136	+0.06	+5	± 8	15
.008	.20	+3	10	11	.160	.24	−6	8	15
.012	.21	−2	7	21	.188	.25	0	12	7
.015	.31	+4	11	8	.214	.30	−1	12	7
.019	.34	+1	9	12	.232	.26	+6	11	8
.024	.46	+5	8	16	.251	.37	−3	12	7
.028	.40	−6	14	5	.266	.39	−3	12	7
.033	.47	+1	7	23	.286	.42	−4	14	5
.037	.46	+3	9	14	.306	.32	+6	8	14
.044	.31	−5	12	6	.333	.43	−5	9	13
.056	.30	+3	8	16	.353	.27	+4	10	11
.068	.22	+2	9	13	.361	.27	−4	10	9
.083	.12	+1	8	15	.368	.11	+1	11	8
.104	− .07	+5	8	16	0.374	+0.07	−5	±14	5
0.120	+0.10	−5	±12	7					

TABLE V

MEAN VISUAL LIGHT-CURVE AND COLOR-CURVE

Phase	Vis. Mag.	C.I.	Phase	Vis. Mag.	C.I.	Phase	Vis. Mag.	C.I.
0.000	10.29	+0.30	0.055	10.04	0.00	0.220	10.62	+0.44
.005	10.20	.20	.060	10.07	+ .03	.230	10.64	.45
.010	10.12	+ .10	.065	10.10	.05	.240	10.65	.45
.015	10.05	.00	.070	10.13	.08	.250	10.66	.46
.020	9.97	− .09	.075	10.16	.10	.260	10.67	.46
.025	9.90	.12	.080	10.19	.12	.270	10.68	.47
.026	9.88	.13	.090	10.24	.17	.280	10.69	.48
.027	9.87	.14	.100	10.28	.21	.290	10.70	.49
.028	9.86	.14	.110	10.32	.24	.300	10.70	.49
.029	9.85	.15	.120	10.37	.28	.310	10.70	.50
.030	9.85	.15	.130	10.41	.31	.320	10.70	.51
.031	9.85	.14	.140	10.44	.34	.330	10.70	.51
.032	9.85	.13	.150	10.47	.36	.340	10.69	.52
.033	9.86	.13	.160	10.50	.38	.350	10.65	.52
.034	9.86	.12	.170	10.53	.39	.355	10.62	.52
.035	9.87	.12	.180	10.55	.40	.360	10.57	.51
.040	9.92	.09	.190	10.57	.41	.365	10.49	.50
.045	9.97	.06	.200	10.59	.42	.370	10.41	.45
.050	10.01	− .03	.210	10.61	.43	.375	10.32	.35
0.055	10.04	0.00	0.220	10.62	+0.44	0.380	10.24	+0.25

Phases are counted from instant when $v=b-0.03$ mag. To refer them to the instant of maximum brightness, subtract 0.031 from the tabular values. To obtain the ordinates of the mean photographic light-curve, add the color-index to the corresponding visual magnitude.

individual deviations (last column, Table I). The error varies considerably from point to point, which accounts for the differences in the weights given in the last column of Table IV.

The reference to the international zero-point is based upon $b=10.32$ mag., a value found by comparing the mean curve defined by Table IV with the photo-visual observations after reduction to the international system by means of polar comparisons. The photo-visual observations themselves (residuals in last column, Table XI) are well represented by the mean visual curve.

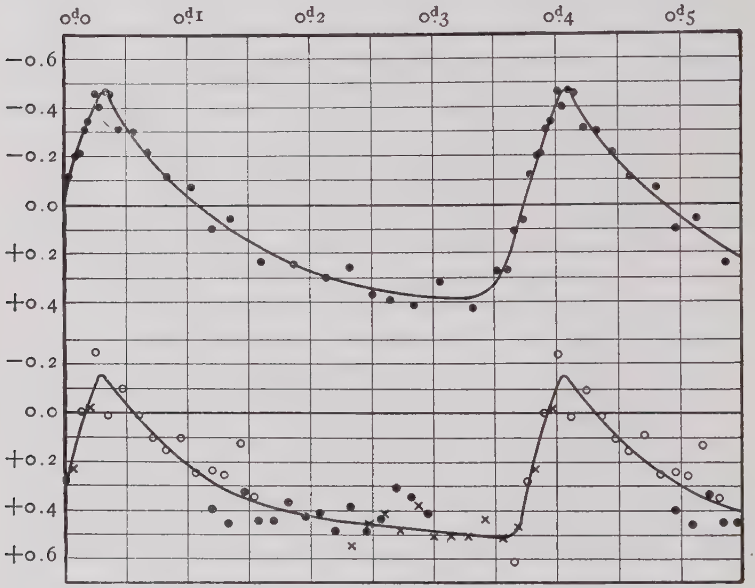

FIG. 1.—Mean visual light-curve of RS Boötis (above) and curve showing the variation of color (below). The ordinates of the lower curve are color-indices. The circles, crosses, and dots represent results obtained on different nights. The zero ordinate of the light-curve corresponds to visual magnitude 10.29.

The phases had been referred, as accurately as possible, to the instant for which $v=b$. The light-curve shows that, in the mean, $v=b$ for phase -0.002; further, that the maximum visual brightness, 9.85 mag., occurs for phase 0.031. The subtraction of this quantity will therefore refer the tabulated phases to the epoch of maximum; finally, from (3) we obtain the adopted elements

$$\text{Max.} = \text{J.D. } 2418115.624 + 0^{d}.37733506 \text{ E, G.M.T.} \qquad (4)$$

THE PHOTOGRAPHIC AND PHOTO-VISUAL OBSERVATIONS

The photographic and photo-visual observations collected in Table XI were obtained by the methods commonly used with the 60-inch reflector.[1] The plates were Seed 27's and Cramer Instantaneous Isos, respectively, the latter exposed behind a yellow filter. Relative magnitudes referred to arbitrary zero-points were determined for the comparison stars by means of images registered on the same plate with diaphragms of different aperture. The results were subsequently reduced to the international zero-point by comparisons with the North Polar Standards.[2]

The photographs of the variable were as follows:

1914, July 16. Plates 1613–1641

Photographic (odd-numbered plates): five 1^{m} exposures; apertures 32, 32, 32, 14, 9 inches; 32 and 14 used to establish scale.

Photo-visual (even-numbered plates): three 2^{m} exposures; apertures 9, 14, 32 inches; images with 9 and 14 invisible or too faint for measurement; 32-aperture images reduced with relative magnitudes of comparison stars in Table VII.

1914, July 17. Plates 1648–1677

Photographic (even-numbered plates): same as July 16.

Photo-visual (odd-numbered plates): three 2^{m} exposures; apertures 60, 40, 32 inches; 60 and 40 used to establish scale.

1914, July 18. Plates 1684–1709

Photographic (even-numbered plates): six 1^{m} exposures; apertures 32, 32, 32, 14, 14, 14 inches, with order reversed on alternate plates; reduced with relative magnitudes of comparison stars in Table VII.

Photo-visual (odd-numbered plates): four 2^{m} exposures; apertures 60, 40, 40, 60; used to establish scale.

To illustrate the method used for such data, the reduction for plate No. 1648 is given in Table VI. The first six columns contain star number and its distance from the optical axis of the instrument, the aperture, the scale reading (mean of two measurements), the distance correction, and the corrected reading *s*. The seventh column contains Δs, the difference in *s* for the apertures used to establish the scale (14 and 32, corresponding to a magnitude difference of 1.01).[1] By a process illustrated in Fig. 2, we find the relation between scale reading and magnitude necessary for the determination of the relative magnitudes.

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[1] Seares, *Mt. Wilson Contr.*, No. 80; *Astrophysical Journal*, **39**, 307, 1914.

[2] Seares, *Mt. Wilson Contr.*, No. 97; *Astrophysical Journal*, **41**, 206, 1915.

TABLE VI

REDUCTION OF PHOTOGRAPHIC PLATE NO. 1648

Star	Dist.	Ap.	Mean *s*	D.C.	Corr. *s*	Δs 14–32	Mag.	Mean Mag.	Zero-Point	Final Mag.	Resid.
1.....	3.2	32	10.0	−0.1	9.9	1.8	1.35	1.40	8.36	9.71	+0.05
		32	10.0	−0.1	9.9		1.35				
		32	10.0	−0.1	9.9		1.35				
		14	11.8	−0.1	11.7		1.37				
		9	14.2	+0.1	14.3		1.60				
2.....	3.4	32	11.6	0.0	11.6	2.1	2.30	2.31	8.29	10.62	−0.02
		32	11.4	−0.1	11.3		2.15				
		32	11.6	0.0	11.6		2.30				
		14	13.6	0.0	13.6		2.27				
		9	16.8	−0.1	16.7		2.53				
3.....	2.6	32	12.4	0.0	12.4	2.3	2.71	2.83	8.27	11.14	−0.04
		32	12.8	−0.1	12.7		2.86				
		32	12.7	−0.1	12.6		2.82				
		14	14.7	+0.2	14.9		2.82				
		9	18.0	−0.1	17.9		2.96				
5.....	3.6	32	15.0	+0.3	15.3	2.7	3.97	3.85	8.33	12.16	+0.02
		32	14.6	+0.2	14.8		3.79				
		32	14.6	+0.2	14.8		3.79				
		14	17.8	−0.1	17.7		3.86				
		9	tf	......	......		......		Mean		
Var...	4.0	32	12.7	−0.1	12.6	.....	2.82	2.80	8.31	11.11	......
		32	12.4	0.0	12.4		2.71				
		32	12.4	0.0	12.4		2.71				
		14	14.8	+0.3	15.1		2.89				
		9	17.7	−0.1	17.6		2.87				

We first plot the value of Δs for each star against the mean *s* for the three 32-aperture images—the largest aperture used. This gives the lower curve in Fig. 2, whose significance is as follows: Let *s* and Δs represent the abscissa and ordinate of any point on the curve, and m the magnitude corresponding to the scale reading s; $s' = s + \Delta s$ will then be the reading for magnitude $m' = m + \Delta m$, where $\Delta m = 1.01$, the reduction constant of the 14-aperture as compared with that of 32 inches. All readings refer to the 32-aperture.

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[1] Seares, *Mt. Wilson Contr.*, No. 80, p. 11; *Astrophysical Journal*, **39**, 317, 1914.

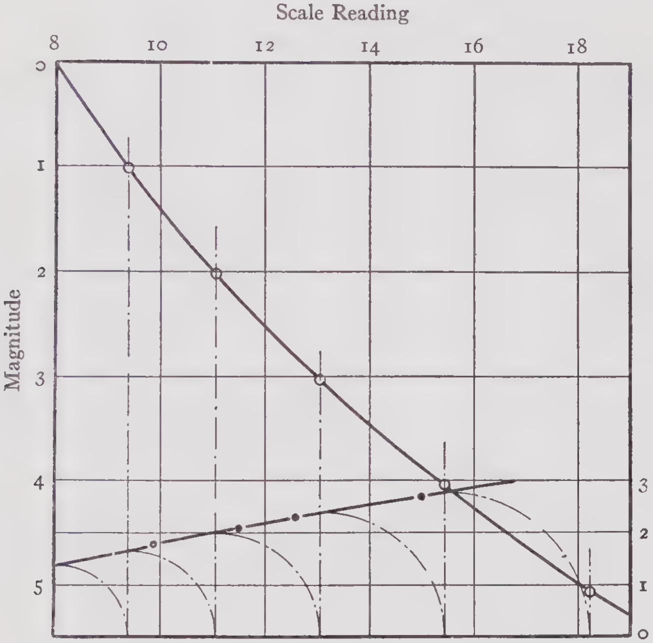

FIG. 2.—The ordinates of the lower curve are values of Δs, indicated in scale intervals on the right. This curve is used to construct the upper curve, which shows the relation between scale reading and magnitudes for Plate 1648.

With the aid of this curve we next construct the scale reading and magnitude curve in the upper part of Fig. 2. Since for the present the zero-point is arbitrary, we assume $m = 0$ for $s = 8$ and thus begin the accompanying tabulation. From the Δs curve, for abscissa 8 we find the ordinate 1.40, and derive $m' = 0.00 + 1.01 = 1.01$ as the magnitude for scale reading $s' = 8.00 + 1.40 = 9.40$. Inserting these numbers in the second line of the table, the process is repeated with m' and s' as a new m and s. Thus, for abscissa 9.40, we have the ordinate 1.65, which gives 11.05 as the scale reading for $m'=2.02$. A plot of the corresponding values of s and m thus found gives the required curve. The operations can be even more quickly accomplished with a pair of dividers and a ruler, by a process sufficiently obvious from the figure.[1]

m	s	Δs
0.00	8.00	1.40
1.01	9.40	1.65
2.02	11.05	2.00
3.03	13.05	2.38
4.04	15.43	2.78
5.05	18.21	....

From this curve we now read the magnitudes entered in the eighth column of Table VI, care being taken in the case of exposures with the 14- and 9-apertures to subtract the relative reduction constant—1.01 or 1.97—in order that all the results may refer to the 32-aperture.

Each photographic plate for July 16 and 17 and each photo-visual plate for July 17 and 18 gave a series of relative magnitudes similar to that in the ninth column of Table VI. The results for the comparison stars were reduced to a common, though still arbitrary, zero-point and combined into the means which appear in Table VII. The internal agreement is illustrated by Table VIII, which shows the deviations of the individual plates for July 16 from the weighted means in the seventh and last columns of

TABLE VII

COMPARISON STARS—RELATIVE MAGNITUDES

STAR	B.D. No.	α 1855	δ 1855	PHOTOGRAPHIC MAG.			PHOTO-VISUAL MAG.		
				July 16	July 17	Mean	July 17	July 18	Mean
1...	$+32^{\circ}2485$	$14^{h}26^{m}\ 8^{s}.9$	$+32^{\circ}16'.8$	0.30	0.24	0.27	0.47	0.50	0.48
2...	....	26 7.7	16.7	1.06	1.10	1.08	1.23	1.19	1.21
3...	32 2487	26 17.3	22.1	1.58	1.59	1.59	1.87	1.89	1.88
4...	32 2486	26 15.2	25.1	2.75	2.81	2.77	2.11	2.10	2.10
5...	....	26 51.6	27.6	2.64	2.63	2.64	2.97	3.03	3.00

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[1] A numerical process, equivalent to the graphical methods given here, is described in *Mt. Wilson Contr.*, No. 80, p. 24; *Astrophysical Journal*, **39**, 330, 1914.

Table VII. Except for very faint stars, the average residual ranges from 0.04 to 0.06 mag. The systematic deviations are naturally one-half the differences between the mean results for the separate nights given in Table VII.

TABLE VIII

COMPARISON STAR RESIDUALS FOR JULY 17, 1914

Plate	Photographic					Plate	Photo-visual				
	1	2	3	4	5		1	2	3	4	5
1648.....	+ 7	− 3	−4	.....	− 1	1649.....	+ 3	−1	− 3	+ 4	+ 1
1650.....	− 3	+ 2	+5	−13	− 4	1651.....	− 2	+2	− 7	+ 5	0
1652.....	+ 6	+ 1	−5	.....	− 2	1653.....	−10	−1	+ 8	+ 3	+28
1654.....	0	− 3	+2	−13	+ 1	1655.....	−19	0	− 1	+20	+30
1656.....	+ 1	+ 3	−4	− 4	+ 2	1657.....	+ 2	+2	− 4	− 1	+ 5
1658.....	+ 1	− 2	+4	− 7	− 1	1659.....	+ 3	+2	− 7	+ 1	+ 9
1660.....	0	− 8	0	+ 4	+ 9	1661.....	+12	+2	− 5	− 9	−11
1662.....	+ 1	− 3	+4	.....	− 3	1663.....	0	−6	+ 5	0	+ 7
1664.....	+ 4	−10	0	+10	− 3	1665.....	+ 6	−2	+ 2	− 6	.....
1666.....	+ 4	0	−9	.....	+ 5	1667.....	+14	+8	− 9	−14	−17
1668.....	+13	− 6	−5	.....	0	1669.....	− 2	0	+ 4	− 2	+ 2
1670.....	+ 7	− 2	−5	.....	− 1	1671.....	− 4	−5	+ 4	+ 6	.....
1672.....	+ 2	− 2	+5	.....	− 7	1673.....	− 1	−9	+15	− 5	−21
1674.....	+ 9	+ 7	−4	.....	−11	1675.....	+ 8	−7	+ 3	− 5	.....
1676.....	+ 3	− 5	+7	.....	.....	1677.....	+11	−7	+ 4	− 6	.....
Syst. diff.	+ 4	− 2	−1	− 4	− 1	Syst. diff.	+ 1	−1	+ 1	− 1	+ 3
Av. dev..	± 4	± 4	±4	± 8	± 4	Av. dev..	± 6	±4	± 5	± 6	±12

REDUCTION TO INTERNATIONAL ZERO-POINT

The reduction to the international zero-point depends on four pairs of comparisons with the Polar Standards, made on August 25 and 26, 1917. The results for the comparison stars in the second and third columns of Table IX are based directly on the scale and zero-point of the Standards themselves. By comparison with the mean relative magnitudes in Table VII, we find for the latter the zero-point corrections in the fifth and sixth columns of Table IX. Adding the mean correction to the relative magnitudes, we have, finally, a second series of magnitudes, given in the last three columns of the table. These refer to the international zero-point, but depend upon the scales established for the individual plates. We thus have for the comparison stars two series of results, quite independently derived. Table X exhibits the agreement and gives the adopted magnitudes, which are the means, with equal weight, of the two determinations.

Incidentally, attention may be directed to the color-indices of the comparison stars. The values in Table IX found by comparing photographic and photo-visual magnitudes agree well with each other and with those in the last column of Table X derived by the exposure-ratio method (three plates),[1] which affords a valuable control.

TABLE IX

MAGNITUDES OF COMPARISON STARS

Star	From N.P. Comp.			Zero-Point		From Rel. Mags.		
	Pg.	Pv.	C.I.	Pg.	Pv.	Pg.	Pv.	C.I.
1......	9.73	9.18	0.55	9.46	8.70	9.78	9.24	0.54
2......	10.62	10.01	0.61	9.54	8.80	10.59	9.97	0.62
3......	11.10	10.63	0.47	9.51	8.75	11.10	10.64	0.46
4......	12.23	10.89	1.34	9.46	8.79	12.28	10.86	1.42
5......	12.22	....	....	9.58	....	12.15	11.76	0.39
Means	....	....	....	9.51	8.76	....	....	....

TABLE X

INTERNAL AGREEMENT AND ADOPTED MAGNITUDES

Star	N.P. Comp. *minus* Rel. Mag.			Adopted Mag. and Color			C.I. from Exp. Rat.
	Pg.	Pv.	C.I.	Pg.	Pv.	C.I.	
1..............	−5	−6	+1	9.76	9.21	0.55	0.56
2..............	+3	+4	−1	10.60	9.99	0.61	0.66
3..............	0	−1	+1	11.10	10.64	0.46	0.30
4..............	−5	+3	−8	12.26	10.88	1.38	1.28
5..............	+7	....	....	12.18	11.76	0.42	0.37

FINAL MAGNITUDES—THE COLOR-CURVE

We are now in a position to finish the reductions in Table VI. Subtracting the magnitudes of the comparison stars in the ninth column from their adopted values in Table X, we find the mean zero-point correction for the plate, namely, 8.31 mag., and, finally, the magnitudes and residuals in the last two columns of Table VI. The main result, however, is the magnitude of the variable, 11.11, which has been transferred to Table XI, where it appears with similar results from the other plates.

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[1] Seares, *Mt. Wilson Communication*, No. 33; *Proceedings of the National Academy of Sciences*, **2**, 521, 1916. For a test of the colors of the North Polar Standards, see *Mt. Wilson Communication*, No. 38; *Proceedings of the National Academy of Sciences*, **3**, 29, 1917.

The magnitudes in Table XI—both photographic and photo-visual—have been compared with the mean visual light-curve in Table V. The photo-visual residuals in the last column of Table XI (unit = 0.01 mag.) are appreciably smaller than the visual residuals (see last column, Table I, and probable errors in Table IV); as already stated, they show that the photo-visual observations are very satisfactorily represented by the adopted visual curve.

The comparison of the photographic measures with this curve gives the color-indices of the variable in the eighth column of Table XI. These are plotted in the lower part of Fig. 1 and show at a glance the change in color paralleling the variation in light. The ordinates of the mean color-curve are in Table V alongside the visual light-curve. The deviations of the color-indices from the mean color-curve are appropriately entered in Table XI under the heading "Photographic O − C," for they are numerically equal to the deviations of the observed photographic magnitudes from the mean photographic light-curve obtained by adding the two series of ordinates in Table V. These residuals are even smaller than those in the last column of the table, owing, probably, to the larger number of exposures made on the plates from which photographic magnitudes have been determined.

The maximum and minimum brightness are: photographic, 9.70 and 11.21; visual, 9.85 and 10.70. The amplitudes, 1.51 and 0.85, respectively, are in the ratio of 1.78 to 1.

SYSTEMATIC DEVIATIONS FROM MEAN LIGHT-CURVE

The systematic deviations of the photographic and photo-visual results from the mean curves are negligibly small—in no case exceeding 0.03 mag. per night. The variation on the three nights involved seems therefore to have been quite normal.

The residuals of the much longer series of observations in Table I, on the other hand, present numerous instances of

TABLE XI

Photographic and Photo-visual Observations

Pg. and Pv. Plate Nos.	G.M. Date		Phase		Photographic		C.I.	Photo-visual	
	Pg.	Pv.	Pg.	Pv.	Mag.	O−C		Mag.	O−C
1914, July 16. J.D. 2420330									
1613–1614....	.670	.679	120	129	10.77	−12	+0.40	10.41	0
1615–1616....	.683	.690	133	140	10.88	−14	.46	10.61	−17
1617–1618....	.697	.704	147	154	10.79	+ 2	.33	10.55	− 7
1619–1620....	.708	.716	158	166	10.94	− 7	.45	10.40	+12
1621–1622....	.720	.727	170	177	10.98	− 6	.45	10.47:	+ 7
1623–1624....	.731	.738	181	188	10.92	+ 3	.37	10.64	− 7
1625–1626....	.746	.754	196	204	11.01	− 1	.43	10.70	−10
1627–1628....	.758	.766	208	216	11.03	+ 1	.42	10.59	+ 3
1629–1630....	.770	.778	220	228	11.11	− 5	.49	10.70	− 6
1631–1632....	.782	.792	232	242	11.03	+ 6	.39	10.65	0
1633–1634....	.796	.804	246	254	11.15	− 3	.49	10.71	− 5
1635–1636....	.808	.815	258	265	11.11	+ 2	.44	10.74	− 7
1637–1638....	.820	.828	270	278	10.99	+16	.31	(10.30)	(+39)
1639–1640....	.832	.842	282	292	11.04	+13	.35	invis.	......
1641.........	.846	......	296	......	11.12	+ 7	+0.42	....	......
1914, July 17. J.D. 2420331									
1648–1649....	.670	.676	365	371	11.11	−12	+0.62	10.48	− 9
1650–1651....	.682	.688	0	6	10.57	+ 2	+ .28	10.14	+ 4
1652–1653....	.695	.700	13	18	10.08	+ 7	.00	9.96	+ 4
1654–1655....	.706	.712	24	30	9.67	+13	− .24	9.71	+14
1656–1657....	.716	.723	34	41	9.88	−14	+ .02	9.93	0
1658–1659....	.729	.735	47	53	9.90	+ 4	− .09	9.96	+ 7
1660–1661....	.742	.747	60	65	10.09	+ 1	+ .02	10.01	+ 9
1662–1663....	.753	.758	71	76	10.25	− 3	.11	9.90	+27
1664–1665....	.764	.770	82	88	10.36	− 3	.16	10.25	− 2
1666–1667....	.776	.782	94	100	10.35	+10	.09	10.27	+ 1
1668–1669....	.789	.796	107	114	10.56	− 2	.25	10.37	− 3
1670–1671....	.802	.807	120	125	10.61	+ 4	.24	10.40	− 1
1672–1673....	.812	.818	130	136	10.67	+ 5	.26	10.31	+12
1674–1675....	.824	.830	142	148	10.58	+21	.13	10.54	− 8
1676–1677....	.836	.841	154	159	10.83	+ 2	+0.35	10.58	− 8
1914, July 18. J.D. 2420332									
1684–1685....	.670	.678	233	241	11.19	−10	+0.55	10.71	− 6
1686–1687....	.684	.690	247	253	11.12	0	.46	10.63	+ 3
1688–1689....	.697	.703	260	266	11.09	+ 4	.42	10.70	− 2
1690–1691....	.710	.718	273	281	11.16	− 1	.48	10.62	+ 7
1692–1693....	.725	.731	288	294	11.08	+11	.38	10.65	+ 5
1694–1695....	.737	.744	300	307	11.22	− 3	.52	10.79	− 9
1696–1697....	.751	.757	314	320	11.22	− 2	.52	10.76	− 6
1698–1699....	.766	.772	329	335	11.21	0	.51	10.89	−19
1700–1701....	.778	.784	341	347	11.13	+ 8	.44	10.80	−14
1702–1703....	.793	.800	356	363	11.13	− 1	.52	10.52	0
1704–1705....	.806	.812	369	375	10.90	− 1	.47	10.33	− 1
1706–1707....	.820	.827	6	13	10.41	− 5	+ .23	9.92	+16
1708–1709....	.834	.841	20	27	9.95	− 7	−0.02	9.72	+15

systematic divergence; for example, those for J.D. 7660, 7667, which are prevailingly positive, and for J.D. 7994, 8118, 8120, 8122, 8131, which are negative. These can scarcely be attributed to variability in the comparison stars, nor is it likely that they are the result of systematic errors of observation. The residuals are larger than the errors normally associated with observations made with the wedge photometer. Moreover, many of the measures, those for J.D. 7660, 7667, 7994, among others, were referred simultaneously to two comparison stars, one equal to or a little brighter, the other appreciably fainter than the variable, with results sensibly the same in both cases. Another peculiarity is found in the fact that divergences of a given sign tend to persist for some time; thus during the interval J.D. 7660–6716 they were positive, while for J.D. 8118–8131 they were negative. The explanation is probably to be found in abnormal fluctuations in the light of the variable, a phenomenon now apparently well established for several of the cluster-type variables.[1]

COMPARISON WITH SPECTROSCOPIC RESULTS

The large change in color—more than 0.6 mag. in the index—led naturally to an inquiry into the spectroscopic behavior of RS Boötis. Mr. Pease kindly placed the star on his program, and quickly found a change in spectral type[2] occurring simultaneously with the fluctuations in light and color which very nearly equals that to be inferred from the well-known relation between color-index and spectrum.

A detailed comparison of Pease's spectra with the color-curve of RS Boötis is given in Table XII. The date, G.M.T. of the middle of exposure, and the spectrum are from his data. The color-indices in the fourth column were interpolated from the curve in Table V, with phases counted from instants for which $v=b$. Those in the sixth column were derived from the spectra by the formula C.I. $=0.4\,S$, in which for B0, A0, F0, etc., S has the numerical values −1, 0, +1, etc.[1] The differences in the two series of results—the color-excess[2]—are in the last column of the table. Ten of the thirteen values are less than a tenth of a magnitude. The amplitude of the color-variation derived from spectra is somewhat smaller than that indicated by the color-curve; but the spectra near minimum are few in number and less accordant than the others, so that conclusive results are not to be expected. Notwithstanding the slight excess of negative signs, the mean color-excess is sensibly zero.

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[1] Shapley and Shapley, *Mt. Wilson Contr.*, No. 104; *Astrophysical Journal*, **42**, 159, 160, 1915; Shapley, *Mt. Wilson Contr.*, No. 112; *Astrophysical Journal*, **43**, 217, 1916; also various references cited in the latter of these investigations.

[2] *Publications of the Astronomical Society of the Pacific*, **26**, 257, 1914.

TABLE XII

COMPARISON OF COLOR-CURVE WITH SPECTRA

Date	G.M.T. Middle of Exposure	Sp.	C.I. from Curve	Phase	C.I. from Spectrum	Color-Excess
1914						
July 20......	$17^h\ 16^m$	B9	−0.05	0.018	−0.04	−0.01
20......	17 57	A0	− .05	.047	.00	− .05
20......	18 40	A5	+ .11	.077	+ .20	− .09
21......	16 35	A6	+ .45	.235	+ .24	+ .21
21......	18 2	F0	+ .49	.296	+ .40	+ .09
21......	19 17	A5	+ .52	.349	+ .20	+ .32
22......	16 28	A7	+ .21	.099	+ .28	− .07
22......	18 5	A8	+ .39	.167	+ .32	+ .07
23......	17 22	A4	+ .22	.004	+ .16	+ .06
23......	18 7	B8	− .11	.036	− .08	− .03
23......	18 52	A6	+ .06	.067	+ .24	− .18
Aug. 17......	15 57	A0	− .08	.042	.00	− .08
17......	16 52	A5	+0.12	0.079	+0.20	−0.08

The negligible character of the color-excess was found from a preliminary reduction, based upon approximate zero-point determinations made in 1914, and was then a result of special interest as one of the earliest indications that the coefficient of space-absorption must be extremely small. Hertzsprung[3] and Russell[4] had found the Cepheids, as a class, to be very bright, and, in general, stars with early type spectra were known to be highly luminous and to show only a small dispersion in their absolute magnitudes. Since the median apparent brightness of RS Boötis is only 10.3, this meant that its distance must be relatively great.[1] The color, however, is only that normally associated with similar spectra in the case of much less distant stars, thus leaving no appreciable excess to be accounted for by scattering of the star's light in its passage to the earth.[2] This conclusion disregarded the influence of the star's absolute magnitude on its color; but the indications were that for stars of early-type spectra this must be very small, and, moreover, that for a highly luminous object like RS Boötis any effect of absolute magnitude would probably increase the color-excess and thus make the estimated upper limit for the absorption coefficient too large.[3]

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[1] Seares, *Mt. Wilson Communication*, No. 16; *Proceedings of the National Academy of Sciences*, 1, 481, 1915.

[2] *Ibid.*

[3] *Zeitschrift für wissenschaftliche Photographie*, **5**, 107, 1907; *Astronomische Nachrichten*, **196**, 201, 1913.

[4] *Science*, N.S., **37**, 652, 1913.

Subsequent observations by Shapley[4] have revealed variations similar to those of RS Boötis in the spectra of other objects, including several of the longer-period variables which by themselves were formerly supposed to constitute the Cepheid class, as well as a number of the short-period cluster-type variables now also commonly classed with the Cepheids. The spectrograms by Pease and Shapley, however, were of low dispersion. For those of Pease $H\gamma$ to K was $2\frac{1}{4}$ mm and their classification could be based only on the hydrogen lines and to some extent on the K line of calcium. Thus the presence of types B8 and B9 in Table XII is not to be interpreted as meaning that the helium lines characteristic of these types were actually observed in the spectra. The classifications by Shapley are also based mainly on the lines of hydrogen, although calcium lines and the G band were used to some extent for types F and G. Still later observations by Adams and Joy,[5] with higher dispersion, confirm the foregoing results, so far as changes in the hydrogen lines are concerned, for a number of stars with periods greater than a day, but show little or no corresponding change in the less conspicuous features of the spectra, which are also characteristic of the various spectral classes.

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[1] A recent determination by Shapley gives absolute magnitude $M=-0.3$, parallax $\pi=0\overset{''}{.}00076$. *Mt. Wilson Contr.*, No. 153; *Astrophysical Journal*, **48**, 1918.

[2] The same principle and others analogous to it have recently been applied by Shapley to the enormously distant globular clusters, with the result that the coefficient of absorption must be far below the upper limit fixed by this early inference.

[3] See summary of results by Adams and by van Rhijn given by Kapteyn in *Mt. Wilson Contr.*, No. 83; *Astrophysical Journal*, **40**, 187, 1914.

[4] *Mt. Wilson Contr.*, No. 124; *Astrophysical Journal*, **44**, 273, 1916, and elsewhere.

[5] *Mt. Wilson Communication*, No. 53; *Proceedings of the National Academy of Sciences*, **4**, 130, 1918.

Presumably this peculiar behavior occurs also in the case of the cluster-type variables. But, in spite of the many similarities connecting the two subclasses of Cepheids, the singularities in their periods and distribution, and possibly also in their motions,[1] suggest some reservation of judgment in the case of stars such as RS Boötis which thus far have not been observed with the dispersion necessary for a detailed study of their spectral characteristics. For the present, the significant result is the suggestion that the color of the Cepheid variables is more closely correlated with the behavior of their hydrogen lines than with other spectral phenomena, even though the latter may be important criteria for classification.

For several of the longer-period Cepheids the question thus raised can be put to a direct test. Adams and Joy have given spectral classifications corresponding to maximum and minimum light, based both on the hydrogen lines and on the "general spectrum"; and from various sources we can derive values of the color-variation for comparison with the spectral changes. Supplementary evidence indicating the intimate connection of color and spectral type as derived from the hydrogen lines is also available for various stars whose general spectrum, in the sense used by Adams and Joy, has not yet been observed.

The data are in Table XIII. The results by Adams and Joy for the third, fourth, and fifth stars are from *Mt. Wilson*

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[1] The frequency-curve of the periods shows two maxima, one for twelve hours and one for seven days, corresponding to the two subclasses of Cepheids. The longer-period stars are closely confined to the plane of the Galaxy, while the cluster-type variables are widely scattered throughout space. So far as known, the proper motions and radial velocities of the long-period Cepheids are small or moderate; of the short-period Cepheids, relatively large. Two recent spectrograms of RS Boötis by Adams give $V = -52$ km per sec. See Hertzsprung, *Astronomische Nachrichten*, **179**, 376, 1909; **192**, 262, 1912; **196**, 205, 1913; and Shapley, *Mt. Wilson Contr.*, No. 153; *Astrophysical Journal*, **48**, December, 1918.

Communication, No. 53;[1] those for η Aquilae and δ Cephei are from spectrograms specially selected by Adams on account of their close agreement with the instants of maximum and minimum brightness, while those for XZ Cygni are unpublished results from two low-dispersion spectra, one at minimum, the other near but not actually coinciding with the maximum. The limiting spectral types by Shapley are from the curves given in *Mt. Wilson Contribution*, No. 124.[2] The changes in color given in the last three columns, corresponding to the observed spectral variations, were obtained as usual from the relation C.I. = 0.4 *S*.

TABLE XIII

CHANGES IN COLOR AND SPECTRUM FOR VARIOUS CEPHEIDS

STAR	AMPLITUDE		OBSD. CHANGE IN C.I.	GENERAL SPECTRUM ADAMS AND JOY	HYDROGEN LINES		CHANGE IN C.I. FROM SP.		
					Adams and Joy	Shapley	General Spectrum	Hydrogen Lines	
	Pg.	Vis.						A.and J.	Sh.
η Aquilae*...	1.09	0.67	0.42	F9–G1	F1–F9	F0–G1	0.08	0.32	0.44
δ Cephei†....	1.25	.76	.49	Gc–G0	F0–G0	F0–G2	.00	.40	.48
ζ Gemin.‡ ...	1.00	.62	.38	F9–G0	F2–F3	....	(.04)	(.04)	......
RT Aurigae§.	1.20	.85	.35	F7–G0	F0–F8	A9–F9	.12	.32	.40
SU Cass.‖....	0.47	.33	.14	F7–F8	F3–F6	A9–F4	0.04	0.12	.20
SU Cygni¶...	1.15	.80	.35	....	....	A7–F6	......	......	.36
T Vulpec.¶...	1.20	.80	.40	....	....	A9–G0	......	......	.44
S Sagitt.¶....	1.30	.77	.53	....	....	F4–G2	......	......	.32
U Vulpec.¶..	1.1	.68	.4	....	....	F8–G4	......	......	(0.24)
XZ Cygni**..	1.62	1.04	0.58	....	A0–A8	....	......	(0.32)	......

* Kohlschütter, Pg.; Luizet, Vis. *Astronomische Nachrichten*, **183**, 265, 1910.

† Wirtz, Pg. *Astronomische Nachrichten*, **154**, 327, 1901; Stebbins, Vis., *Astrophysical Journal*, **27**, 188, 1908.

‡ Wirtz, Pg. *Loc. cit.;* Vis. is the value from Argelander's observations as given by Wirtz. The only Cepheid of normal amplitude thus far observed whose hydrogen lines do not indicate an appreciable change of type.

§ Kiess, *Laws Observatory Bulletin*, **2**, 99, 1915.

‖ Parkhurst, Pg., *Astrophysical Journal*, **28**, 279, 1908; Müller and Kempf, Vis., *Astronomische Nachrichten*, **173**, 307, 1907.

¶ Pg. data by Wilkins; Vis., by various observers, collected by Wilkins, *Astronomische Nachrichten*, **172**, 316, 1906. The comparison for U Vulpeculae is unreliable; the star is probably subject to large fluctuations (cf. *Laws Observatory Bulletins*, **1**, 150, 1907); moreover, the spectra are difficult and not well distributed over the curve.

** Photometric data by Martha Betz Shapley, *Mt. Wilson Contr.*, No. 128; *Astrophysical Journal*, **45**, 182, 1917. The spectrum A0 corresponds to an instant following the maximum by 0.03 day. Since the period is only 0.47 day, the change in color derived from the spectrum is probably too small.

For the general spectrum there is little change in passing from maximum to minimum and the inferred color-changes in the eighth column are very small, whereas those derived from the hydrogen lines (ninth and tenth columns) are sensibly equal to the color-variation found from the photometric data (fourth column). Disregarding values in parentheses because of the peculiarities and uncertainties mentioned in the notes, the mean results are:

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[1] *Proceedings of the National Academy of Sciences*, **4**, 130, 1918.

[2] *Astrophysical Journal*, **44**, 273, 1916.

High-dispersion, four stars, Adams and Joy:	
Color-change from general spectrum	+0.06 mag.
Color-change from hydrogen lines	+0.29
Low-dispersion, same stars, Shapley:	
Color-change from hydrogen lines	+0.38 mag.
Observed change in color-index	+0.35

For the seven stars observed by Shapley which have trustworthy results the means are

Color-change from hydrogen lines	+0.38 mag.
Color-change from color-index	+0.38

The intimate connection between color and spectral type as derived from the hydrogen lines is shown clearly enough by these results; in fact, the closeness of the agreement for individual stars is rather surprising in view of the heterogeneous character of the photometric data and the uncertainties of classification, especially for low-dispersion spectrograms. On the other hand, it is equally clear that there is little or no correlation between the color of Cepheids and numerous other features of their spectra, which, for the stars in general, commonly follow the hydrogen lines in their changes throughout the normal succession of spectral types.

SUMMARY

Two independent series of observations on the cluster-type variable RS Boötis—one visual, the other photographic and photovisual—made with different instruments and separated by an interval of several years, have been discussed for the revision of the elements and the determination of mean light- and color-curves. There is some evidence of irregular fluctuations in the light, but no certain indication of a variation in the period, the adopted value for which is based on nearly 7000 cycles of variation.

The photo-visual observations are well satisfied by the mean curve based on the visual observations. The amplitude of the photographic variation is 1.78 times that of the visual. The color-index varies continuously from −0.15 mag. at maximum to +0.52 mag. at minimum.

The change in color agrees closely with that to be inferred from spectrograms by Pease. Bearing in mind certain anomalies in the spectroscopic behavior of other Cepheids, this result indicates that the color of these variables is more closely correlated with the characteristics of their hydrogen lines than it is with less conspicuous phenomena of the general spectrum also used for spectral classification. An examination of all the data available confirms this conclusion for a number of Cepheids.

Mount Wilson Solar Observatory
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STUDIES BASED ON THE COLORS AND MAGNITUDES IN STELLAR CLUSTERS

THIRTEENTH PAPER: THE GALACTIC PLANES IN 41 GLOBULAR CLUSTERS

By HARLOW SHAPLEY and MARTHA B. SHAPLEY

From evidence presented in earlier papers there can be little doubt that the observed elliptical distribution of stars in the photographic images of globular clusters is really an indication of flattening with respect to more or less symmetrical equatorial planes. The analogy with other flattened sidereal organizations—such as the Galaxy, the spiral nebulae, the solar and planetary systems—leads us to interpret these projected elongations of clusters as indicative not of prolate spheroids but of oblate spheroids or ellipsoids. It is of some interest to know how general this phenomenon is among globular clusters; and since their relation to the galactic system has been demonstrated, it is important to see if their orientation in space betrays any effect of the dominance of the greater system.

A study of star counts by Pease and Shapley[1] has shown the elliptical form for five clusters and has suggested that three others are sensibly circular. The ellipticity of Messier 5 has also been determined from Mount Wilson plates.[2] Further discussion by Shapley[3] has indicated the relation of the elongation in ω Centauri to its variable stars, and in Messier 13 to the stars of bluish color. The present paper continues the work, increasing the number of clusters known to have appreciable elongation from six to about thirty, and showing that several others are sensibly circular, either because of their orientation in space or because of actual sphericity. All globular clusters sufficiently bright (and in other respects satisfactory) to give definite results have been investigated. The accumulated material, though not extensive, is complete enough to permit statistical examination.

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[1] *Mt. Wilson Contr.* No. 129; *Astrophysical Journal*, **45**, 225, 1917; *Mt. Wilson Communications*, No. 39; *Proceedings of the National Academy of Sciences*, **3**, 96, 1917.

[2] Shapley and Davis, *Publications of the Astronomical Society of the Pacific*, **30**, 164, 1918.

[3] *Mt. Wilson Communications*, No. 45; *Proceedings of the National Academy of Sciences*, **3**, 276, 1917.

TABLE I

COMPARISON OF COUNTS AND ESTIMATES

N.G.C.	MESSIER	POSITION-ANGLE OF MAJOR AXIS		
		Counts	Estimates	Difference
5024	53	160°	170°	−10°
5139	....	105	120	−15
5272	3	Asym.	Asym.	....
5904	5	55	50	+ 5
6121	4	115	Indeterm.	....
6205	13	125	130	− 5
6218	12	Appr. circ.	Appr. circ.	....
6254	10	See note	Appr. circ.	....
6266	62	75	65:	+10:
6273	19	15	15	0
6341	92	Indeterm.	25	....
6402	14	110	70:	+40:
6626	28	50	45	+ 5
6656	22	25	25	0
6779	56	Appr. circ.	150:	....
7078	15	35	20	+15
7089	2	135	150	−15

N.G.C. 5139 (ω Centauri). The direction of the major axis, as estimated from photographs made at Harvard, is 105°.

N.G.C. 5272. Cf. tabulated counts in *Mt. Wilson Contr.* No. 129.

N.G.C. 5904. Cf. discussion of asymmetry in *Publications of the Astronomical Society of the Pacific*, **30**, 164, 1918.

N.G.C. 6205. The image of the Hercules cluster on the Franklin-Adams chart is too distant from the center and too diffuse to permit dependable estimates. The tabulated value is from photographs by Pease.

N.G.C. 6254. Perhaps the counts given in *Mt. Wilson Contr.* No. 129, show a trace of ellipticity, major axis in position-angle 150°.

The Franklin-Adams charts, which were of much value in determining the relative diameters of globular clusters, are also highly useful in the present problem. Although the ellipticity of clusters cannot be accurately determined from ordinary photographs without a special study of the distribution of the individual stars, and generally then only when many faint stars are included in the counts, it is found that direct estimates based upon photographs of small scale give values of the direction of elongation as definite as those resulting from the extensive counts of stars. The direct estimates do not give the degree of elongation, but that property is also indefinite from counts, for it apparently depends in an uncertain manner upon the brightness of the stars enumerated and upon other factors.

In Table I the direction of elongation, when measurable, is given for 17 clusters, both from estimates and counts, the two determinations being entirely independent of each other. The average difference[1] between the position-angles thus derived is 8°. Better agreement than this is not obtainable from counts of two plates of the same cluster, or from the counts on one plate of stars of different magnitudes, colors, or distances from the center.[2] The consistent agreement therefore justifies the substitution of direct estimates for laborious counts. We are therefore enabled to extend the investigation expeditiously to the bright southern objects and complete the survey of the form of globular clusters.

Ten of the values of the third column of Table I are from published results. Those for N.G.C. 6121, 6266, 6273, 6341, 6402, 6626, and 6656, based on Mount Wilson plates, are from recent counts made by Miss Davis subsequent to the estimations from the charts. New counts have also been made of the stars in N.G.C. 5139 (ω Centauri) on plates kindly lent by the Harvard College Observatory. Some of the more interesting results of the unpublished counts are noted below.

1. The decided asymmetry of N.G.C. 6266 (Messier 62), particularly noted by Sir John Herschel[3] and later by Bailey,[4] is shown by the following values derived from a Mount Wilson plate:

Sector.......	15°	45°	75°	105°	135°	165°	195°	225°	255°	285°	315°	345°
No. stars....	67	56	67	45	35	42	49	56	68	79	72	72

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[1] Omitting N.G.C. 6402, which is very open and ragged in outline on the Franklin-Adams chart.

[2] Cf. curves for Messier 13 in *Mt. Wilson Contr.* No. 129.

[3] *Cape Results* (London, 1847), p. 23.

[4] *Harvard Annals*, **76**, 74, 1915.

Stars within 30″ of the center could not be counted. Combining opposite sectors, the position-angle of the major axis appears to be about 75°.

2. Probably the most conspicuously elongated globular cluster is N.G.C. 6273 (Messier 19), whose relative stellar density is shown in Fig. 1. The radial co-ordinate is number of stars for each 30° sector of position-angle. Without correction for superposed field stars, there are more than twice as many stars in the direction of the major axis as perpendicular thereto. The plates show no evidence of a double nucleus. We probably see this flattened cluster more nearly edgewise than is usually the case. The major axis is approximately parallel to the Milky Way, the galactic latitude is +9°, and it appears likely, therefore, that the plane of symmetry is nearly parallel to the galactic plane. Apparently the flattening of a cluster, if we judge from this extreme case, is of a much lower order than that of spirals and of the galactic system.

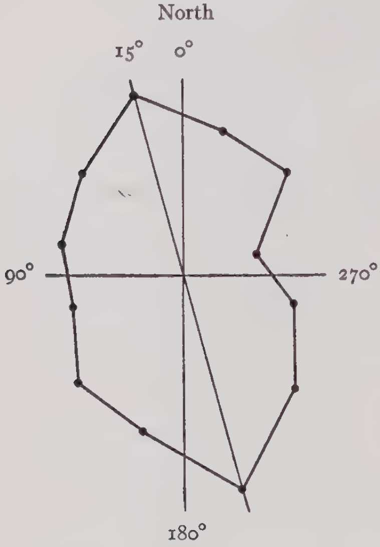

Fig. 1.—Curve of star-density for Messier 19. Radial co-ordinates show the number of stars for each 30° of position-angle.

3. The cluster N.G.C. 6626 (Messier 28) is also conspicuously elongated, with major axis roughly parallel to the Milky Way, but it is hardly as symmetrical as Messier 19.

4. A photograph made with the 60-inch reflector of the bright southern cluster N.G.C. 6656 (Messier 22), which shows more than 70,000 stars, has been counted by Miss Davis.

Nearer the center than 8 mm (218″) the density is too great to give reliable results; under such conditions the large images of the brighter stars occult so many faint images that counts of the latter may show the ellipticity obliterated or even reversed.

Outside the circle of 14 mm (381″) radius the proportion of field stars becomes so large that the amount of ellipticity is masked, and its observed direction may be affected by local irregularities in the field. Neither of these factors—the interior crowding or the exterior field—is troublesome in estimating the direction of elongation of bright clusters from small-scale plates.

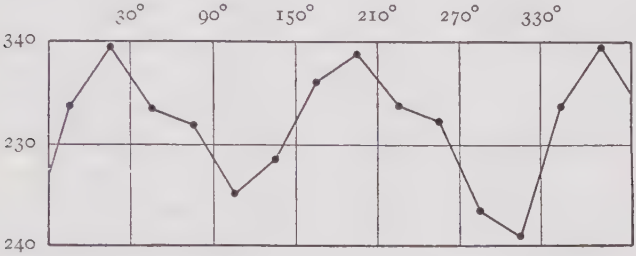

FIG. 2.—Distribution of stars in Messier 22. Ordinates are numbers of stars; abscissae are position-angles.

Counts within the limits 8 and 14 mm give the distribution of stars shown in Fig. 2, the ordinates being numbers of stars in 30° sectors and the abscissae position-angles. A correction for superposed field stars has been made by deducting 30 stars per square minute. Combining opposite sectors we have the following means for the smooth ellipticity-curve shown in Fig. 3:

Sector	15°	45°	75°	105°	135°	165°
No. stars	336	308	300	262	263	314

The number of stars in the direction of the major axis exceeds the number in the direction of the minor axis by nearly 30 per cent of the latter.

Within the errors of measurement the major axis of Messier 22 is exactly parallel with the Galaxy. The high ellipticity suggests that we see the cluster edgewise, and, since it is in low galactic latitude, the result further indicates that the plane of symmetry is approximately parallel with the galactic plane. This point may be significant in connection with the loose structure of the cluster and its proximity to the central plane of the galactic system.

Table II contains data for 41 globular systems. When the direction of the major axis has been derived by two methods a weighted mean is adopted. The inclination of the observed major axis to the galactic plane, in the sixth column, has been computed from the data of preceding columns and checked graphically. The linear distances from the galactic plane, $R \sin \beta$, expressed in units of 100 parsecs, are taken from Tables V and VIII of *Mt. Wilson Contr.* No. 152.

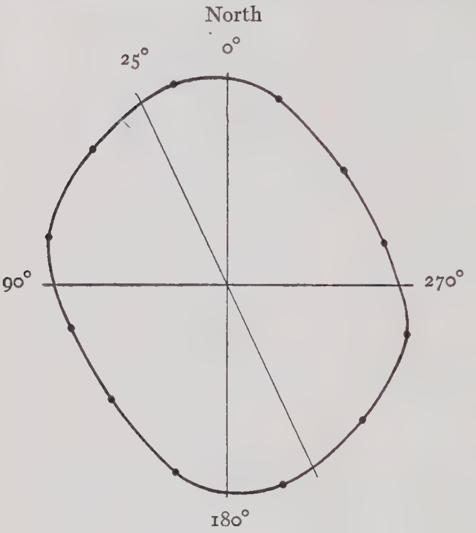

Fig. 3.—Curve of star-density for Messier 22. The radial co-ordinates show the number of stars for each 30° of position-angle.

In order that the equatorial plane of a cluster and the galactic plane may be parallel, the parallelism of the major axis with the galactic circle is a necessary though not sufficient condition. Without knowing another dimension of the inclination we can say little as to the possible influence of the greater system upon the orientation of its secondaries. There are 15 clusters whose major axes are inclined less than 35° to the galactic circle; some of these may be parallel with the Galaxy, but none of the others can be considered as even approximately so.

If the equatorial planes of globular clusters are distributed at random we should find both large and small inclinations of the major axes at all distances. This condition holds for the 25 clusters more distant than 2000 parsecs from the plane, their inclinations ranging from 0° to 85° with an average of 45°, as shown in Table III. The major axes of all the 8 clusters within 3000 parsecs of the plane, however, are inclined less than 35°, except for N.G.C. 4372, which is very loose and apparently is partially obstructed by dark nebulosity.[1]

TABLE II

RESULTS FOR 41 CLUSTERS

N.G.C.	Messier	Galactic Longitude	Galactic Latitude	Adopted Position-Angle of Major Axis	Inclination of Major Axis to Galactic Circle	Distance from Galactic Plane (Unit is 100 Parsecs)
104	....	272°	−44°	35°	45°	−47
362	....	268	−46	110	15	−109
1851	....	211	−34	Appr. circ.	....	−96
1904	79	195	−28	20	30	−120
2808	....	249	−11	20	65	−32
4372	....	269	−9	30:	65:	−18
5024	53	307	+79	165	75	+186
5139	....	277	+16	110	30	+18
5272	3	8	+77	Asym.	....	+135
5286	....	279	+11	Appr. circ.	....	+37
5904	5	333	+45	55	15	+88
5986	....	305	+13	Appr. circ.	....	+47
6093	80	320	+18	165	55	+62
6121	4	319	+15	115	70	+30
6171	....	331	+22	65:	30:	+60
6205	13	26	+40	125	65	+71
6218	12	344	+25	Appr. circ.	....	+52
6254	10	343	+22	Appr. circ.	....	+45
6266	62	320	+7	70	35	+19
6273	19	324	+9	15	20	+25
6284	....	325	+10	Appr. circ.	....	+64
6293	....	325	+8	35	0	+37
6304	....	323	+5	Appr. circ.	....	+28
6341	92	35	+34	25	15	+69
6362	....	293	−17	110	80	−38
6397	....	304	−12	35	5	−17
6402	14	349	+14	100	70	+56
6541	....	316	−11	45	20	−28
6584	....	309	−16	Appr. circ.	....	−73
6626	28	336	−7	45	20	−23
6656	22	338	−9	25	0	−13
6681	70	329	−13	150:	55:	−41
6715	54	333	−16	95	70	−44
6723	....	327	−18	Appr. circ.	....	−39
6752	....	303	−26	125:	65:	−39
6779	56	30	+7	Appr. circ.	....	+30
6809	55	335	−24	100	80	−41
7006	....	32	−20	115	85	−228
7078	15	33	−29	30	10	−71
7089	2	22	−37	140	70	−94
7099	30	356	−48	20	5	−128

---

[1] Cf. p. 22, n. 2, of *Mt. Wilson Contr.* No. 152.

There is some evidence, therefore, that the equatorial planes in clusters may be roughly parallel to the galactic plane when near it, but that when distant they are oriented at random in space.

TABLE III

AVERAGE INCLINATION TO GALAXY FOR DIFFERENT DISTANCES FROM GALACTIC PLANE

Interval of $R \sin \beta$	Number of Clusters	Mean $R \sin \beta$	Mean Inclination
0 to ±20........	5	17	23°
±20 to ±40........	8	31	41
±40 to ±60........	5	46	64
±60 to ±80........	5	67	36
> ±80........	7	136	42
> ±20........	25	72	45°

SUMMARY

1. The good agreement of results from counts on large-scale photographs with results from estimates on Franklin-Adams charts justifies the use of the latter for detecting the ellipticity of clusters (Table I).

2. Thirty globular clusters show an apparent elongation due to the discoidal distribution of their stars; a number of others appear circular in outline, due either to actual sphericity or, more probably, to the high inclination of their planes of symmetry to the line of sight (Table II).

3. Probably the most elongated cluster on record is Messier 19, for which the stars are more than twice as numerous in position-angles 15° and 195° as in directions perpendicular to the major axis of the projected ellipse (Fig. 1).

4. The flattening of a globular cluster, while appreciable, is obviously very small in comparison with that of spiral nebulae or with what now appears to be the form of the general galactic system.

5. The equatorial plane of the globular cluster nearest the galactic plane, Messier 22, appears to be parallel to the Galaxy (Figs. 2 and 3).

6. In general, the clusters nearest the dense stellar regions are most nearly parallel to the galactic plane.

MOUNT WILSON SOLAR OBSERVATORY
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STUDIES BASED ON THE COLORS AND MAGNITUDES IN STELLAR CLUSTERS

FOURTEENTH PAPER: FURTHER REMARKS ON THE STRUCTURE OF THE GALACTIC SYSTEM

By HARLOW SHAPLEY AND MARTHA B. SHAPLEY

The present paper contains miscellaneous notes relating to or extending the results and arguments of the twelfth paper of this series. To economize space the observations are treated synoptically, and the discussion of interpretations is abbreviated in so far as consistent with a fair presentation. The subjects discussed are the following:

I. The indication of a direct relationship between globular and open clusters afforded by the galactic distribution.

II. The seventeen clusters which recent studies have added to the original lists in *Mount Wilson Contributions* No. 152.

III. Diagrams of the positions in space of all globular clusters.

IV. Evidence that the observed absence of globular clusters from the equatorial segment is not due to obstructing cosmic clouds in the Milky Way.

V. The diameter-parallax relation for globular clusters as derived from other sources than the Franklin-Adams charts.

VI. Holetschek's magnitudes of 40 globular clusters and the possibility of determining relative parallaxes from measures of the integrated light.

VII. Observational data supporting the view that the motions and distribution of spiral nebulae depend upon dynamical causes quite different from those prevailing in clusters of stars.

VIII. The bearing of the cluster studies on the formulation of a tentative hypothesis of the origin and development of the galactic system.

IX. Evidence from the *Henry Draper Catalogue* concerning the extent and inclination of the local cluster defined in *Mount Wilson Contributions* No. 157.

I. THE COMPLEMENTARY DISTRIBUTION IN GALACTIC LATITUDE OF GLOBULAR AND OPEN CLUSTERS

Although we have not as yet determined individual distances for open clusters we may safely accept that, generally speaking, they are much nearer the sun than the globular systems. If we should assume that the two kinds are similarly distributed with respect to the galactic plane,[1] most open groups would appear in relatively high galactic latitude, because of their proximity to the sun. With the exception of such near and luminous groups as the Pleiades and Praesepe, however, they occur only in low galactic latitudes, and they do not show the avoidance of the Milky Way exhibited by globular clusters. In fact, when actual positions in space are considered, they appear to occur in the dense stellar regions where globular clusters are not found; and they do not occur in the extra-galactic realms where the globular clusters are. This apparently complementary distribution of the two types of clusters is of high importance for the hypothesis which derives the galactic system originally from stellar clusters and obtains the open groups of the Milky Way from the extra-galactic globular organizations.

The continuous gradation of the compact groups into the open clusters of the Milky Way might be more clearly shown if we could correlate compactness with $R \sin \beta$;[2] but lacking parallaxes of the open clusters, we must for the present content ourselves with pointing out the complementary character of the frequencies of galactic latitude. In Fig. 1 the full line refers to open clusters—the objects of Classes II and III of Melotte's catalogue.[3] A few clusters have been withdrawn from his classes and two or three added, on the basis of Mount Wilson plates. Table I contains the assembled results.

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[1] Cf. *Contributions from the Mount Wilson Solar Observatory*, No. 152, Figs. 4, 5, and 6; No. 157, Fig. 2.

[2] R is the radial distance from the earth, β is the galactic latitude.

[3] *Memoirs of the Royal Astronomical Society*, **60**, Part V, 1915. Diagrams by Melotte illustrate the well-known difference for open and globular clusters of the distribution in galactic longitude.

The dotted line in Fig. 1 refers to globular clusters; the data are from *Mount Wilson Contributions* No. 152,[1] supplemented by the additional material in Table II of this paper, omitting only the five unproved clusters. Fig. 2 gives the frequency when the results are treated irrespective of the sign of β, and shows more clearly the apparent relation of open and globular clusters. A few globular systems in high galactic latitude are not represented in the diagrams, and for $\beta \geqq 10^\circ$ the numbers of Table I for both globular and open clusters have been combined into small groups in drawing the curves.

TABLE I

FREQUENCY IN GALACTIC LATITUDES OF OPEN AND GLOBULAR CLUSTERS

Galactic Latitude	Number of Open Clusters		Number of Globular Clusters	
	North	South	North	South
0°– 2°	21	25	0	0
2 – 4	19	19	1	1
4 – 6	10	13	4	2
6 – 8	7	4	3	5
8 –10	3	7	4	5
10 –12	5	0	4	3
12 –14	4	2	2	3
14 –16	0	3	4	2
16 –18	3	3	2	3
18 –20	0	0	1	1
20 –22	2	0	0	3
22 –24	0	1	2	0
24 –26	0	0	1	1
26 –28	1	0	0	1
28 –30	0	0	0	3
30 –32	0	0	1	0
32 –34	2	0	0	0
34 –36	0	0	1	2
36 –38	0	0	1	1
38 –40	0	0	1	0
40 –42	0	0	1	0
42 –44	0	0	0	0
44 –46	0	0	0	1
46 –48	0	0	1	1
48 –50	0	0	1	1

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[1] A few of the latitudes of Table V (*op. cit.*) have been slightly revised as noted in the third section of the present contribution.

If for β, the angular distance from the plane, we were able to substitute the linear distance, $R \sin \beta$, the mid-galactic maximum for the open clusters would be closely restricted around the vertical axis; and the two maxima of the dotted curve would be relatively much steeper on the side nearer the vertical axis, showing that the galactic zone is entirely clear of globular clusters.[1]

Clusters of Melotte's Class II are more compact and populous than those of Class III, but the distribution in galactic latitude is essentially the same for both types. Choosing from all the data the twenty richest and most regular open clusters, we find the following distribution in galactic latitude:

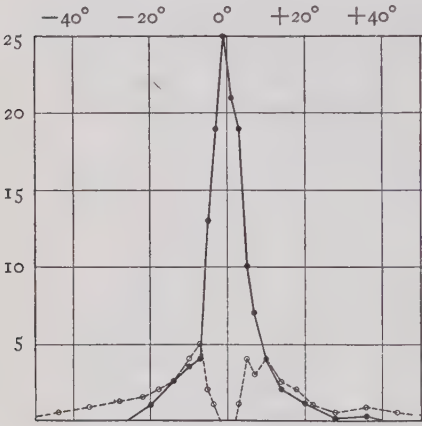

FIG. 1.—Frequency in galactic latitude of open clusters (full line) and globular clusters (broken line). Ordinates are numbers of clusters; abscissae are angular distances from the galactic plane.

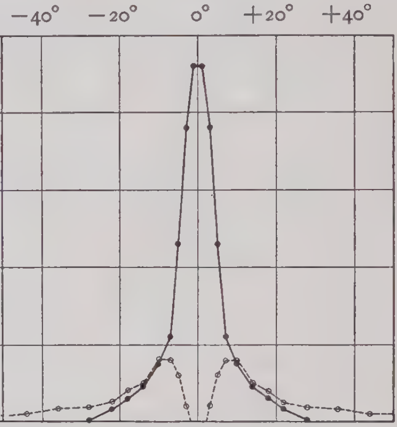

FIG. 2.—Reflected frequency-curve of galactic latitudes for open clusters (full line) and globular clusters (broken line).

β	$<-4°$	$-4°,-3°$	$-2°,-1°$	$-0°,+0°$	$+1°,+2°$	$+3°,+4°$	$>+4°$
Number	2	4	4	3	5	2	0

This certainly does not show a concentration to the medial plane; but perhaps it does not indicate as marked an avoidance of that plane as might be expected if we assume that globular systems are more closely allied to rich clusters than to the average open group.

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[1] Cf. Fig. 6 of the seventh paper of this series.

II. THE DISTANCES OF 17 ADDITIONAL GLOBULAR CLUSTERS

A year ago 69 clusters were accepted as definitely globular.[1] Since that time special efforts have been made to examine with the 60-inch reflector the suspected and doubtful cases north of declination -30° that had not been admitted to the earlier lists. The experience gained as a consequence of photographing faint and abnormal systems has also afforded a basis for better judgment in accepting or rejecting the clusters too far south for observation at Mount Wilson. Some unpublished observations of clusters with the Crossley reflector at the Lick Observatory were generously placed at our disposal by Professor Curtis. While at Mount Wilson during the past summer Professor Duncan of Wellesley College kindly made some long exposures on clusters with the 60-inch reflector.

Results of the survey of doubtful cases are summarized in Table II. In addition a number of clusters concerning which some doubt existed,[2] such as N.G.C. 4147, 6144, 6656, 6712, and 7006, have been proved during the year to be entitled to places in the lists; none of the 69 original clusters should be withdrawn, as far as the present observations go.

As in *Mount Wilson Contributions* No. 152, the treatment of individual cases must be omitted. The distances, which in general cannot claim as high accuracy as those previously studied, depend for the most part upon diameters, since the objects in many cases are so faint that accurate analysis of the magnitudes would have been too laborious. Exposures of several hours were occasionally necessary to test for globularity. Frequently systems that shorter exposures had shown with some definiteness to be open[3] proved upon more persistent observation to be globular. At present there is hardly another object, north of -30° at least, that is suspected with good reason of being a globular cluster.

Three points relative to the large distances are worthy of comment. (1) Practically all of these additional systems are more distant than 30,000 parsecs (100,000 light-years), thereby confirming an earlier surmise as to the completeness of the survey of globular clusters within that distance of the sun. (2) Every recognized globular cluster except one bears a number from the *New General Catalogue*, thus testifying that, in spite of distance and faintness, all of these remote objects were known prior to 1888. In fact, all but this one exception were known before 1864, the date of the *General Catalogue*, and all but three or four had already been catalogued by the Herschels or earlier observers more than eighty years ago. This is further evidence of the completeness of our lists of globular clusters. (3) N.G.C. 7006, with adopted distance of 67,000 parsecs, still holds its place as the most remote sidereal object of definitely estimated distance.

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[1] Tables V and VIII of *Mt. Wilson Contr.* No. 152, 1917.

[2] *Op cit.*, p. 9, n. 2.

[3] *Op. cit.*, p. 14, n. 2.

TABLE II

PARALLAXES OF 17 CLUSTERS

N.G.C.	R. A. 1900	DECL. 1900	GALACTIC		DISTANCE (UNIT IS 100 PARSECS)		
			Long.	Lat.	Radial	Projected on Galactic Plane	From Galactic Plane
*5466	$14^h\ 1^m.0$	+29° 0′	10°	+72°	195:	60	+180
†I.C. 4499	14 45.0	−81 49	274	−20	250	235	− 85
*5927	15 20.8	−50 19	294	+ 5	185	185	+ 16
‡5946	15 28.2	−50 19	295	+ 4.0	415	415	+ 29
‡6355	17 17.8	−26 15	327	+ 4.4	500	500	+ 38
*6366	17 22.4	− 4 59	346	+16	320	310	+ 89
*6426	17 39.9	+ 3 13	356	+15	570	550	+150
*6440	17 42.9	−20 19	336	+ 2.5	525	525	+ 23
‡6496	17 51.8	−44 14	315	+11	305:	300	+ 58
*6517	17 56.4	− 8 57	347	+ 5	625	625	+ 55
‡6535	17 58.7	− 0 18	354	+10	370:	365	+ 64
†6539	17 59.4	− 7 35	348	+ 6	400	400	+ 42
†6553	18 3.2	−25 56	332	− 3.4	320	320	− 19
‡6558	18 3.8	−31 47	327	− 6	455	450	− 48
*6569	18 7.2	−31 51	328	− 7	400	395	− 49
†6760	19 6.1	+ 0 52	3	− 4.6	525	525	− 42
*7492	23 3.1	−16 10	21	−64	285	125	−255

* Certainly a globular cluster.

† Almost certainly a globular cluster.

‡ Probably a globular cluster but as yet not proved.

Further observations are likely to prove that N.G.C. 6355 and 6535 are globular, but present photographs of them are hardly conclusive. The other three unproved systems are too far south for the 60-inch reflector.

Perhaps the most striking result of this special survey is the evidence that every faint, little-condensed cluster in galactic latitude higher than 15° or 20° is really globular, although short exposures and visual observations had in several cases heretofore recorded few stars. On the other hand, the similar faint clusters along the galactic equator, without exception, are open groups with no condensed background of faint stars appearing on long exposures. N.G.C. 7492, for instance, was formerly considered an exception[1]—an open cluster outside the galactic segment—but it is actually globular, containing thousands of stars. The evidence grows continually stronger that open and globular clusters occupy regions of space that are mutually exclusive.

There is also some evidence that an abnormal type of globular cluster exists, one in which the brighter stars are fainter and more scattered than is usually the case. In their luminosity-curves a distinct break appears to occur between the brighter and fainter stars, and for such systems the parallax-diameter relation may not be strictly applicable. The known examples of this type are N.G.C. 5466, 6366, and 7492; it might be well also to place N.G.C. 4372, 5897, and 6144 in this group, although for two of them an alternative interpretation of the apparent discrepancy between diameter and magnitude may be available.[2] The abnormal form possibly represents an early or late stage, or a disturbed condition of a typical globular cluster. In N.G.C. 7492, for which some preliminary colors are available, the brightest stars are yellow.

III. ON THE DISTRIBUTION IN SPACE OF 86 GLOBULAR CLUSTERS

For the reasons apparent from the following remarks, it seems worth while to revise the plots and extend the discussion of the distribution in space of globular clusters. Fig. 3 shows the projection on a plane perpendicular to the Galaxy and oriented to include galactic longitude 325°. It gives the appearance of the system of clusters as seen from longitude 235° and is thus merely a revision of Fig. 4 of the seventh paper of this series.[1] The values of the abscissae, $R \cos\beta \cos(\lambda-325^\circ)$, were determined graphically for

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[1] *Publications of the Astronomical Society of the Pacific*, **30**, 50, 1918.

[2] *Mt. Wilson Contr.* No. 152, p. 14, n. 2, 1917.

TABLE III

SPACE CO-ORDINATES OF GLOBULAR CLUSTERS

N.G.C.	$R \sin\beta$	$R \cos\beta$-$\cos(\lambda-325^\circ)$	$R \cos\beta$-$\sin(\lambda-325^\circ)$	N.G.C.	$R \sin\beta$	$R \cos\beta$-$\cos(\lambda-325^\circ)$	$R \cos\beta$-$\sin(\lambda-325^\circ)$
104....	− 47	+ 29	− 39	6341....	+ 69	+ 33	+ 96
288....	−189	− 3	− 7	6352....	− 28	+216	− 66
362....	−109	+ 58	− 89	6355*...	+ 38	+500	+ 18
1261....	−199	+ 8	−161	6356....	+ 67	+375	+ 59
1851....	− 96	− 58	−131	6362....	− 38	+105	− 66
1904....	−120	−145	−173	6366....	+ 89	+290	+111
2298....	− 63	− 88	−219	6388....	− 34	+269	− 62
2808....	− 32	+ 40	−162	6397....	− 17	+ 76	− 29
3201....	+ 26	+ 23	−143	6402....	+ 56	+207	+ 92
4147....	+514	− 23	−107	6426....	+150	+471	+283
4372....	− 18	+ 63	− 94	6440....	+ 23	+516	+100
4590....	+ 97	+ 70	−108	6441....	− 40	+452	− 32
4833....	− 23	+ 96	−132	6496*...	+ 58	+296	− 52
5024....	+186	+ 34	− 11	6517....	+ 55	+579	+234
5139....	+ 18	+ 41	− 46	6535*...	+ 64	+319	+177
5272....	+136	+ 20	+ 21	6539....	+ 42	+368	+156
5286....	+ 37	+134	−139	6541....	− 28	+142	− 22
5466....	+180	+ 42	+ 42	6553....	− 19	+318	+ 39
5634....	+229	+191	− 55	6558*...	− 48	+450	+ 16
I.C. 4499	− 85	+148	−183	6569....	− 49	+395	+ 21
5897....	+ 75	+125	− 31	6584....	− 73	+243	− 70
5904....	+ 90	+ 86	+ 11	6624....	− 40	+282	+ 25
5927....	+ 16	+159	− 95	6626....	− 19	+181	+ 32
5946*...	+ 29	+359	−208	6637....	− 41	+209	+ 15
5986....	+ 47	+191	− 69	6638....	− 48	+337	+ 60
6093....	+ 65	+188	− 16	6652....	− 65	+305	+ 16
6101....	− 55	+156	−135	6656....	− 12	+ 80	+ 18
6121....	+ 31	+109	− 13	6681....	− 41	+177	+ 12
6144....	+ 63	+235	− 25	6712....	− 32	+274	+145
6171....	+ 60	+148	+ 16	6715....	− 44	+153	+ 22
6205....	+ 71	+ 40	+ 75	6723....	− 39	+121	+ 4
6218....	+ 52	+106	+ 34	6752....	− 39	+ 73	− 30
6229....	+274	+ 82	+328	6760....	− 42	+414	+323
6235....	+112	+487	+ 8	6779....	+ 30	+105	+225
6254....	+ 45	+106	+ 32	6809....	− 41	+ 90	+ 16
6266....	+ 19	+150	− 13	6864....	−206	+376	+152
6273....	+ 25	+157	− 3	6934....	−114	+180	+256
6284....	+ 64	+364	0	6981....	−164	+192	+150
6287....	+ 83	+428	+ 15	7006....	−228	+245	+577
6293....	+ 37	+261	0	7078....	− 69	+ 49	+121
6304....	+ 28	+320	− 11	7089....	− 94	+ 70	+104
6316....	+ 46	+524	0	7099....	−128	+101	+ 56
6333....	+ 43	+244	+ 34	7492....	−255	+ 70	+104

*Probably a globular cluster but not definitely proved.

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[1] *Mt. Wilson Contr.* No. 152, 1917. The point for M 75 (N.G.C. 6864) is erroneously plotted in the earlier figure.

the earlier plot; they are now computed and entered in the third column of Table III. All globular clusters are represented in the new diagram, including the five unproved objects marked with the double dagger in Table II. The open circles designate clusters for which the provisional distances are marked in Table II by a colon.

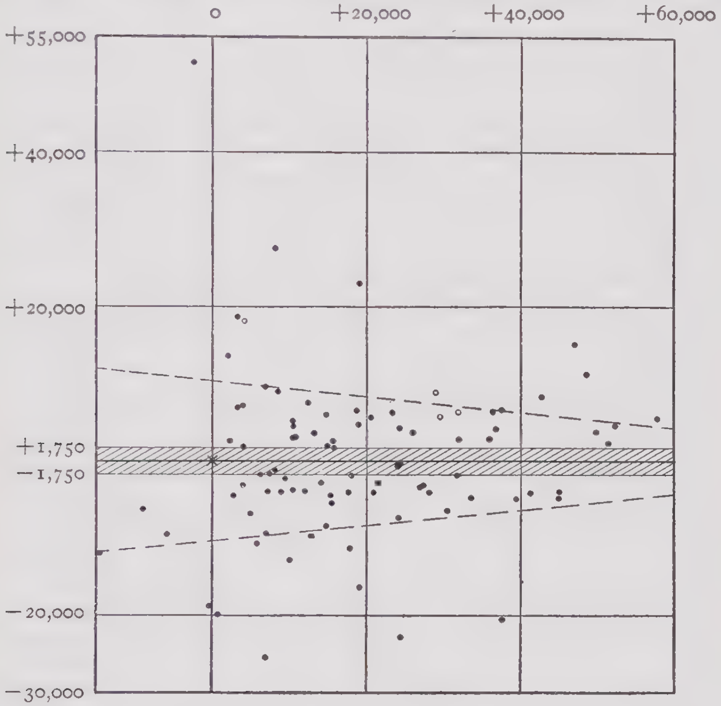

FIG. 3.—Projection of the complete system of globular clusters on a plane perpendicular to the Galaxy and oriented to include the line from the sun to the center of the system. Ordinates are $R \sin \beta$; abscissae are $R \cos \beta \cos (\lambda - 325^\circ)$; unit of distance is one parsec. The sun at the origin of co-ordinates is marked by a cross. See Fig. 4 of *Mt. Wilson Contr.* No. 152.

The parallax of Messier 22 (N.G.C. 6656) has been increased slightly over the value previously adopted. The cluster is fairly open and in a rich field, and it is now found that the stars selected for the study of magnitudes were so near the center that an error of 0.15 mag., due to the Eberhard effect, crept into the earlier results. The values of the galactic co-ordinates for some of the clusters in Table V of the seventh paper were taken from the catalogues of Bailey, who used a position of the North Galactic Pole slightly different from that adopted for this work. The computations in Table III are all based on the co-ordinates in Melotte's catalogue, with the result that the new values of $R \sin \beta$ differ in a few cases from those previously computed. The revised galactic positions, however, never differ by more than a degree in either co-ordinate, except for the longitudes in high galactic latitude.

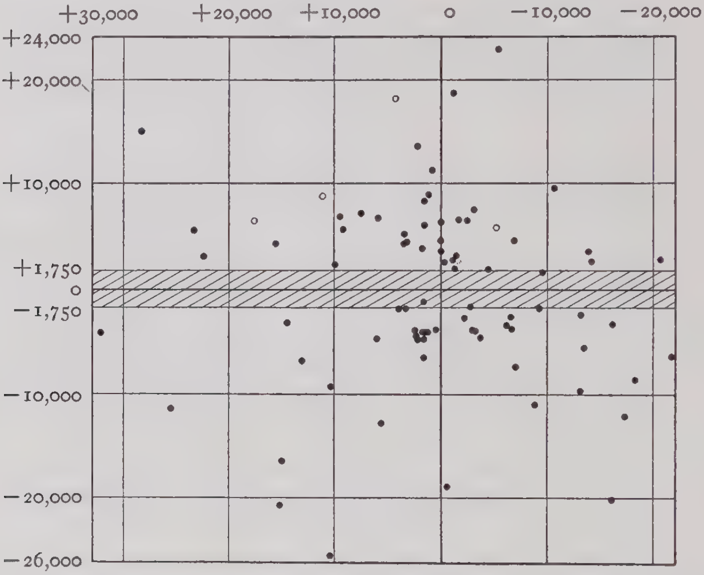

FIG. 4.—Projection of the system of globular clusters on a plane perpendicular to the Galaxy and to the plane of the preceding diagram. Ordinates are $R \sin \beta$; abscissae are $R \cos \beta \sin (\lambda - 325°)$.

Three points formerly not definite are now emphasized by Fig. 3: (1) There is no sensible increase in the minimum distance from the galactic plane with increasing distance from the sun. (2) The additional results have filled in the gaps of the earlier work, and show that the provisional estimate of a distance of 20,000 parsecs to the center of the system is not too great. (3) There is evidence that the globular clusters occupy a space shaped somewhat like a split wedge, the base of which passes nearly through the sun and contains the great circle defined by galactic longitudes 55° and 235°. Only a part of the tapering of the wedge beyond the center of the system is to be attributed to a lack of observations. The inclined broken lines in Fig. 3 suggest the degree of tapering away from the extremely broad and sharply defined base.

Fig. 4 shows the projection of globular clusters on a plane perpendicular to the Galaxy and to the plane of Fig. 3. It represents the appearance of the system of clusters as seen from a great distance in longitude 145°, indicating approximately the apparent distribution as seen from the earth. The co-ordinates for this plot are in the second and fourth columns of Table III. The close approach to symmetry when viewed from this angle is interesting; the numbers of clusters in the four quadrants are 20, 23, 23, and 20. N.G.C. 4147, 6229, and 7006 are outside the limits of the figure.

It is a striking fact that the system of 86 globular clusters listed in Table III is divided into exactly equal numbers by the plane of the Milky Way.

IV. NOTE ON THE ABSENCE OF GLOBULAR CLUSTERS FROM MID-GALACTIC REGIONS

In the seventh and twelfth papers of this series[1] comments have been made upon the absence of globular clusters from the equatorial region of the galactic system. The importance of the phenomenon necessitates a full consideration of its reality and meaning. That the condition is real is attested by such evidence as: (1) the consistent agreement of the results for clusters at various intervals of distance along the galactic plane; (2) the presence of blue stars and open clusters on the galactic equator at distances equal to those of the nearer globular clusters; (3) the absence of appreciable light-scattering in space; and, finally, (4) the apparent dynamical relation of globular to open clusters and their complementary distribution.

Probably the best evidence that dark nebulosity does not obscure globular clusters in the equatorial segment (the nebulous clouds assumed, for instance, to be analogous to the peripheral ring of absorbing matter observed in some spiral nebulae) is afforded by the diagram of Fig. 5. Let us consider first the 31 globular clusters for which the distance projected on the plane of the Milky Way, $R \cos \beta$, does not exceed 15,000 parsecs. Fifteen are north of the galactic plane and sixteen are south. Their frequency in distance from the plane, $R \sin \beta$, is as follows, the unit of distance being 100 parsecs:

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[1] *Mt. Wilson Contr.* No. 152, p. 22, 1917; No. 157, pp. 6, 10, 1918.

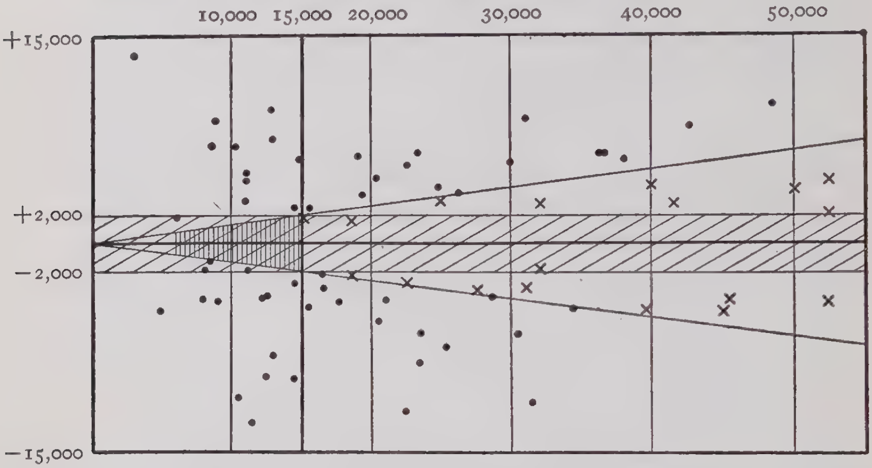

FIG. 5.—Diagram to illustrate that the equatorial segment may not be devoid of globular clusters because of clouds of obstructing matter in the Milky Way. Ordinates are $R \sin \beta$; abscissae are $R \cos \beta$; the unit of distance is one parsec. Twelve clusters fall outside the limits of this diagram.

$R \sin \beta$	$> \pm 200$	± 200 to ± 150	± 150 to ± 100	± 100 to ± 75	± 75 to ± 50	± 50 to ± 25	$< \pm 25$	-12 to $+18$
Number of Clusters	3	2	3	5	5	9	4	0

This tabulation, as well as the part of the diagram to the left of the heavy vertical line, shows definitely the absence from the Milky Way of the nearer clusters.

If we attribute the absence of globular clusters to the obscuration by dark matter in low latitudes, we may indicate this hypothetical obscuring material by the vertical shading across the equatorial segment. If the nearby clusters are missing because of these clouds of obstructing matter, then certainly the faint and more distant clusters behind the clouds should be wanting. That is, the light from every visible cluster with galactic latitude less than ±8° would pass through this region, which, by supposition, is capable of concealing twenty or thirty systems that are relatively near. To maintain the assumption, no clusters should be found within the diverging lines. Observation shows nearly a score. The supposition, therefore, that the mid-galactic regions are not transparent appears untenable.

During the six months that have elapsed since the preceding paragraphs were written some doubt has been thrown on the foregoing conclusion, notwithstanding the amount and character of the evidence in its favor. A discussion of the new arguments and the observational data will be given elsewhere in connection with a treatment of the parallaxes of open clusters. For the present the reality of the avoidance of the galaxy by globular clusters must be considered an open question.

We cannot suppose that the 19 clusters[1] within the diverging lines have been estimated too distant because of partial obstruction of their light, and that they are actually the clusters missing from the nearer equatorial segment. The distances of these clusters are based upon angular diameters, some upon magnitudes as well, and we have found that the parallax-diameter relation holds whether the clusters are near or far from the galactic plane.[2]

V. ON THE PARALLAX-DIAMETER RELATION FOR GLOBULAR CLUSTERS

The relation between the distances, as determined both from variable stars and from mean magnitudes, and the apparent diameters, as measured on the Franklin-Adams charts, is unexpectedly definite, if we consider the various constitutional differences among globular clusters.[3] The relatively small deviations from the parallax-diameter curve[4] appear to mean that abnormal clusters are rare. That the correlation of distance and apparent diameter is

[1] One cluster with $R \sin \beta = +5{,}500$ parsecs falls outside the diagram to the right.

[2] In the sixth section of this paper the additional parallax-diameter curves are definite, notwithstanding the large differences in exposure-time and in the brightness of the stars appearing in the photographs.

[3] For instance, the occasional contrast in concentration of the brighter stars, noted in the second section of this paper.

[4] Fig. 1 of *Mt. Wilson Contr.* No. 152.

actually fundamental, however, and is therefore reliably applicable to the estimation of parallaxes not otherwise obtainable, is further attested by the supplementary data represented in the curves of Fig. 6, which we shall presently describe.

The significance of these fairly uniform results must be that, for the large majority of globular clusters, the linear dispersion of the central nucleus[1] is nearly constant, although we cannot as yet decide definitely whether that condition connotes approximately simultaneous origin of all known extra-galactic cluster systems, or rather an essentially permanent dynamical condition in spheroidal stellar groups. The second alternative is supported by the evidence of numerous open galactic clusters. Such groups appear to maintain dimensions of the same order as those of globular systems, but they show, in the scarcity of their red giants and the preponderance of highly luminous blue stars, indications either of much greater age or of more expeditious development.

Without doubt the most homogeneous photographic survey now available for the whole sky is that initiated by Mr. Franklin-Adams;[2] the uniform length of exposure, the quality of stellar images, and the scale of the photographic charts are all particularly suitable for the study of the composite images of clusters, reported in *Mount Wilson Contributions* No. 152.

We have a second comprehensive photographic survey in the Harvard Map of the Sky, but the copies (positive prints on glass) of the originals show considerable lack of homogeneity in quality and limiting magnitude;[3] in particular, the scale of the plates is so small (aperture 1 inch, focal length 13 inches, exposures 39 to 75 minutes) that high accuracy cannot be expected in determining the apparent diameters of the nearly starlike images of globular clusters.[4] Notwithstanding the difficulties due to faintness, small size, and the resulting confusion with the surrounding fields, the mean results show, in the upper curve of Fig. 6, a definite progression of apparent diameter with adopted parallax. Each point represents the mean of five values. Many of the most distant clusters could not be certainly identified on this scale; and ω Centauri, with diameter 22ʹ.6, parallax 0ʺ.00015 is also not plotted. The deviations for the individual clusters average nearly 25 per cent of the adopted parallaxes, a decidedly lower accuracy than shown by results from the Franklin-Adams charts[1] but still of considerable value as a further justification of the method and as a check of the separate values.

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[1] The estimates of diameter on the Franklin-Adams charts "refer actually to what appears to be a central core of each system. The scale of the photographs does not permit close differentiation of the outlying members of a cluster from the stars of its surrounding field," p. 25, n. 1, *Mt. Wilson Contr.* No. 152.

[2] *Memoirs of the Royal Astronomical Society*, **60**, Part 3, 1915.

[3] Cf. Nort, *Recherches Astronomiques de l'Observatoire d'Utrecht*, VII, 1917.

[4] The Harvard Map is described in *Harvard Circular*, No. 71, 1903.

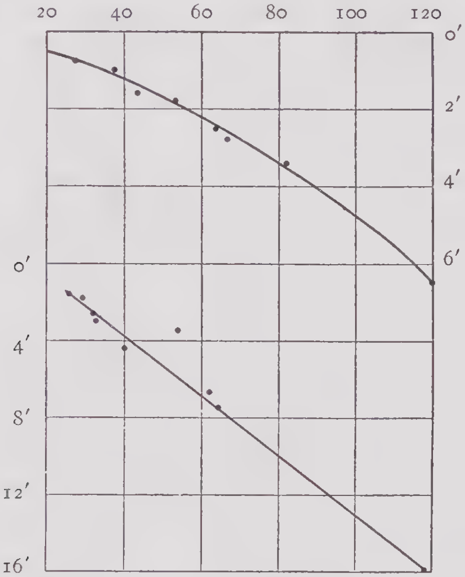

FIG. 6.—Parallax-diameter curves for globular clusters. Above: diameters from plates of Harvard Map of the Sky; below: diameters from Palisa-Wolf charts; ordinates are diameters; abscissae are parallaxes in units of 0ʺ.000001.

The photographs of the Milky Way by Bailey[2] are also serviceable for this work, since many of the globular clusters lie within 10° of the galactic circle. The scale of his photographs is about the same as for the Harvard Map, and a casual examination gives comparable results.

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[1] Cf. Table VII of *Mt. Wilson Contr.* No. 152.

[2] *Harvard Annals*, 72, No. 3, 1913; 84, No. 4, 1916.

The scale of the photographic charts published by Palisa and Wolf is more than double that of the Franklin-Adams charts, but the series is incomplete and the exposure times for the few plates that contain clusters vary from 1^h40^m to 3^h30^m. The results for the nine clusters available are in the lower part of Fig. 6; they suggest the high value of a complete series of long exposures on the larger scale, and they again emphasize the validity of this manner of estimating distance. The discordant point refers to N.G.C. 6626, a compact cluster in declination −25°.

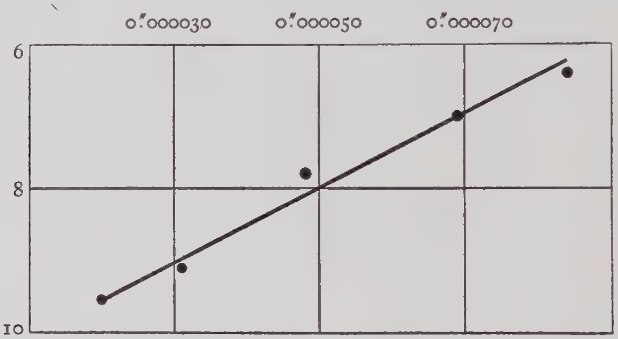

FIG. 7.—Parallax and integrated visual magnitude. Ordinates are Holetschek's magnitudes; abscissae are adopted parallaxes.

Nearly all of the values of diameter used in this section are based upon two independent series of measures by Miss Davis.

VI. TOTAL LIGHT AS A MEASURE OF THE DISTANCES OF CLUSTERS

The mean apparent magnitude of the 25 brightest stars in a globular cluster has been used with satisfactory results as a criterion of distance.[1] The integrated apparent brightness of the same stars, or even of the hundred or so brightest stars in each system, would probably be an equally good measure of the distance. The integrated light of all the thousands of stars in a globular cluster, however, does not greatly exceed that due to its brightest stars alone;

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[1] See the fifth section of the sixth paper of this series, *Mt. Wilson Contr.* No. 151, 1917.

and if it were possible, after overcoming some of the obvious minor difficulties, to measure accurately the total apparent brightness of such systems, possibly a valuable method would result for deriving the relative parallaxes of all typical globular groups.

A study has been made by Holetschek[1] of the integrated visual magnitudes of the brighter clusters and nebulae visible at Vienna. His adopted magnitudes for globular clusters, given in Table IV, are of very unequal merit owing to disparity of observation, varying diffuseness of images, and availability of comparison stars. Magnitudes of the comparison stars were taken in general from the B.D. catalogues, and high accuracy is not claimed for the results. The parallaxes in Table IV are the adopted values from *Mount Wilson Contributions* No. 152; the unit is 0″.000001.

TABLE IV

HOLETSCHEK'S INTEGRATED MAGNITUDES OF GLOBULAR CLUSTERS

N.G.C.	Mag.	Parallax	N.G.C.	Mag.	Parallax	N.G.C.	Mag.	Parallax
1904	8.0	39	6229	8.6	23	6637	9.0	47
4147	9.4	19	6235	9.7	20	6656	6.2	118
4590	8.2	62	6254	6.9	83	6681	9.5	55
5024	7.8	53	6266	7.0	66	6712	8.9	32
5272	6.6	72	6273	6.8	63	6760	10.5	19
5466	8.5	51	6284	9.5	27	6779	8.3	40
5634	9.7	33	6287	9.2	23	6864	8.0	22
5897	10.2	67	6293	8.5	38	6934	9.0	30
5904	6.7	80	6333	7.3	40	6981	9.5	34
6093	7.8	50	6341	6.2	81	7006	9.7	15
6121	6.8	88	6356	8.5	26	7078	6.2	68
6171	9.0	62	6402	7.8	43	7089	6.7	64
6205	5.8	90	6626	7.9	54	7099	8.5	58
6218	6.8	81						

Omitting the magnitudes for clusters south of declination −20° because of uncertainties connected with low altitude and short observing season, we have combined the remaining twenty-three values into normal groups in order of parallax. The plot of the normal points in Fig. 7 shows the expected uniform decrease of brightness with increasing distance. Obviously a systematic comparison with recognized magnitude standards, using a telescope of extremely short focus, or a method that is independent of the angular diameters of the images compared, would contribute important material to the problem of the relative parallaxes of star clusters.

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[1] *Annalen der k.k. Universitäts-Sternwarte in Wien*, **20**, 114, 1907.

VII. THE CONTRASTED MOTIONS AND DISTRIBUTION OF GLOBULAR CLUSTERS AND SPIRAL NEBULAE

The known radial velocities of globular clusters are predominantly negative.[1] These enormous stellar systems, moving under gravitational attraction with an average speed of more than 100 km/sec., are apparently as a class approaching the sun; and, remembering their extra-galactic positions, we also infer that they are approaching and falling into the dense stellar strata of the general galactic system. To be sure, the spectroscopic study of clusters has not gone far as yet, but the above inference does not rest upon observed velocities alone. The composition of the galactic system and the distribution of clusters in space, especially the relation of globular systems to the open groups of mid-galactic regions,[2] yield more important evidence than is afforded by radial velocities that the galactic system absorbs globular clusters.

On the other hand, the brighter spiral nebulae as a class, apparently regardless of the gravitational attraction of the galactic system, are receding from the sun and from the galactic plane—a remarkable condition that has been little emphasized heretofore. From the published spectroscopic work of Adams, Campbell, Moore, Paddock, Pease, Sanford, Wolf, Wright, and particularly of V. M. Slipher at Flagstaff, we obtain the radial velocities in the fifth column of Table V. The Andromeda nebula (N.G.C. 224) and its companion are treated as a single object. The radial velocity of the nucleus of Messier 33, as determined by Pease from absorption lines, is given in preference to the earlier values derived by Pease and Slipher from bright lines.[3]

Limited as this material is, it yields some important results, which emphasize the contrast between clusters and spiral nebulae and bear directly upon the structure and present status of the galactic system. These preliminary results, appearing in the seven numbered divisions below, are the more worthy of mention at the present time because the increase of observational data for spirals will necessarily be slow from now on, due to the extreme faintness of the nebulae not already observed for radial velocity.

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[1] See *Mt. Wilson Contr.* No. 157, 1918, Table I and Section 6.

[2] Sections I and III of this paper.

[3] *Publications of the Astronomical Society of the Pacific*, **28**, 33, 1916.

TABLE V

Positions and Radial Velocities of 25 Spiral Nebulae

N.G.C.	Messier	Galactic		Radial Velocity	Apical Distances, θ				Vis. Mag. (Holetschek)
		Long.	Lat.		$\omega=90^\circ$	$\omega=105^\circ$	$\omega=120^\circ$	$\omega=150^\circ$	
				km					
224	31	89°	−20°	− 316	20°	25°	36°	63°	5.0
598	33	102	−30	− 70	32	30	35	55	7?
1023	....	112	−19	+ 300	29	20	21	42	9.7
1068	77	141	−52	+1120	67	60	55	52	8.7
2683	....	158	+40	+ 400	73	63	53	41	9.2
3031	81	109	+42	− 30	45	42	43	56	8.0
3115	....	216	+38	+ 600	118	106	95	71	9.0
3379	....	218	+59	+ 810	108	102	94	79	9.1
3521	....	225	+54	+ 730	115	107	99	81	9.3
3623	65	209	+64	+ 800	102	96	90	77	8.9
3627	66	211	+64	+ 650	103	97	90	78	8.6
4151	....	118	+76	+ 940	78	76	76	78	10.7
4258	....	103	+68	+ 500	69	68	69	75	8.7
4526	....	262	+71	+ 580	109	107	105	97	10.0
4565	....	215	+88	+1100	91	91	90	89	9.4
4594	....	267	+52	+1180	128	126	121	106	8.7
4649	60	265	+75	+1090	105	104	102	96	8.6
4736	94	76	+86	+ 290	86	86	87	89	7.7
4826	64	295	+84	+ 150	95	96	96	95	8.6
5005	....	64	+78	+ 900	79	81	83	89	9.1
5055	63	69	+74	+ 450	75	77	80	88	9.2
5194	51	68	+71	+ 270	72	75	78	87	8.4
5236	83	283	+31	+ 500	147	148	145	126	9.5
5866	....	59	+52	+ 650	58	65	73	91	10.3
7331	....	62	−22	+ 500	35	47	61	88	9.3

1. Since the greater number of bright spirals are north of the galactic plane and their positions are more favorable for northern observers, only five negative galactic latitudes appear in Table V. To these should be added that of N.G.C. 1700, for which Sanford estimates a large positive velocity, though, in the absence of a definite numerical value, it is not included in the table. Only one of the twenty velocities for spirals north of the plane is negative,

while two of the spirals on the south side are approaching. But the three negative values depend, as we shall see, on low galactic latitude and high apparent brightness rather than on the sign of the latitude, and hence we conclude that essentially without exception, on both sides of the Galaxy, spiral nebulae recede.

2. The speed of spiral nebulae is dependent to some extent upon apparent brightness, indicating a relation of speed to distance or, possibly, to mass. The six spirals with smallest radial velocity, including all of those with negative values, are not exceeded in brightness by any spirals in Holetschek's list of visual magnitudes:[1]

N.G.C.	224	598	4736	3031	5194	4826
Integrated magnitude	5.0	7?	7.7	8.0	8.4	8.6
Radial velocity	−316	−70	+290	−30	+270	+150 km

The arithmetical mean of these six velocities is ±188 km/sec.; their algebraic mean is +49 km/sec., while for the other 19 spirals of Table V it is fifteen times as large—that is, +726 km/sec. Only three spirals besides these six bright ones are now known to have radial velocities of less than +500 km/sec.

3. Forming means of five in order of galactic latitude, we derive from Table V the first two lines of the following tabulation, which indicate that speed may be related to angular distance from the galactic equator:

Mean galactic latitude, β	24°	45°	59°	72°	82°
Mean radial velocity, V_r	+183	+548	+834	+578	+676km
$V_p = V_r$ cosec β	+450	+775	+975	+610	+680km

If we should assume that the motion is wholly perpendicular to the galactic plane, the mean velocities would be as given in the last tabulated line above.

4. The correlation of velocity and the latitude co-ordinate, although not very definite, may be of some significance for theories of the spiral nebulae; but, guided by the provisional hypothesis described in Section VIII of this paper, we find some evidence of a more striking relation between the velocities of spirals and a new position co-ordinate. Let λ and β denote the galactic

[1] *Annalen der k.k. Universitäts-Sternwarte in Wien*, **20**, 114, 1917. N.G.C. 4826 is equaled in brightness by two other spirals, according to Holetschek's estimates.

co-ordinates of any spiral, and ω the longitude of an origin on the galactic circle. Then the angular distance, θ, of the spiral from this origin is given by

$$\cos\theta = \cos\beta \cos(\lambda - \omega) \tag{1}$$

In the last four columns of Table V we give for each spiral the values of θ for $\omega = 90°$, 105°, 120°, 150°. The angle θ may lie between 0° and 180°, and at the galactic poles is 90°; in the table, however, no value of θ is less than 20° because of the avoidance of the Milky Way by spiral nebulae, and few values exceed 120° because the most southern nebulae have not been observed for velocity.

TABLE VI

THE PROGRESSION OF THE MEAN VELOCITY OF SPIRAL NEBULAE WITH DISTANCE FROM THE GALACTIC APEX

INTERVAL OF θ	$\omega=90°$			$\omega=105°$			$\omega=120°$		
	Mean θ	Mean V_r	Number	Mean θ	Mean V_r	Number	Mean θ	Mean V_r	Number
$\leqq 50°$ . .	32°	+ 77	5	33°	+ 77	5	34°	− 29	4
51°– 75 . .	69	+565	6	66	+588	5	62	+634	5
76 –100 . .	86	+676	5	88	+660	8	88	+641	12
101 –125 . .	109	+751	7	105	+762	5	109	+950	3
>125 . .	138	+840	2	137	+840	2	145	+500	1

For the first three values of ω the progression of the observed radial velocity of spirals, V_r, with increasing θ is shown in Table VI for equal intervals of θ. The interval for θ less than 50° contains in all cases the three bright nebulae with negative radial velocities, and the mean V_r is correspondingly affected. The correlation, if real, is about equally definite for $\omega = 90°$ and $\omega = 105°$. Its meaning would be that, regardless of galactic latitude, the average radial velocity of spiral nebulae increases with the angular distance from a point in the northern Milky Way—a point which, in anticipation of an explanation proposed in Section VIII, we may call the galactic apex.

In these progressions we have a suggestion that average radial velocity may be roughly predicted from position; but before this relation can be definitely established from spectroscopic results alone, we must have more data, for the wide range of peculiar velocity in Table V is inadequately reflected by the means of Table VI. Other groupings could alter or even conceal the uniform progression of mean velocities.

If we omit the two brightest nebulae for the sake of greater homogeneity (see p. 20) and combine the others into four groups in order of increasing θ, we obtain for $\omega=105°$:

Mean θ	46°	74°	95°	116°	
Mean V_r	+458	+618	+633	+780 km	(2)
Number of spirals	5	6	6	6	

5. On the basis of this more homogeneous material we may make the following computation, which is perhaps to be considered more as an interesting illustration than as an approach to definite cosmic fact. Let V_s be the average systematic recessional velocity of spiral nebulae in the line of sight, and let V_g be a quantity which we may call the velocity of the galactic system toward $\lambda=105°$, $\beta=0°$. Then

$$V_s-V_g\cos\theta=V_r \tag{3}$$

and from the values of θ and V_r in (2) we obtain, provisionally,

$$\left.\begin{aligned}V_s&=+650 \text{ km/sec.}\\ V_g&=+300 \text{ km/sec.}^{[1]}\end{aligned}\right. \tag{4}$$

Employing these provisional values of V_s and V_g, and setting up conditional equations of the form (3) for each of the 23 spirals in the tabulation (2), we derive the following normal equations:

$$23\Delta V_s-1.96\Delta V_g-120=0$$
$$-1.96\Delta V_s+4.34\Delta V_g+\ 80=0$$

The solution gives for corrections to (4), $\Delta V_s=+8$ km/sec., $\Delta V_g=+17$ km/sec. The adopted values with their probable errors are

$$V_s=+660\pm\ 45 \text{ km/sec.}$$
$$V_g=+320\pm100 \text{ km/sec.}$$

The average difference between an observed velocity and that computed by putting the foregoing values in formula (3) is ±240 km/sec., a quantity that is to be taken as representing, not observational errors, but rather the peculiar velocities of spirals.

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[1] By assuming $V_g=+300$ km/sec., the mean value of V_s derived from *all* the material of Table VI is +635 km/sec. for $\omega=105°$, in close agreement with the result above.

From the above result we infer that one way of accounting for the observed increase of average V_r with distance from the so-called galactic apex is by assuming that the galactic system moves toward longitude 105°, approximately, with a velocity of some three hundred kilometers a second, while the spirals of the brightness here involved recede with an average systematic radial velocity of six or seven hundred kilometers a second.[1]

6. If we should assume that the average systematic motion is perpendicular to the galactic plane, rather than radial from the center of the galactic system, or radial from the sun as is observed and assumed above, then we would have for this perpendicular velocity,

$$V_p = (V_r - V_g \cos\theta) \operatorname{cosec}\beta$$

and a solution of the four mean observational equations derivable from (2) gives, provisionally,

$$V_p = +775 \text{ km/sec.,} \quad V_g = +140 \text{ km/sec.}$$

The least-squares solution of the 23 conditional equations then leads to the following results:

$$V_p = +765 \pm 55 \text{ km/sec.}$$
$$V_g = +110 \pm 100 \text{ km/sec.}$$

and the average value of O − C is ± 250 km/sec. The probable errors give little choice between assumptions involving radial and perpendicular systematic motion, but the conception of radial motion is distinctly preferable from the standpoint of physical probability.

When sufficient data become available, not only should V_g and V_s (or V_p) enter the computations, but also the co-ordinates of the origin of θ.

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[1] Following the procedure commonly used to determine the apex of the solar motion, Truman, Young and Harper, and Slipher have computed from the radial velocities of spirals a motion of the galactic system toward a rather vaguely defined southern apex; but they have assumed, explicitly, a random peculiar motion for spiral nebulae and, implicitly, the "island universe" hypothesis. Their result is an obvious consequence of the preferential recession and of the absence of spirals of known velocity from the southern sky.

7. From the present evidence we see that there is a magnitude effect for the very brightest objects; in addition the observed velocities of spirals may be divided into three parts, the first two of which appear to be very definite: (*a*) a systematic radial recession of more than 600 km/sec., which is increased by one-sixth if the systematic motion is assumed to be perpendicular to the galactic plane; (*b*) the peculiar velocities, which average about ±250 km and are represented by the deviations from the means and formulae; (*c*) a component of the radial velocity whose effect decreases on the average with distance from the "galactic apex," and which may be interpreted as a drift of the whole galactic system with respect to the brighter spiral nebulae. This last component is the least definite, quantitatively, but on the other hand we shall see that its existence seems to be affirmed by the distribution of spirals, and by other considerations discussed in the last part of Section VIII.

The apparent distribution of spiral nebulae appears to be sufficiently known for a general statement, though much remains to be done in extending the recent work of Hardcastle, Fath, Sanford, and Curtis. That the spirals approach nearest to the Milky Way in two hours of right ascension ($\omega = 100°$) has been noted by Hinks, Stratonoff, and others. Globular clusters are wholly absent from that region, and it is very significant that in the opposite region of the sky, where globular clusters and clouds of stars most abound, the avoidance of the Milky Way by spirals reaches its maximum.[1]

Thus the region avoided, at least by the brighter spirals, is roughly defined by a wedge, symmetrical in relation to the galactic plane, with its base in the general direction now adopted as the center of the galactic system. A wedge-shaped arrangement (analogous to the wedge-shaped avoidance by spirals) has been pointed out for globular clusters in Section III of this paper, but the base of the wedge lies in a direction approximately opposite to the direction of the center.

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[1] See the diagrams by Hardcastle and Hinks (*Monthly Notices*, **74**, 699, 1914) and the bibliography and discussion by Sanford (*Lick Observatory Bulletins*, **9**, 82, 1917).

There is, accordingly, in the contrasted distribution of spirals and globular clusters, as well as in the opposing directions of their motions, an indication that the compelling forces act in opposite directions.

VIII. DATA AND INFERENCES FOR A PROVISIONAL COSMOGONY

The observational results discussed in the foregoing sections of the present contribution and in the thirteen preceding papers of this series permit the statement of five of the conditions that must be considered in attempting to account for the origin of the galactic system and its present relation to clusters and spiral nebulae: (1) the similarity of globular clusters in dimensions and content; (2) the complementary distribution of open and globular clusters; (3) the existence of thousands of suborganizations in the galactic system; (4) the contrasted distribution of spiral nebulae and globular clusters; and (5) the opposed directions of preferential radial motion for spirals and clusters.

In the following paragraphs we offer a brief summary of the observational data bearing on each of these five conditions, and a suggestion as to the general interpretation of the evidence.

1. The parallax-diameter curves in *Mount Wilson Contributions* No. 152 and in Section V of this paper indicate that with few exceptions globular clusters have approximately the same linear diameters. Observations of (*a*) the integrated light of clusters, (*b*) their general luminosity-curves, and (*c*) the phenomena of color and absolute magnitude of their giant stars show also that the stellar content is much the same from system to system.

2. For globular clusters the frequency-curve of galactic latitudes has a distinct minimum at the galactic plane, whereas the corresponding curve for open clusters shows a pronounced maximum in low latitudes (Figs. 1 and 2 of this paper). The evidence suggests that the two kinds of clusters occupy regions of space that are mutually exclusive (Sections I and III of this paper). There is some indication that a transition from one kind to the other occurs along the outskirts of the equatorial galactic segment.

3. Numerous conditions suggest that the Galaxy is a heterogeneous assemblage of unequally organized parts. The remarkably

wide pairs of stars of common motion, the local moving groups, the open clusters, the clouds of the Milky Way, the star-streams[1]—all these combinations indicate that the galactic system may be largely composed of disintegrating clusters. The testimony of the distribution and radial motion of globular clusters indirectly supports this view, and suggests further that the system is now growing and has gradually grown throughout the past from a much less complicated state.

4. While both globular clusters and spiral nebulae appear to be mainly if not wholly outside the equatorial galactic segment, they occur in general in different parts of the sky. In the Southern Hemisphere the globular clusters crowd in close to the Milky Way and the bright spirals widely avoid it; in the Northern Hemisphere the spirals approach their nearest to the galactic circle and globular clusters are almost wholly absent.[2]

5. Globular clusters as a class appear to be rapidly approaching the galactic system; spiral nebulae as a class are receding with high velocities. The relation of the velocities of spirals to brightness and to position in the sky has been dealt with above in Section VII. Some of the reasons for not considering spiral nebulae to be separate galactic systems have been outlined in the introduction to the twelfth paper of this series. For the present we shall accept that the distances of globular clusters and spirals are of the same order, and that, with the possible exception of a few of the very brightest, none is within mid-galactic stellar regions.

The foregoing conditions, when considered in connection with previously accepted stellar and nebular results, suggest as a preliminary hypothesis that the discoidal galactic system originated from the combination of spheroidal star clusters and has long been growing into its present enormous size at their expense. The evidence further suggests that the galactic system now moves as a whole through space, driving the spiral nebulae before it and absorbing and disintegrating isolated stellar groups. Apparently the suggested interpretation requires that two types of sidereal organization prevail generally throughout extra-galactic space: spiral nebulae, and stars of known types assembled for the most part into globular clusters;[1] and while the globular clusters now known are, at least potentially, members of the galactic system, the spirals are not members, rather being general inhabitants of extra-galactic space. The hypothesis demands that gravitation be the ruling power of stars and star clusters,[2] and that a repulsive force, radiation pressure or an equivalent, predominate in the resultant behavior of spiral nebulae.

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[1] Accepting the evidence presented in *Mt. Wilson Contr.* No. 157 (see also the ninth section of this contribution) bearing on the existence of a local cluster, we note that from material now available the circum-solar cluster appears to be both larger and more oblate than typical open or globular systems; it seems to be more like the Magellanic Clouds in dimensions. Possibly before it became a member of the galactic system it represented the discoidal combination of several smaller clusters, the residual nuclei of which are still shown in the Orion, Perseus, and Scorpio-Centaurus groups of B stars.

[2] See the last three paragraphs of Section VII of this paper.

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[1] The apparent limitation of the size and mass of a globular cluster (the first of the five conditions) suggests, for example, the narrow range in masses of stars, for which the limiting factor, according to Eddington's theory, is the balance of attractive and dispersive forces. Judging by the galactic system, and perhaps by the local cluster and the Magellanic Clouds, the discoidal form permits a greater mass; and if, as seems likely, the Magellanic Clouds recently passed through the galactic system (cf. p. 12, n. 1, of the twelfth paper) a stellar discoid possibly shows much greater stability than is possessed by the compact spheroidal distribution.

[2] Jeans has considered mathematically the effect of the encounter of two clusters, showing the transformation from the globular to the oblate form (*Monthly Notices*, **76**, 552, 1916). His analysis may direct the way to an understanding of the beginning of a flattened stellar system of growing mass.

If such a growth be theoretically possible, we may suppose that at first the combination of separate clusters would proceed very slowly to the formation of composite systems of increasing mass, the galactic system that finally results representing an advanced stage in the survival of the most massive and stable. Once an enormous mass had been acquired, subsequent accretions would be numerous and almost inevitable if ordinary unit spheroidal clusters were encountered rather than growing discoids of multiple mass. The distinctly limited galactic star clouds, some of which are not mid-galactic, and the rapidly receding Magellanic Clouds as well, might represent partially assimilated and controlled systems of greater mass. Undoubtedly some globular clusters would merely describe orbits around the growing Galaxy and outside the limits of the system would cross the galactic plane, but our observations do not show them; it may be the true "orbital" clusters are constrained to keep far from the galactic system, all near approaches ending in disorganization. (Compare the tentative hypothesis sketched in the seventh section of *Mt. Wilson Contr.* No. 147, 1918.)

Relative to spiral nebulae, two additional conditions that seem not to have been considered heretofore may be pointed out as significant in connection with the proposed hypothesis.

a) In the midst of a field of stars the effect of repulsive forces would be largely nullified by the symmetrical distribution of the sources of repulsion (assumed to be the stars); above or below a discoidal stellar field the action is of course wholly one-sided.

b) The extremely high velocities of recession indicate that, if the galactic system had remained stationary, most of the brighter spirals would have been among the stars in low galactic latitudes within recent cosmic times, for instance within the last twenty million years. That scarcely any spiral (on either side of the galactic plane) is approaching or is now among the stars in low galactic latitudes, not only suggests that a repulsive force is predominant, but also indicates either (1) a movement of the entire Galaxy through space or (2) recently accelerated motions of spirals, or (3) distances enormously greater than now seem at all probable.

The last supposition, though extending the time alloted, would not remove the difficulty of accounting for the absence of spirals from the equatorial segment at the present time. For example, four bright spirals in high galactic latitude are receding with a velocity in excess of 1/300 that of light. Suppose they are as much as two million light-years away—ten times the distance of the remotest globular cluster known. Then, with constant velocities throughout the past, they would have been at the galactic plane less than 600 million years ago—an interval of time so short in the life-history of a stellar system that in the case of the sun, for instance, it has not sufficed to show an appreciable change in radiation.[1]

It is more in keeping, however, with observations of apparent rotation, and with the luminosity of novae in spirals, to divide by one hundred the distance supposed above. Only a few million years are therefore involved in the problem, and it seems all the more necessary to accept, in view of the observed distribution and radial motion, either (1) that the Galaxy is moving or (2) that the velocities have been rapidly developed. The suggestion (2) that the position of the Galaxy has remained stationary while the velocities of spirals have but recently become large, does not commend itself for various reasons, including the consequent necessity of seeking the origin of spirals in the galactic regions where now they are not observed.

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[1] *Publications of the Astronomical Society of the Pacific*, **30**, 283, 1918.

The remarkable progression of average velocity with distance from the "galactic apex," discussed in Section VII of this paper, is also simply explained by assuming that the galactic system moves in the direction and with the velocity required by these phenomena of distribution and motion. We conclude, therefore, that the most plausible interpretation of the present arrangement and motions of the brighter spirals implies repulsion by the moving discoidal galactic system;[1] but we keep in mind the alternative hypotheses.

IX. NOTE RELATIVE TO THE MORE DISTANT B STARS OF THE LOCAL SYSTEM

The stars of spectral type B, because of their relatively small dispersion in absolute magnitude and their tendency to clustering and to community motion, are particularly valuable in studying the form and position of the local star cluster whose existence and general properties have been discussed provisionally in the last part of the twelfth paper of this series. Although the overwhelming majority of brighter B stars treated in *Mount Wilson Contributions* No. 157 belong to the cluster, a few appear from the evidence of space positions to be field stars, in corroboration of Kapteyn's result that occasionally B stars are found in the Second Stream.[1]

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[1] Is it possible that the spirals represent the failure to form stars from the original condensing nebulosity through the presence of too much material? According to Eddington's theory of the structure of a giant star (*Monthly Notices*, **77**, 16, 596, 1917), the pressure of radiation nearly counterbalances gravitation if the condensing gaseous mass exceeds some 10^{35} grams. A mass 100 times that of the sun would incompletely condense, possibly with the result of a diffuse pseudo-stellar nucleus, grading off into the extensive, low-density envelopes of gas that are unable to fall into the center because of the balance of radiation. Such a body would not readily disintegrate (since the ratio of radiation pressure to gravitation cannot exceed unity), but would present to external repulsive forces a surface relatively large for the mass involved, and composed, it may be, of particles of molecular dimensions peculiarly susceptible to the pressure of such radiation as is emitted by the stars. The existence of powerful electrical fields in spiral nebulae would clarify the problem, and already Slipher has suggested (*Lowell Observatory Bulletin*, No. 80, 1917) that such may occur in Messier 77 and Messier 1. Spirals of greatest mass or density would be least repelled by the galactic system, and if their individual velocities directed them toward a radiant source they might not be easily turned aside. In his study of the rôle of rotation in cosmogony Jeans has shown the probable development of spiral arms when the condensing body greatly exceeds the sun in mass (*Monthly Notices*, **77**, 186, 1917; *Scientia*, **24**, 270, 1918).

It has frequently been assumed, mainly on the basis of an extrapolation, that all B-type stars are included in the earlier catalogues of spectra—that fainter than the seventh or eighth visual magnitude no stars of that class of spectrum would be found. Such a condition would place a sharp limit to the extent of the local cluster, assuming it to be outlined by stars of type B. Extremely faint and distant blue stars, however, have been found in the galactic clouds far beyond the limits of the local cluster, and the data given below show that Class B is also represented in considerable numbers among stars from the seventh to the tenth apparent magnitude.

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[1] The complete identification of the local cluster with Stream I involves some difficulties that were not sufficiently appreciated when the earlier paper was written. The two most important obstacles in the way of the suggested interpretation, which made star-streaming simply the result of uniform motion of the local cluster through the galactic field, appear to be the high relative velocity of the two streams and the probably different stream velocities for different spectral types. There seems to be no doubt of the existence of a definitely organized local cluster, and it certainly contains practically all of the brighter B stars. Kapteyn's studies also leave little doubt that the B stars as a whole, and therefore this cluster, move toward an apex approximately the same as the apex of the first stream as determined from stars of other types. The moving cluster must give rise to a true stream-motion. It happens that the position of the sun with respect to the local cluster's center (the direction of which is shown by F-G-K stars as well as by the B's) is such that we may have, in addition to and hardly distinguishable in direction from the true stream-motion, a pseudo-stream-motion due to internal circulation (according to Strömberg's suggestion, for instance).

These two possible sources of the observed preferential drift of stars were recognized from the first, but in the provisional discussion the internal motion was considered relatively of minor importance. Now it appears that the two difficulties mentioned above, and other minor ones, may be avoided if necessary by inverting the relative importance of the two sources of stream-motion. The cluster accordingly contains not only all the stars of Stream I but a considerable part of those of Stream II, and the cluster's direction and speed of motion are best measured by the mean drift of the early B stars. The average internal velocity may of course vary with spectral type without disrupting the cluster. Professor Eddington suggests in a letter that the field should probably be identified with Halm's O-stream, which contains a great many second-type stars formerly assigned to the first and second streams.

The modified view sacrifices little of the simplicity of the former statement and at the same time uses in a rational manner the large local cluster, whose existence must play an important part in the problem of star-streaming. Except in this matter of motion and content, none of the earlier conclusions referring to the local cluster appears to require appreciable modification.

The first volume of the *Henry Draper Catalogue*,[1] which covers the first four hours of right ascension, contains nearly 26,000 stars, of which 832 belong to Class B. It contains 600 B stars fainter than the seventh magnitude, corresponding to about 3 per cent of the total for all types. For the naked-eye stars over the whole sky the percentage is of course much larger, and, because of the galactic concentration of B stars, it will likely be larger also for the faint stars in those instalments of the new catalogue that include larger portions of the Milky Way. Practically all the B stars of this first volume are in Cassiopeia and Perseus.

Stars of the subdivisions B8 and B9 are generally discussed in connection with A-type stars. In the present note we shall only call attention to the following tabulation of the total number of such stars brighter than successive half-magnitude limits:

Visual magnitude	4.5	5.0	5.5	6.0	6.5	7.0	7.5	8.0	8.5	9.0	9.5
Total No. stars...	7	17	32	56	95	156	235	317	430	537:	591::
Ratio............		4.6	3.3	3.0	2.8	2.5	2.0	1.8	1.7:	1.4:	

The ratio N_{m+1}/N_m in the last line shows the decrease of density with distance.

The numbers of stars in the first five divisions of Class B are shown for different intervals of visual magnitude in Table VII. The limit to which the catalogue is complete is not specifically stated, but is probably in the neighborhood of visual magnitude 8.5. The 38 B stars with undetermined subdivision are more likely to belong in this table than with the stars of types B8 and B9.

Evidence discussed below indicates that the mean absolute magnitudes may be accepted as the same for all these faint stars as for the brighter B's, whose motions and parallaxes have led to the evaluation of their average intrinsic luminosities. Table VII indicates, therefore, that B stars are found continuously to a distance of possibly 1000 parsecs in the direction of Perseus and Cassiopeia.

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[1] *Harvard Annals*, **91**, 1918.

TABLE VII

NUMBERS OF B-TYPE STARS IN *Harvard Annals*, 91

Spectral Type	Visual Magnitude							Total
	<7.0	7.0–7.5	7.5–8.0	8.0–8.5	8.5–9.0	9.0–9.5	>9.5	
B0	6	3	2	2	3	2	0	18
B1	4	2	1	0	1	0	0	8
B2	7	1	5	5	7	0	0	25
B3	26	11	7	5	7	1	2	59
B5	35	9	5	11	4	2	3	69
B0–B5	78	26	20	23	22	5	5	179
Undefined B	0	0	4	12	8	10	4	

Table VIII affords evidence that the 101 stars fainter than the seventh magnitude and of spectral types B0–B5 are possibly in large part members of the local cluster. The galactic latitudes, derived graphically from charts prepared by Kapteyn, are tabulated in order of decreasing brightness, the horizontal lines in each column marking the magnitude intervals of Table VII. All stars in the table are north of the celestial equator except the eight with galactic latitudes in excess of 41°.

With respect to the galactic plane, the descending node of the central plane of the local cluster is in galactic longitude 70°, approximately, as is clearly shown by Fig. 4 of *Mount Wilson Contributions* No. 157. The galactic longitudes of the stars involved in Table VIII are almost exclusively between 85° and 130°, and, therefore, if they belong to the cluster rather than to the general galactic field, their latitudes should be predominantly negative. This is seen to be the case, and in fact we are led to believe that most stars of types B0, B1, B2, brighter than the tenth magnitude, may be members of the local system.

The area of the sky between right ascension 0^h and 4^h is mainly in the southern galactic hemisphere. Hence, for a fair comparison of the number of stars in positive and negative galactic latitudes, we should consider only a narrow belt along the galactic circle—the region within $\beta \leqq 5°$, for instance. We have then the following indication that the distribution of the early B-type stars of Table VIII may have little or nothing to do with the galactic

TABLE VIII

Galactic Latitudes of B-Type Stars Fainter than Magnitude 7

B0	B1	B2	B3		B5	
+8°	−4°	+2°	−3°	0°	−8°	−6°
+8	−3	−3	−4	+4	+2	+7
+1	−4	−12	+10	−2	−5	−4
+6	−4	+1	−21	−2	−5	−4
−2		−5	−50	−2	−12	−6
−4		−6	−3	−3	−23	+2
−4		−2	−2	−5	−82	+4
−2		−3	−2	−42	−3	+5
−4		−12	−1	−5	+1	−5
−3		−5	−9	−2	−12	−64
−4		−5	−15	−4	−16	−6
−2		−4	+2	−5	+6	−4
		−3	+2	−4	−4	−63
		−19	+6	−4	−5	−5
		−4	−3	−6	−46	−54
		−4	−54		−7	−6
		−5	−16		+1	
		−5	+2		+2	

plane—rather, these stars appear to be condensed to a circle that in this region of the sky is three or four degrees south of the galactic circle:

Spectrum		B0	B1	B2	B3	B5	B0–B5
Number of stars	β negative	8	4	12	18	10	52
	β positive	1	0	2	4	7	14

The maximum frequency of the galactic latitudes for the 101 faint stars of types B0–B5 is about −4° according to the following tabulation, which is based on Table VIII:

β	$\geqq +10°$	$+9°, +8°$	$+7°, +6°$	$+5°, +4°$	$+3°, +2°$	$+1, 0°$	$-1°, -2°$
No. of stars	1	2	4	3	7	5	11
β	$-3°, -4°$	$-5°, -6°$	$-7°, -8°$	$-9°, -10°$	$-11°, -12°$	$-13°, -14°$	$< -14°$
No. of stars	28	19	2	1	4	0	14

It is easy to show that the dip of the equatorial circle of the cluster, due to the sun's position to the north of its central plane, is less than −1° for stars at the mean distance of those concerned above.

Allowing for the dip, and assuming that in the mean these B-type stars are about 25° from the descending node, we may make a rough determination of the angle between the central plane of the local cluster and that of the Galaxy. The result is 8°, in fair agreement with the earlier value of 12°, but naturally of much lower weight. Accordingly, we may conclude, as previously assumed, that the existence and inclination of the local cluster does not depend upon the accidental positions of a few groups of brighter B stars.

The most significant feature of the preceding tabulation, however, is the high concentration to the central plane. More than half of all these stars fall into the interval of latitude $-2°$ to $-6°$, and three-fourths of the B0, B1, B2 stars are within those limits. For B-type stars brighter than those considered here the dispersion is decidedly greater, both in this part of the sky and in general.

Three interesting conclusions may be drawn from the above result: (1) The failure to find the fainter B stars heretofore may be due to the extremely narrow belts within which they are to be found—one belt, moreover, apparently standing well out of the lowest galactic latitudes for regions of small declination. (2) The local cluster is exceedingly flat, at least as far as the B-type stars are concerned. It may be more than five times as extended in its plane as at right angles. Table VIII shows how infrequent are the faint B stars in high galactic latitudes. (3) The greatly increased concentration for the fainter stars may be taken as proof that these objects, rather than peculiar B stars of abnormally low intrinsic brightness, are normal stars at a greater distance from the sun.

The completion of the *Henry Draper Catalogue* will afford a good basis for testing and extending the results outlined above.[1] With data for the whole sky we shall be able to define more accurately the position of the local cluster in the galactic system and perhaps determine its form completely.

MOUNT WILSON SOLAR OBSERVATORY
November 1918

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[1] *Note added to proof:* Data derived from the second instalment of the *Henry Draper Catalogue* fully verifies the existence of a secondary galaxy as outlined by early B-type stars. The faintest B's, however, show a decided preference for the primary galaxy, in contrast to the stars discussed above. Possibly a branching of the main Milky Way stream in Perseus and Cassiopeia, or the wide extent of the open clusters in Perseus, is largely responsible for the great preponderance of negative latitudes in the foregoing discussion; and accordingly these faint B's may be only in part members of the local cluster.

Contributions from the Mount Wilson Solar Observatory, No. 162
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DISCUSSION OF SOME EVIDENCE ON THE ORIGIN OF RADIATION IN THE TUBE-RESISTANCE FURNACE

By ARTHUR S. KING

In two recent papers[1] which give his very interesting observations with the tube-resistance furnace, Dr. Hemsalech has reached some conclusions based on evidence which is not in agreement with experimental results obtained by the writer. In the interest of a proper correlation of all available data, so that we may the sooner see what conclusions may be safely drawn, a brief account of these points of difference will be given.

Dr. Hemsalech, in observing the iron spectrum produced in a carbon-resistance tube at atmospheric pressure, did not find certain lines, notably the group near λ 4900 which is strong in the arc, until his furnace reached a temperature of about 2500° C. At this temperature he considers that the strong ionization known to take place within the tube, together with the potential difference of a few volts at its extremities, give a conduction of electricity through the vapor equivalent to a low-tension arc, and that lines of this type require such an electrical condition for their production in the furnace.

The effect of the ionized state within the tube on the radiation of the inclosed vapor is a matter which needs full investigation; but sufficient data on the electrical side are not yet available to justify the conclusion that the inclosed vapor carries a considerable part of the current, as compared with the highly conducting graphite tube, under the conditions which I have found to give spectra favorable for examination. In a former contribution[2] I reported some observations in which I repeated the experiments of Harker and Kaye[3] on the ionization current between exploring electrodes and then measured the resistance between insulated electrodes placed within the tube as compared with that of the tube itself, titanium vapor at 2600° C. being present. The results showed the resistance of the vapor to be relatively high.

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[1] *Philosophical Magazine* (6), **36**, 209, 281, 1918.

[2] *Cantributions from the Mount Wilson Observatory*, No. 73; *Astrophysical Journal*, **38**, 315, 1913.

[3] *Proceedings of the Royal Society*, **86A**, 379, 1912.

Aside from the question as to how nearly the furnace at high temperature approaches a low-tension arc, the high-temperature iron lines referred to by Dr. Hemsalech have been obtained by me at temperatures much below those at which he considers that the electrical conductivity of the vapor becomes important. In my paper[1] on the furnace spectrum of iron all of the lines of the λ 4900 group are recorded as appearing with fair intensity at temperatures between 2100° and 2300°, while the two strongest were perceptible at a temperature between 1800° and 1900°. Since the publication of that paper I have occasionally photographed the iron spectrum under the more favorable conditions of improved furnace construction and the use of the 15-ft. concave grating spectrograph, the first order of which gives very bright spectra with a dispersion of 1 mm = 3.7 A. The group at λ 4900 proved more sensitive than the former results indicated, the two strongest lines, λ 4921 and λ 4958, being visible at 1650° with an exposure of 110 min., while the entire group, including the weaker components of the doublets, was well developed at 2000° with an exposure of 15 min. In fact the later experiments have shown that the classification which I have adopted does not require temperatures which Dr. Hemsalech regards as giving a low-tension arc. Iron lines of Classes I, II, and III may be classified according to their behavior at 1700° and 2000°, while 2350°, in conjunction with the arc intensities, serves to place the lines of Classes IV and V. This latter temperature was used as the high-temperature stage in a study of the ultra-violet portion of the iron spectrum now being completed. Higher temperatures increase the width of reversals and may bring out a very few extra lines, but they soon give difficulty on account of the stronger continuous ground, the emission of which finally balances the emission of most of the metallic lines, while the more reversible ones appear in absorption.

As the furnace of Dr. Hemsalech was operated at atmospheric pressure, which might affect the vaporization point of iron, I have made a series of tests with air present, but the differences between Dr. Hemsalech's observations and mine are not to be ascribed to this feature. I obtained all of the lines of the λ 4900 group at atmospheric pressure, the temperature not rising above 1850°. An interesting effect, however, was a much increased chemical action with air present. Iron, when fused in the furnace, forms a carbide in vacuum, which sticks to the tube but does not penetrate deeply. With air present this action was much stronger. The iron ate into the tube and, when the latter burned thin through long use, dropped through the wall in several places. The tube was used in air without protection other than being in the steel chamber, the window-holder of which was removed. When operated first in vacuum and then with air present the change to atmospheric pressure caused a temperature increase of from 50° to 100° with the same impressed voltage, though the thinning of the tube gave a lower average current. The iron spectrum was very bright, but the most notable difference relative to the vacuum spectrum was a high intensity of the more reversible lines. The cyanogen (nitrogen?) bands were strong at 1850°.

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[1] *Mt. Wilson Contr.* No. 66; *Astrophysical Journal*, **37**, 239, 1913.

Obviously it must be assumed that there is no large discrepancy between the temperature measurements of Dr. Hemsalech and the writer. The Wanner type of pyrometer was used in both cases, and with ordinary care this instrument, when properly adjusted with the standard lamp, and the current through the pyrometer lamp kept constant by means of a milammeter, gives very consistent readings. The extreme upper and lower temperatures measured by us were nearly the same, my photographs showing, like his, that a few iron lines are emitted at as low a temperature as 1500° C., the formation of the carbide evidently lowering the melting-point of iron.

Dr. Hemsalech in his latest paper[1] describes an experiment in which he avoids exposing the vapor to a potential difference by constructing a chamber resting on a graphite plate, the latter carrying the heating current. Judging from the drawing, the maximum temperature given by the hot plate was applied along only about one-seventh of the periphery of the four-sided chamber. The fusion of both iron and copper in this inclosure failed to show a spectrum for either, and Dr. Hemsalech concludes from this that their line spectra are not to be excited by purely thermal means. Granting, however, that a potential difference was thus avoided, it would seem that if the temperature were high enough (pyrometer readings are not given) and the means of observation sufficient a large group of iron lines which are subject to thermo-chemical excitation should have appeared. This is the stimulus which Dr. Hemsalech believes produces the spectrum shown by the tube furnace up to 2500°, and there was full opportunity for the formation of iron carbide and for chemical action resulting from the presence of oxygen.

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[1] *Op. cit.*, p. 290.

I must further call attention to two investigations in which furnaces, which seem to differ in no essential respect from this "hot plate" furnace, produced line spectra. The first case was when the writer[1] vaporized substances within a bored-out carbon placed horizontally and heated at its middle by an arc formed between the side of this carbon and a vertical electrode beneath. With this crude furnace, some twenty-eight lines of iron and three of copper were photographed. I may say in passing that the lines from the various elements studied in this furnace have corresponded fully with their low-temperature spectra when produced by the tube-resistance furnace at present in use.

The second furnace of this type was constructed by Hale, Adams, and Gale[2] in the course of their investigation of sun-spot spectra. A powerful arc passed between two horizontal carbons and heated the upper side of a carbon tube containing the substance to be vaporized. The temperature inside the tube was probably below 2000°, as titanium could not be melted, but a fairly rich iron spectrum consisting of the low-temperature lines was photographed in the green-yellow region. In both of these arc-heated furnaces a potential difference in the iron vapor is avoided.

A third point in the last paper[3] by Dr. Hemsalech on which I wish to present additional data is the failure of his tube-resistance furnace to show the spectra of copper, silver, and zinc. He was unable to obtain any lines of these elements even at high temperature and concludes that in addition to showing no spectroscopic reaction at moderate temperatures their vapors are so poorly conducting that they do not respond to the potential gradient when more highly heated.

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[1] *Astrophysical Journal*, 21, 236, 1905.

[2] *Mt. Wilson Contr.* No. 11; *Astrophysical Journal*, 24, 185, 1906.

[3] *Op. cit.*, p. 288.

As to the appearance of copper lines in the furnace, I must again refer to my observations with the tube heated by an external arc. To obtain further data on the question raised by Dr. Hemsalech I have used copper and silver in the tube-resistance furnace and have photographed some of the stronger lines of each element, both in vacuum and at atmospheric pressure. The temperatures ranged from 2000° to 2150° and the spectrum was examined from λ 3600 to λ 5800. With silver, λλ 3682, 4055, 5209, 5466 appeared distinctly and with copper, not only λλ 5106, 5700, 5782, observed previously with the external-arc furnace, but also λ 5153 and λ 5218 were now photographed. Some strong arc lines in this region failed to appear in the furnace at the moderate temperature used, but these are mainly of diffuse structure, a type which in general requires higher temperature. The period of strong emission is brief, the line λ 5106 of copper narrowing during a ten-minute exposure from a width of over an angstrom to a sharp maximum. This is readily explained by the behavior of the copper and silver, which, instead of sticking to the carbon and remaining in the hot portion, condense near the cool ends of the tube. This could be avoided by a spraying apparatus, giving a continuous supply. Zinc was not tried with this furnace, but in previous experiments I have found it to give the same trouble, forming filmy slag across the opening of the tube.

Summarizing these experiments, it is seen that for the high-temperature lines of iron, for the production of the iron spectrum without a potential gradient, and for the spectra of copper and silver in the tube furnace, some feature of Dr. Hemsalech's experiments has prevented the observation of spectrum lines which can nevertheless be produced under conditions which I believe invalidate the conclusions which he draws from his results. I make no plea for the existence of a purely thermal radiation, which I have long maintained is not to be proved by means of the tube furnace; still a clear disproof of the possibility of producing spectra by this means is yet to be presented. Dr. Hemsalech's work has done much to show the importance of possible chemical actions, and I believe this feature of the furnace radiation deserves to be taken into account at every point. But whatever processes of this nature may take place within the tube in any given experiment, they are controlled by the temperature, and according to present evidence it appears legitimate to employ the electric current to produce this temperature.

Mount Wilson Solar Observatory
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THE MOTIONS IN SPACE OF SOME STARS OF HIGH RADIAL VELOCITY

By WALTER S. ADAMS AND ALFRED H. JOY

The determinations of radial velocity made at the Lick and Mount Wilson observatories during the past few years have added greatly to the number of stars known to have large velocities in the line of sight. Most of these stars have well-determined proper motions and for many of them parallaxes have been measured by numerous observers. The motions in space of these stars are of interest because of the velocities with which they move and the wide range in spectral type and absolute magnitude which is found among them.

A list of all stars with radial velocities exceeding 80 km for which proper motions and parallaxes (derived either by the trigonometric or the spectroscopic method) are known is given in Table I. The stars are numbered consecutively, and the successive columns give the positions for 1900, their apparent visual magnitudes, spectral types, proper motions and direction of proper motion, radial velocities, and radial velocities corrected for the solar motion. The latter are derived from the apex determined by Strömberg,[1] $A_0 = 270\overset{\circ}{.}9$, $D_0 = +29\overset{\circ}{.}2$, $V_0 = 21.5$ km. The apparent magnitudes are from Harvard determinations, the spectral types from Mount Wilson observations, and the proper motions have in most cases been taken from the catalogues of Boss and Porter.[2] The radial velocities are from the Lick and Mount Wilson determinations, the values being combined when the same star has been investigated at both observatories.

The parallaxes of all of the stars in the list with the exception of five have been derived from the absolute magnitudes determined spectroscopically at Mount Wilson. Small systematic corrections

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[1] *Mt. Wilson Contr.* No. 144; *Astrophysical Journal*, **47**, 7, 1918.

[2] *Publications of the Cincinnati Observatory*, No. 18, Parts 1–4, 1915–1918.

TABLE I

No.	Name	α (1900)	δ (1900)	*m*	Spec.	μ	ψ	*V*	*V′*
								km	km
1	ε Andromedae	$0^h33^m.3$	+28°46′	4.5	G5	0″.336	222°	− 84	− 81
2	μ Cassiopeiae	1 1.6	+54 26	5.3	G5	3.760	114.5	− 97	− 91
3	Lalande 1966	1 3.3	+61 1	7.9	F5	0.638	86	−325	−318
4	Lalande 4855	2 32.6	+30 24	7.2	F8	0.625	231	− 99	−103
5	Lalande 5761	3 2.5	+25 58	8.0	A3p	0.861	194	−144	−151
6	W.B. 3^h617	3 35.3	− 3 32	7.2	F6	0.745	107	+114	+ 99
7	A.G.Berl. 1366	4 8.6	+22 6	8.9	F0	0.54	125	+339	+328
8	Groom. 864	4 34.5	+41 56	7.3	G2	0.690	127	+105	+ 99
9	Cordoba 5^h243	5 7.7	−44 59	9.2	G-K	8.75	131	+242	+222
10	Lalande 9960	5 14.1	− 3 11	8.5	K5	0.721	78	+ 86	+ 68
11	ϕ^2Orionis	5 31.4	+ 9 14	4.4	G7	0.321	163	+ 99	+ 82
12	δ Leporis	5 47.0	−20 53	3.9	G7	0.696	160	+ 99	+ 78
13	A.G.Berl. 1866	5 57.3	+19 23	9.0	F9	0.76	129	−190	−204
14	Boss 1511	5 59.2	−26 17	5.2	K3p	0.104	24	+183	+162
15	θ Can. Maj.	6 49.5	−11 55	4.2	K4	0.137	264	+ 97	+ 77
16	Groom. 1281	7 8.4	+47 25	5.6	G0	0.184	168	+ 88	+ 83
17	Lalande 15290	7 47.2	+30 55	8.2	F7	1.962	158	−242	−251
18	Lalande 19821	10 6.3	+24 15	8.5	G2	0.414	279	+ 81	+ 77
19	Lalande 21185	10 57.9	+36 38	7.6	Ma	4.779	187	− 87	− 85
20	Groom. 1830	11 47.2	+38 26	6.5	G8	7.047	145	− 97	− 92
21	Lalande 23995	12 47.9	−17 57	8.2	F3	0.877	159	+144	+144
22	Lalande 27274	14 54.2	−21 36	8.3	F4	0.785	229	+160	+168
23	A. Oe. 14318. A. Oe. 14320	15 4.7 15 4.7	−15 59 −15 54	9.6 9.2	K0 G8	3.682	196	+300	+310
24	Lalande 28607	15 37.7	−10 36	7.3	A2p	1.178	256	−170	−157
25	A.G.Leid. 5734	16 11.4	+32 24	(8.3)	K4	0.035	145	−164	−144
26	Boss 4188	16 22.3	− 7 22	5.4	Mb	0.176	168	+ 97	+112
27	Σ 2173	17 25.2	− 0 59	5.9	G6	0.214	215	− 81	− 63
28	W.B. 17^h514	17 29.9	+ 6 4	8.6	F5	0.623	311	−148	−129
29	Barnard's Star	17 53.0	+ 4 28	9.7	Mb	10.27	356	−106	− 87
30	31 b Aquilae	19 20.2	+11 44	5.2	G7	0.961	49	− 98	− 78
31	α Vulpeculae	19 24.5	+24 28	4.6	Ma	0.170	227	− 86	− 66
32	Lalande 37120	19 29.7	+32 59	6.6	F9	0.517	294	−162	−142
33	A. Oe. 20452	20 17.7	−21 40	8.1	F5	1.182	154	−179	−168
34	η Cephei	20 43.3	+61 27	3.6	G8	0.826	7	− 87	− 71
35	Boss 5482	21 16.6	+23 26	5.8	G9	0.252	123	− 88	− 73
36	W.B. 21^h502	21 24.5	−12 56	9.1	K5	1.052	105	− 90	− 81
37	W.B. 21^h594	21 26.0	+45 27	7.5	G8	0.56	50	− 83	− 67

have been applied to these values in accordance with an unpublished investigation by Strömberg. These corrections are as yet provisional and in nearly all cases their influence is slight. The directly measured parallaxes have been used in the case of stars numbered 5, 9, 23, 24, and 29. For star No. 27, which is a close visual binary, the apparent magnitude of the brighter component has been employed. The derivation of the parallaxes is from the formula $\log \pi = 0.2(M-m)-1$, and the values of the absolute magnitude and parallax are given in the second and third columns of Table II.

TABLE II

No.	M	π	A	D	x	y	z	v	L	B
					km	km	km	km		
1	+ 0.9	+0.019	211°	−29°	+ 26	− 70	+ 44	87	290°	+30°
2	+ 5.7	0.120	112	−42	−115	−103	− 28	156	222	−10
3	+ 3.9	0.016	152	−47	−155	−326	+ 49	364	245	+ 8
4	+ 4.4	0.028	275	−40	+117	− 92	− 35	153	322	−13
5	+ 6.0	0.039	247	−54	+ 89	−155	− 13	179	300	− 4
6	+ 4.4	0.028	103	−13	−148	− 34	− 11	152	193	− 4
7	+ 3.0	0.007	101	− 8	−489	− 63	− 28	494	187	− 3
8	+ 6.0	0.055	94	+29	− 93	+ 52	+ 14	108	151	+ 7
9	+11.7	0.319	123	−59	−121	−219	− 57	257	241	−13
10	+ 7.0	0.050	121	+12	− 87	+ 3	+ 38	95	178	+24
11	+ 1.0	0.021	94	−26	− 92	− 35	− 29	102	200	−16
12	+ 1.4	0.032	126	−66	− 45	−116	− 31	128	249	−14
13	+ 5.6	0.021	230	−36	+134	−212	+ 74	262	302	+16
14	+ 0.6	0.012	95	−13	−163	− 26	− 30	168	189	−10
15	− 0.4	0.012	69	−10	− 77	+ 7	− 49	91	175	−33
16	+ 5.5	0.095	116	+54	− 47	+ 54	+ 43	84	131	+31
17	+ 5.0	0.023	217	−70	+ 73	−457	− 70	467	279	− 9
18	+ 6.0	0.032	115	+35	− 73	+ 38	+ 41	91	153	+27
19	+10.8	0.437	328	−61	+ 31	− 58	− 68	95	298	−46
20	+ 6.5	0.100	243	−50	+180	−301	+ 2	351	301	0
21	+ 3.6	0.012	266	−66	+143	−321	−122	372	294	−19
22	+ 3.8	0.013	156	−45	−128	−289	+ 62	322	246	+11
23	+ 7.8 + 7.4	0.044	200	−64	+ 38	−489	− 10	491	274	− 1
24	+ 4.9	0.033	98	0	−221	+ 4	− 8	221	179	− 2
25	+ 0.1	0.002	33	−51	− 40	− 71	−144	166	241	−60
26	− 0.6	0.006	266	−51	+101	−128	− 35	167	308	−12
27	+ 5.1	0.069	86	− 1	− 61	+ 6	− 14	63	174	−13
28	+ 4.4	0.014	130	+31	−182	+ 68	+148	245	159	+37
29	+13.4	0.540	92	+45	− 92	+ 89	+ 30	132	136	+13
30	+ 5.4	0.110	94	+11	− 86	+ 20	− 2	89	167	− 1
31	+ 0.4	0.014	158	−42	− 36	− 78	+ 23	89	245	+15
32	+ 5.1	0.050	134	−24	−116	− 89	+ 40	152	217	+15
33	+ 4.0	0.015	96	−36	−314	−194	−130	391	212	−19
34	+ 3.0	0.076	136	−29	− 60	− 56	+ 19	84	223	+13
35	0.0	0.007	63	−39	−107	− 60	−123	174	209	−45
36	+ 8.7	0.083	112	+ 9	− 88	+ 5	+ 23	91	177	+15
37	+ 5.9	0.048	120	−17	− 73	− 33	+ 12	81	204	+ 9

The computation of the equatorial velocity-components referred to the sun, ξ, η, and ζ, has been made from the formulae

$$\xi = V \cos\alpha \cos\delta - \frac{k}{\pi}(\mu_1 \sin\alpha + \mu_2 \cos\alpha \sin\delta)$$

$$\eta = V \sin\alpha \cos\delta + \frac{k}{\pi}(\mu_1 \cos\alpha - \mu_2 \sin\alpha \sin\delta)$$

$$\zeta = V \sin\delta + \frac{k}{\pi}\mu_2 \cos\delta$$

In these expressions V is the radial velocity, $k=4.737$ and $\mu_1 = \mu_\alpha \cos\delta$, $\mu_2 = \mu_\delta$ where μ_α and μ_δ are the components of the proper motion. To reduce these values to the usually adopted centroid of the stars by eliminating the solar motion, we put:

$$\xi_0 = V_0 \cos A_0 \cos D_0$$
$$\eta_0 = V_0 \sin A_0 \cos D_0$$
$$\zeta_0 = V_0 \sin D_0$$

in which V_0 is the solar motion, and A_0 and D_0 the right ascension and declination of its apex. We then have for the equatorial components of the star's space-velocity,

$$\xi_1 = \xi + \xi_0 = v \cos A \cos D$$
$$\eta_1 = \eta + \eta_0 = v \sin A \cos D$$
$$\zeta_1 = \zeta + \zeta_0 = v \sin D$$

From these equations we calculate not only ξ_1, η_1, and ζ_1, but also v, A, and D, the space-velocity of the star and the co-ordinates of its apex. Using the values already referred to we have

$$\xi_0 = 0, \qquad \eta_0 = -19, \qquad \zeta_0 = +10$$

The components of velocity with reference to the galactic system are found from the equations

$$x = v \cos L \cos B = +0.1846\, \xi_1 - 0.9828\, \eta_1$$
$$y = v \sin L \cos B = +0.4494\, \xi_1 + 0.0844\, \eta_1 + 0.8893\, \zeta_1$$
$$z = v \sin B \qquad = -0.8740\, \xi_1 - 0.1642\, \eta_1 + 0.4573\, \zeta_1$$
$$v^2 = x^2 + y^2 + z^2$$

The numerical coefficients result from the assumed values $\alpha = 190^\circ\!.6$, $\delta = +27^\circ\!.2$ for the pole of the galaxy.

The results of the computations are given in Table II. A and D are the right ascensions and declinations of the apices, x, y, and z the components of velocity referred to the galaxy, v is the total velocity, and L and B the galactic co-ordinates of the apices. The values are shown graphically in Fig. 1, the plane used being that of the galaxy. The apices are shown in projection, and the vectors drawn from the origin represent the direction and amount of motion in this plane.

Some of the conclusions to be drawn from these results may be summarized as follows:

1. The components of velocity normal to the plane of the galaxy are much smaller than those in the plane. Thus we find from Table II the average values:

$$x=115 \text{ km}, \qquad y=119 \text{ km}, \qquad z=46 \text{ km}.$$

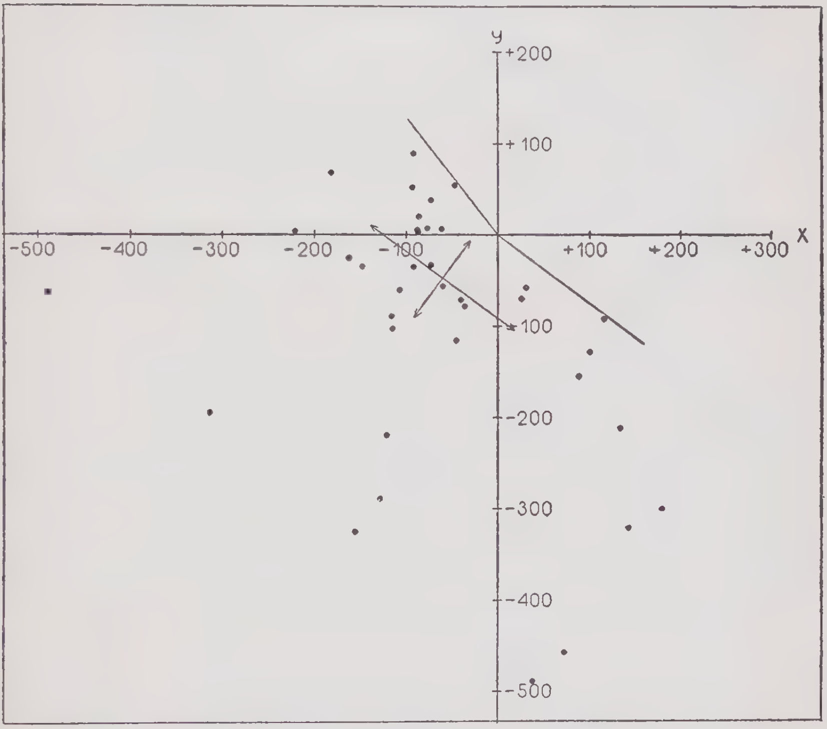

FIG. 1.—Projection on galactic plane of apices of motion of thirty-seven stars of high radial velocity. The vectors drawn from the origin represent the projected velocities in km per sec. The axes of the ellipse of intersection of the velocity-ellipsoid with the galactic plane are indicated by the two arrow-headed lines.

The same result follows from a consideration of the latitudes *B* of the apices. The latitudes of only six of the thirty-seven stars exceed 30°.

2. Nearly an entire hemisphere is devoid of apices, the galactic longitudes all lying between 131° and 322°.

3. The velocity of the centroid of these stars is remarkably high. Thus omitting those with total velocities greater than 300 km we find from the remaining twenty-nine stars:

$\bar{x}=-59.5$ km $\quad\bar{y}=-44.5$ km $\quad\bar{z}=-4.4$ km
$v=74.4$ km $\quad L=217°$ $\quad B=-3°$

4. If we project the velocities of these twenty-nine stars upon axes through this center such that the sum of the squares of the projected velocities is a maximum and a minimum we find the following directions[1] and the mean square dispersion σ:

Maximum: $L=141°$ $\quad B=+9°$ $\quad\sigma=100.6$ km
Minimum: $L=61°$ $\quad B=-49°$ $\quad\sigma=46.1$ km
Intermediate: $L=43°$ $\quad B=+39°$ $\quad\sigma=61.4$ km

The directions of the smaller axes are necessarily uncertain. Thus it is clear that the effect of stream motion among these stars is very marked.

5. It is of interest to note that the direction of the major axis as derived from these stars is in close agreement with that found by Strömberg from a discussion of the radial velocities of 260 dwarf stars ($L=148°$, $B=+14°$), and that of Raymond[2] from 559 stars of large proper motion ($L=146°$, $B=+8°$). All three of these investigations agree in indicating that the galactic longitude of the principal vertex for the stars of high velocity is considerably less than that for stars in general, which is about 170°.

6. An interesting feature of the results, but one which must be accepted with some reserve, is the apparent tendency of the stars to move along a line of galactic longitude about 260°, the assumption being made of a motion of the centroid of $\bar{x}=-100$ km, $\bar{y}=-20$ km. This direction coincides nearly with that of the greatest star-density as determined by Charlier, Walkey, Nort, and Plummer.

7. Reference may perhaps also be made to the tendency shown by the stars of the very highest velocity (over 300 km) to move along a line parallel to that of the major axis, referring the motion

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[1] The intersection of this velocity-ellipsoid with the galactic plane is indicated in Fig. 1 by straight lines which show the axes of the ellipse of intersection.

[2] *Astronomical Journal*, **30**, 191, 1917.

to their common center. Thus all of the stars omitted from the previous discussion show a very fair degree of alignment with the direction of the principal axis as given in Fig. 1.

8. A marked difference is seen in the average galactic latitudes of the apices of the stars of high and low luminosity, the latter lying more nearly in the plane of the galaxy. Thus if we select the stars of absolute magnitude brighter and fainter than 3.0 we find:

	Average *M*	Average *B*
9 stars	+0.4	26°
28 stars	5.9	14°

The galactic latitudes of the apices of only four of the fainter stars exceed 26°.

9. A similar result is found for the average velocities of the stars of high and low luminosity, the latter stars moving more rapidly.

	Average *M*	Average *v*
9 stars	+0.4	130 km
28 stars	5.9	216 km

10. An examination of the spectral types of these stars shows that an extraordinarily large proportion, twenty-six out of thirty-seven, are of types F and G. The stars of type F have by far the largest average space motion, nearly double that of the stars of type G. The comparison, which is of little weight for types A, K, and M, gives:

No. of Stars	Type	Average *v*	Average *V′*
		km	km
2	Ap	200	115
11	F	307	187
15	G	156	104
5	K	122	106
4	M	121	88

Among the stars of type F it is of interest to note that those of earlier spectrum show the largest velocities. Thus the six stars with spectra between F0 and F5 inclusive have space motions of 364, 494, 372, 322, 245, and 391 km, an average value of 365 km. The average value of V′ for these stars is 209 km.

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THE ORBITS OF THREE SPECTROSCOPIC BINARIES

By WALTER S. ADAMS and ALFRED H. JOY

THE ORBIT OF BOSS 593

The two stars Boss 592 and 593 ($\alpha = 2^h\ 31^m\!.2$; $\delta = +24^\circ\ 13'$; 1900) form the well-known double star 30 Arietis, or Σ 5, with a distance of 38″ and a common proper motion of 0″.146 annually. The visual magnitudes of the stars are 7.4 and 6.6 respectively, and the spectral types are very similar, being F5 and F4 according to our most recent determinations. The brighter star of the pair, Boss 593, was found in 1916 to be a spectroscopic binary, and the number of spectrograms is now sufficient to make a determination of its orbit readily possible.

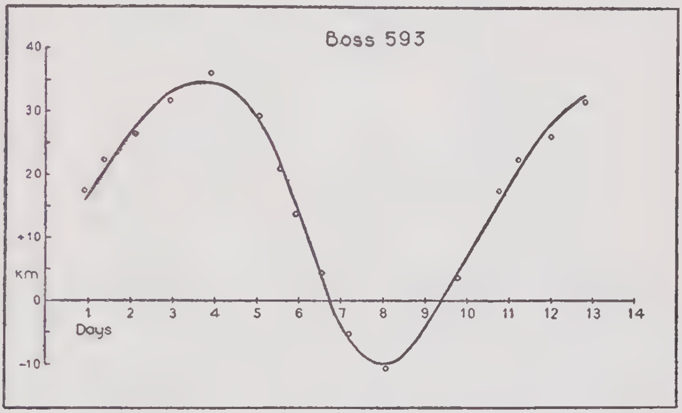

FIG. 1

All of the spectrograms were secured with the Cassegrain spectrograph and the 60-inch reflector, a dispersion of one prism and a camera of 18 in. (45.7 cm) focal length being used throughout the observations. The spectrum is not of the best quality for measurement, and the lines are broad and somewhat hazy. For this reason, and because of the low dispersion employed, the computation of the orbit has been based upon normal positions, the radial velocities for spectrograms taken at nearly the same phase being combined to give mean values.

Since the observations extend over an interval of three years, it has been possible to determine the period with considerable accuracy. This is found to be closely 9.851 days. With the aid of this period the phases have been determined and the radial velocities combined into the normal values given in Table I. The epoch selected is December 20.0, 1917, Julian day 2421583.0 G.M.T.

TABLE I

No.	No. of Plates	Phase	V	$O-C_1$	$O-C_2$	$O-C_3$
		Days	km	km	km	km
1..........	2	0.910	+36.2	+1.1	+2.1	+1.5
2..........	2	2.008	+29.4	+0.8	+1.0	+0.9
3..........	2	2.531	+21.0	−0.4	−0.9	−0.8
4..........	3	2.931	+14.2	−0.4	−1.1	−0.9
5..........	4	3.551	+ 4.3	+0.6	+0.3	+0.5
6..........	4	4.188	− 5.3	0.0	+0.2	+0.5
7..........	1	5.081	−10.5	−0.5	−1.1	−0.4
8..........	2	6.778	+ 3.8	−1.2	−1.3	−0.9
9..........	2	7.759	+17.6	+1.0	+1.6	+1.5
10..........	1	8.207	+22.3	+1.3	+1.7	+1.3
11..........	2	8.976	+26.0	−1.2	−1.2	−1.8
12..........	2	9.784	+31.5	−0.8	−0.7	−1.4

The method of calculation adopted is that involving the use of a Fourier's series, the details of which are given by Plummer,[1] Russell,[2] and others. The mean radial velocities and phases of Table I were plotted, and a smooth curve was drawn through the corresponding points. From this curve were read off the radial velocities $V_0, V_1 \ldots\ldots V_{11}$ for each twelfth part of the period. The following equation was then assumed:

$$V = b_0 + b_1 \cos \tau + b_2 \cos 2\tau + a_1 \sin \tau + a_2 \sin 2\tau .$$

The value of τ being 30°, the computation of the coefficients is made readily from the relationships

$$b_0 = \tfrac{1}{12}\sum_0^{11} V_n ; \quad b_1 = \tfrac{1}{6}\sum_0^{11} V_n \cos n.30^\circ ; \quad b_2 = \tfrac{1}{6}\sum_0^{11} V_n \cos n.60^\circ ; \quad \text{etc.}$$

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[1] *Astrophysical Journal*, **28**, 212, 1908.

[2] *Ibid.*, **15**, 252, 1902.

The resulting value for the series is:

$$V=+14.42+21.31\cos\tau-3.08\cos 2\tau+4.96\sin\tau+15.1\sin 2\tau.$$

The normal values of the radial velocities are well represented by this expression, as the residuals given in Table I under $O-C_1$ show.

Approximate values of the elements may be derived from the coefficients of this series by means of the relationships given by Plummer. Neglecting the second and higher powers of the eccentricity we find:

$$\gamma=+14.4\text{ km}$$
$$K=21.9\text{ km}$$
$$\omega=127^\circ.6$$
$$e=0.157$$
$$T=\text{J.D. }2421586.86$$
$$P\text{ (assumed)}=9.851\text{ days}$$
$$a\sin i=2{,}925{,}000\text{ km}$$
$$\frac{m_1^3\sin^3 i}{(m+m_1)^2}=0.010\odot$$

The values of the radial velocity calculated from these elements show the differences from the observed values given in Table I under $O-C_2$.

Although this representation is satisfactory in view of the degree of accuracy obtained in the measurements of the radial velocity, it seemed desirable to carry out a least-squares solution for the purpose of obtaining the probable errors of the various elements. In this solution the period P was assumed to be constant, and corrections were determined for γ, ω, e, K, and T. The final values of the resulting elements are as follows:

$$\gamma=+14.42\pm0.71\text{ km}$$
$$K=22.48\pm0.47\text{ km}$$
$$\omega=129^\circ.6\pm7^\circ.3$$
$$e=0.1455\pm0.0185$$
$$T=\text{J.D. }2421586.899\pm0.196$$
$$a\sin i=3{,}012{,}700\text{ km}$$
$$\frac{m_1^3\sin^3 i}{(m+m_1)^2}=0.0113\odot$$

The differences between the observed radial velocities and those computed from these elements are shown in Table I under $O-C_3$.

The parallax of Boss 593 has been determined by van Maanen and by Miller. The weighted mean of the results gives an absolute magnitude of 4.2.

THE ORBIT AND ABSOLUTE DIMENSIONS OF W URSAE MAJORIS[1]

The variable star W Ursae Majoris ($\alpha = 9^{h}\ 36^{m}.7$; $\delta = +56^{\circ}\ 25'$; 1900) has attracted much interest since its discovery in 1903 by Müller and Kempf, who found its light to vary in the unusually short period of four hours. The variation was found to be continuous, with very sharp minima and rather flat maxima. The peculiar shape of the light-curve led Russell to apply the theory of an eclipsing binary, and he thus found that the star's light-changes might be accounted for by assuming that the period should be doubled, and that the primary and secondary eclipses produced equal minima. This result has not been as widely accepted as the evidence would warrant, probably on account of the exceptionally short period and the spectral type, which was given by the Harvard observers as G0. Hence the star is still given in some lists as a Cepheid or cluster-type variable.

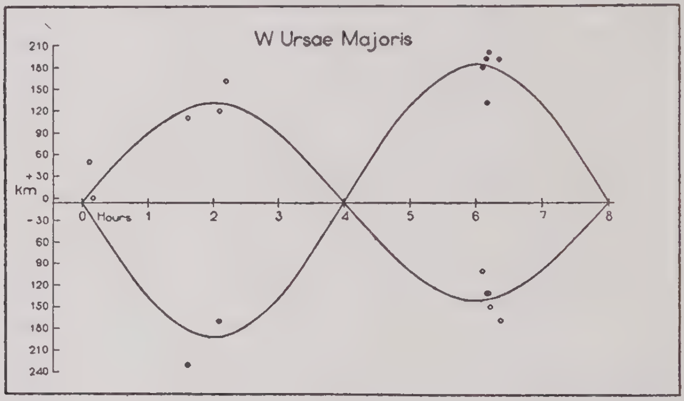

FIG. 2

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[1] An abstract of this paper was read at the meeting of the American Astronomical Society, August 20–22, 1918.

The star was placed on the Mount Wilson spectrographic observing list, together with other stars of the later types having measured trigonometrical parallaxes, for the determination of absolute magnitude and parallax. Ten plates were secured from December, 1917, to March, 1918, with exposures varying from 30^{m} with the 7-inch camera to $2^{h}\ 15^{m}$ with the 18-inch camera. The spectra of both components appear, the lines of the primary being slightly stronger than those of the secondary. The spectrum is most extraordinary. It appears to belong to the solar type and might be classified as F8p. The faint lines are entirely obliterated, and the strong lines are so widened and weakened that measures for velocity and estimates of line-intensity can be made only with the greatest difficulty. The results accordingly are given only as the best approximations which can be determined from the plates. The results of the spectrographic measures, together with the photometric orbits by Russell and Shapley, show that the unusual character of the spectral lines is due partly to the rapid change in velocity during even our shortest exposures but mainly to the rotational effect in each star, which may cause a difference of velocity in the line of sight of as much as 240 km between the two limbs of the star.

TABLE II

Plate	Date	G.M.T.	Exp.	Phase	Velocity of Primary	Velocity of Secondary
					km	km
6417.......	1917 Dec. 1	$22^{h}45^{m}$	135^{m}	$6\overset{h}{.}4$	−170	+190
6433.......	3	22 35	60	6.1	−130	+190
6443*......	5	0 13	115	0.1	.....	.....
6521†......	1918 Jan. 1	1 13	45	0.2	.....	.....
6524.......	1	19 15	30	2.2	+160	(−260)
6526.......	1	23 15	30	6.2	−150	+200
6538.......	2	23 11	38	6.1	−100	+180
6692.......	Feb. 1	19 50	56	2.1	+120	−170
6695.......	1	23 55	60	6.2	−130	+130
6820.......	March 31	20 35	70	1.6	+110	−230

* Lines of two spectra blended. Measured $V=+50$ km.

† Lines of two spectra blended. Measured $V=0$ km.

Table II gives a summary of the observations. Three plates were taken at maximum light when the primary star was receding, five at the opposite maximum when the primary was approaching, and two near the even minima. The phases were computed from the recent light-elements by Shapley and Van der Bilt,[1] using the period of 0.3336392 day. The velocities are corrected to give the effective velocity for the whole exposure and are rounded off to the nearest ten kilometers on account of the character of the spectrum. They are the means of the measures of three and in a few cases of four observers. On plate 6524 the second component was measured by only one observer, and the value is omitted.

The following orbital elements were determined after grouping the observations according to phase and assigning weights according to the character of the lines:

Eccentricity, e (assumed) $=0.00$
Velocity of center of mass, $\gamma=-5\pm 13$ km/sec.
Semi-velocity range of primary, $K_1=134\pm 5$ km/sec.
Semi-velocity range of secondary, $K_2=188\pm 15$ km/sec.
Radius of orbit of primary, $a_1 \sin i=610{,}000$ km
Radius of orbit of secondary, $a_2 \sin i=860{,}000$ km
Mass of primary, $m_1 \sin^3 i=0.67 \odot$
Mass of secondary, $m_2 \sin^3 i=0.48 \odot$
Ratio of masses, $\frac{m_2}{m_1}=0.71 \odot$

With the aid of Russell's solution of the photometric orbit as given by Shapley[2] we may now compute the absolute dimensions of the orbit and the individual stars and find the masses and densities of each star, thus giving data which are known for only a very few stars.

Taking the "uniform" solution which Russell suggests as the most probable because of the color-indexes found by Shapley, we note that the stars are ellipsoids with the same dimensions. The inclination $i=77° 35'$, and the ratio of the axes of the ellipsoids are given, whence we find:

Radius of the primary orbit $=620{,}000$ km
Radius of the secondary orbit $=880{,}000$ km

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[1] *Mt. Wilson Contr.* No. 140; *Astrophysical Journal*, **46**, 290, 1917.

[2] *Contributions from the Princeton University Observatory*, No. 3, 1915.

Radius of the relative orbit = 1,500,000 km
Greatest semi-axis of each star = 548,000 km = 0.78 sun's radius
Intermediate semi-axis of each star = 408,000 km = 0.58 sun's radius
Polar semi-axis of each star = 373,000 km = 0.54 sun's radius
Mass of primary = 0.69 $\odot$
Mass of secondary = 0.49 $\odot$
Density of primary = 2.8 $\odot$
Density of secondary = 1.9 $\odot$

The densities are remarkable as being the highest known in any stellar system. The projected surface area of the two stars is equal to about 90 per cent of the sun. If we should assume that the same surface luminosity prevails on the stars as on the sun, as we should expect in similar spectral types, the absolute magnitude would be about 4.8 (calling the sun 5.0) and the parallax $0\overset{\prime\prime}{.}024$.

The trigonometric parallax has been measured at the Yerkes Observatory and found to be $0\overset{\prime\prime}{.}012$, corresponding to an absolute magnitude of 3.3, and our rough estimates from the very poor spectral lines seem to agree with it. If then we accept the measured parallaxes, we find the surface luminosity of the stars to be four times that of our sun.

THE ORBIT AND ABSOLUTE DIMENSIONS OF Z HERCULIS

The Algol variable Z Herculis ($\alpha = 17^{h}\ 53\overset{m}{.}6$; $\delta = +15^\circ\ 9'$; 1900) is well known because its period is so nearly equal to four days that it has been found difficult to make complete observations of its light and velocity changes at any one observatory. In the course of the observation of stars with measured parallaxes nine spectrograms have been secured at Mount Wilson. These observations have been timed in such a way that they allow a determination of the spectrographic orbit, if it is assumed to be circular in accordance with the photometric orbit computed by Shapley.[1] Both the photometric and spectrographic results indicate that in general the Algol variables move in orbits which are nearly circular. The star R Canis Majoris,[2] $e = 0.138$, is the only one thus far found to have an eccentricity of more than 0.100. Plate 6942, which was taken near minimum of light, was of inferior quality and was not used in the solution.

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[1] *Contributions from the Princeton University Observatory*, No. 3, p. 100, 1915.

[2] Jordan, *Publications of the Allegheny Observatory*, 3, 49, 1912.

Two spectrograms taken by Frost[1] in 1905 give velocities which fall fairly well on our curve and show that the period has been accurately determined. Our photographs show the presence of two spectra of type F2. The lines of the secondary spectrum are only about one-half as strong as those of the primary, and many are poorly defined or not seen at all on some of the plates. The orbit of the secondary is consequently less accurately determined. Although the results show that there is a marked difference in surface luminosity between the two stars, the spectra appear to be essentially of the same type. The rotational effect is so small that the spectral lines are fairly sharp, and measures of the primary show good agreement. The velocity of the center of mass is unusually large.

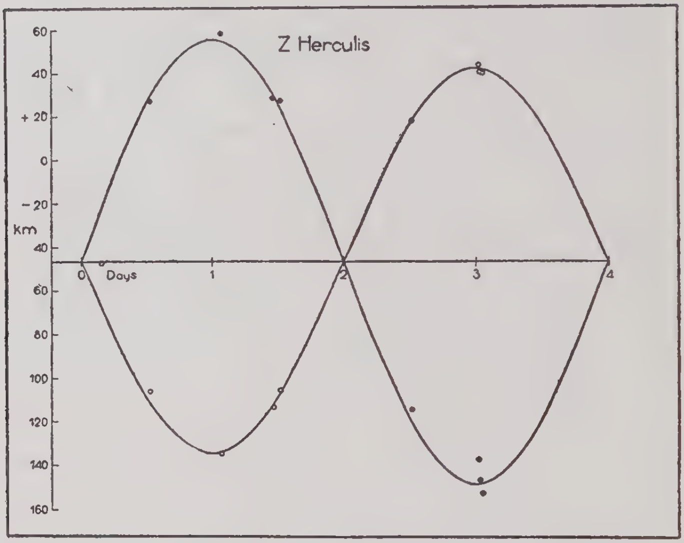

FIG. 3

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[1] *Astrophysical Journal*, **22**, 214, 1905.

The following elements were determined, the orbit being assumed to be circular:

Photometric period, $P=3.992775$ days (Lehnert)
Eccentricity, $e=0.00$ (assumed)
Velocity of center of mass, $\gamma=-46.5\pm0.3$ km/sec
Semi-velocity range of primary, $K_1=88.2\pm0.4$ km/sec
Semi-velocity range of secondary, $K_2=101.8\pm1.4$ km/sec
Radius of primary orbit, $a_1 \sin i=4,800,000$ km
Radius of secondary orbit, $a_2 \sin i=5,600,000$ km
Mass of primary, $m_1 \sin^3 i=1.5$ ⊙
Mass of secondary, $m_2 \sin^3 i=1.3$ ⊙
Ratio of masses, $\frac{m_2}{m_1}=0.86$

The photometric orbit has been computed by Shapley[1] from observations by Wendell at Harvard. On account of the peculiarity of the period the observations of different portions of the light-curve had to be made in different seasons, but the results were considered fairly reliable. The "uniform" solution gives a grazing total eclipse and the "darkened" solution a partial eclipse, with the larger star in front in both cases. The larger star is the fainter. The secondary eclipse is very shallow. Combining the results of the spectrographic observations with the photometric we obtain the absolute dimensions of the orbits and of the individual stars.

TABLE III

Plate	Date	G.M.T.	Phase	V_1	V_2
			Days	km	km
6265.........	1917 Oct. 5	$15^h 46^m$	0.514	−106.0	+ 27.1
6273.........	6	15 44	1.512	−106.8	+ 27.5
6300.........	26	14 13	1.466	−113.0	+ 28.6
6357.........	Nov. 4	14 38	2.507	+ 17.4	−114.7
6933.........	1918 May 25	19 10	1.075	−134.9	+ 58.0
6939.........	27	18 40	3.054	+ 39.5	−152.7
6942.........	28	21 5	0.162	....	− 47.2
7070.........	June 24	16 55	3.031	+ 43.7	−137.0
7134.........	July 18	16 17	3.049	+ 39.8	−146.4

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[1] *Contributions from the Princeton University Observatory*, No. 3, 1915.

The photometric observations are not sufficient to give the relation of the axes of the ellipsoids.

TABLE IV

	"Uniform" Solution	"Darkened" Solution
Inclination of orbit	83° 40′	82° 0′
Radius of primary orbit	4,870,000 km	4,880,000 km
Radius of secondary orbit	5,620,000 km	5,640,000 km
Radius of relative orbit or distance of centers	10,490,000 km	10,520,000 km
Radius of bright primary star	1,020,000 km	1,230,000 km
Radius of faint secondary star	2,170,000 km	2,290,000 km
Mass of primary	1.5 ⊙	1.6 ⊙
Mass of secondary	1.3 ⊙	1.3 ⊙
Density of primary	0.5 ⊙	0.3 ⊙
Density of secondary	0.05 ⊙	0.04 ⊙

The absolute magnitude of the brighter star determined from the spectra is +2.4, indicating a parallax of 0″.010, which is in good agreement with the hypothetical parallax. The trigonometric parallax as determined at the Yerkes Observatory is +0″.033.

Mount Wilson Solar Observatory
February 1919

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THE MAGNETIC POLARITY OF SUN-SPOTS

By GEORGE E. HALE, FERDINAND ELLERMAN,
S. B. NICHOLSON, and A. H. JOY

Whirling storms in the earth's atmosphere, whether cyclones or tornadoes, follow a well-known law which is said to have no exceptions: the direction of whirl in the northern hemisphere is left-handed or counterclockwise, while in the southern hemisphere it is right-handed or clockwise. The theory of terrestrial cyclones is still very obscure, but the direction of whirl is evidently determined by the increase in linear velocity of the air from pole to equator, due to the earth's rotation. The question naturally arises whether storms in the solar atmosphere are also whirlwinds, and, if so, what law governs their direction of whirl in the northern and southern hemispheres.

The first definite evidence bearing on this question was obtained with the spectroheliograph in 1908.[1] Photographs of the hydrogen flocculi made with the Hα line showed clearly marked vortical structure in regions centering in sun-spots. This structure was found to be repeated in hundreds of spots, leaving no doubt as to the generality of the phenomenon. Furthermore, photographs were obtained showing masses of hydrogen in the act of being drawn from a great distance toward the center of sun-spots, as though sucked into a vortex.

These photographs suggested the hypothesis that a sun-spot is a vortex, in which electrified particles, produced by ionization in the solar atmosphere, are whirled at high velocity. This might give rise to magnetic fields in sun-spots, regarded as electric vortices. A search for the Zeeman effect led to its immediate detection, and abundant proofs were soon found of the existence of a magnetic field in every sun-spot observed.[2]

[1] Hale, "Solar Vortices," *Mt. Wilson Contr.* No. 26; *Astrophysical Journal*, **28**, 100, 1908.

[2] Hale, "On the Probable Existence of a Magnetic Field in Sun-Spots," *Mt. Wilson Contr.* No. 30; *Astrophysical Journal*, **28**, 315, 1908.

Subsequent investigations have led to the view that two classes of vortices are involved:

1. High-level hydrogen vortices, centering in sun-spots, which are revealed by the spectroheliograph when monochromatic images of the sun are photographed with the light of the central part of the Hα line. In these vortices, which sometimes cover vast areas of the solar surface, the motion of the hydrogen appears to be spirally inward and downward. Little is known, except by inference, as to the form of the stream-lines in the lower levels close to the photosphere. There is reason to infer, however, that the hydrogen gas, descending toward the solar surface, moves spirally outward in the lower chromosphere above the spots.[1]

2. Low-level electric vortices, formed in the photosphere, which constitute the sun-spots themselves. In these vortices the motion of the gases appears to be spirally upward from within the photosphere and outward along its surface.[2] It is easy to show by laboratory experiments that such a primary vortex formed in water may set up a secondary vortex in a gaseous atmosphere above it, closely analogous to the hydrogen vortices above sun-spots.

The present paper deals with the magnetic polarity of sun-spots, though some reference will also be made to the closely related phenomena exhibited by the vortex structure of the hydrogen flocculi. The purpose in view is to discover the law of magnetic polarities, if such exists, and to consider whether this law is subject to variation in the course of the sun-spot cycle. Subsequent papers will deal with the various magnetic peculiarities of sun-spots and the structure of the hydrogen flocculi, with special reference to the nature of the vortices associated with sun-spots.

METHOD OF OBSERVATION

It is a well-known fact that when a normal Zeeman triplet is observed along the lines of force of a magnetic field the central (p) component is absent and the two side (n) components are circularly polarized in opposite directions. A quarter-wave plate and Nicol prism mounted over the slit of the spectroscope permit either *n*-component to be cut off at will by rotating the Nicol. Furthermore, if the polarizing apparatus be adjusted so as to extinguish one component, reversal of the current through the coils of the magnet will cause this component to reappear, while the other will be extinguished. The method thus offers a simple means of determining the polarity of a magnetic field, which can still be used when the angle between the line of sight and the lines of force is as great as sixty or seventy degrees. In this case, however, the *p*-component of the triplet is present, and the elliptically polarized light of the *n*-components can be only partially extinguished.

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[1] St. John, *Mt. Wilson Contr.* No. 69; *Astrophysical Journal*, **37**, 322, 1913; Evershed, *Kodaikanal Observatory Bulletin*, No. 15; Memoirs, 1, Pt. 1, 1909.

[2] St. John, *loc. cit.*

This method is employed daily for the study of the magnetic polarity of sun-spots with the aid of the 75-foot spectrograph of the 150-foot tower telescope on Mount Wilson. A 12-inch (30.5 cm) visual objective of 150 feet focal length, mounted just below the second mirror of the coelostat near the summit of the tower, produces at the base of the tower an image of the sun about 43 cm in diameter, any portion of which may be brought upon the slit of the spectrograph below by means of the electric motors that control the slow motions of the coelostat and second mirror. The massive circular head of the spectrograph, which carries the slit in a horizontal plane, can be rotated to any desired position angle. After passing through the slit, the light from the sun-spot, or other portion of the solar image, descends vertically into a subterranean well, 10 feet (3.0 m) in diameter, with concrete walls. The collimating lens of 6-inch (152 mm) aperture is 75 feet (22.9 m) below the slit. This lens and the large Michelson grating below it are mounted on a heavy support at the bottom of the well, where the temperature is essentially constant throughout the year. After falling on the grating, the light is returned through the collimating lens, which forms an image of the spectrum at a point near the slit of the spectrograph. Here it can be observed visually, or photographed if desired. There is no connection (other than the walls of the well) between the head of the spectrograph, which carries the slit, and the support for the collimator-camera lens and grating at the bottom of the well. When the slit is rotated in position angle, the grating support is rotated through the same angle by means of an electric motor, thus bringing the lines of the grating again into parallelism with the slit. All of the necessary adjustments, including the rotation of the dome at the summit of the tower, the slow motions of the coelostat and second mirror, the focusing of the sun's image on the slit, the focusing of the collimator-camera lens, the inclination of the grating to bring different orders of spectra into view, and the rotation in azimuth of the grating support, are accomplished by electric motors controlled by push buttons near the hand of the observer.

Most of the observations, both visual and photographic, of the Zeeman effect in sun-spots are made in the second-order spectrum, where the linear dispersion at the region in question is 1 A=2.96 mm. For the daily determination of spot polarities the sharp iron triplet λ 6173.553 is usually employed.

In work of this nature, as already stated, it is necessary to use a Nicol prism, which is supported just over the slit of the spectrograph, and surmounted by a compound quarter-wave plate, so constructed that the principal sections of the successive mica strips (2 mm wide) are normal to one another. The Nicol prism, 130 mm in length, 18 mm high, and 10 mm wide (effective width 5 mm), was built by Werlein (from four sections each 32.5 mm long) for use with the 75-foot spectrograph (see Plate III*a*, *Contribution* No. 71). The compound quarter-wave plate, there shown swung to one side, can be turned into position above the Nicol. In view of the considerable focal length of the spectrograph, the distance of the quarter-wave plate from the slit (65 mm) is not sufficient to reduce materially the sharpness of the dividing lines between adjoining strips of spectra. For the study of plane polarization phenomena, a compound half-wave plate is substituted for the compound quarter-wave plate. Circular half-wave and quarter-wave plates, mounted so that they can be rotated in position angle, are also used for special purposes.

With this apparatus the magnetic polarities and the strength of the magnetic field in all sun-spots are recorded daily (see Fig. 1). A sheet of paper is placed in the focal plane of the telescope and all spots visible are traced in outline upon it. The polarity of each spot is then indicated by a "V" or an "R," according as the violet

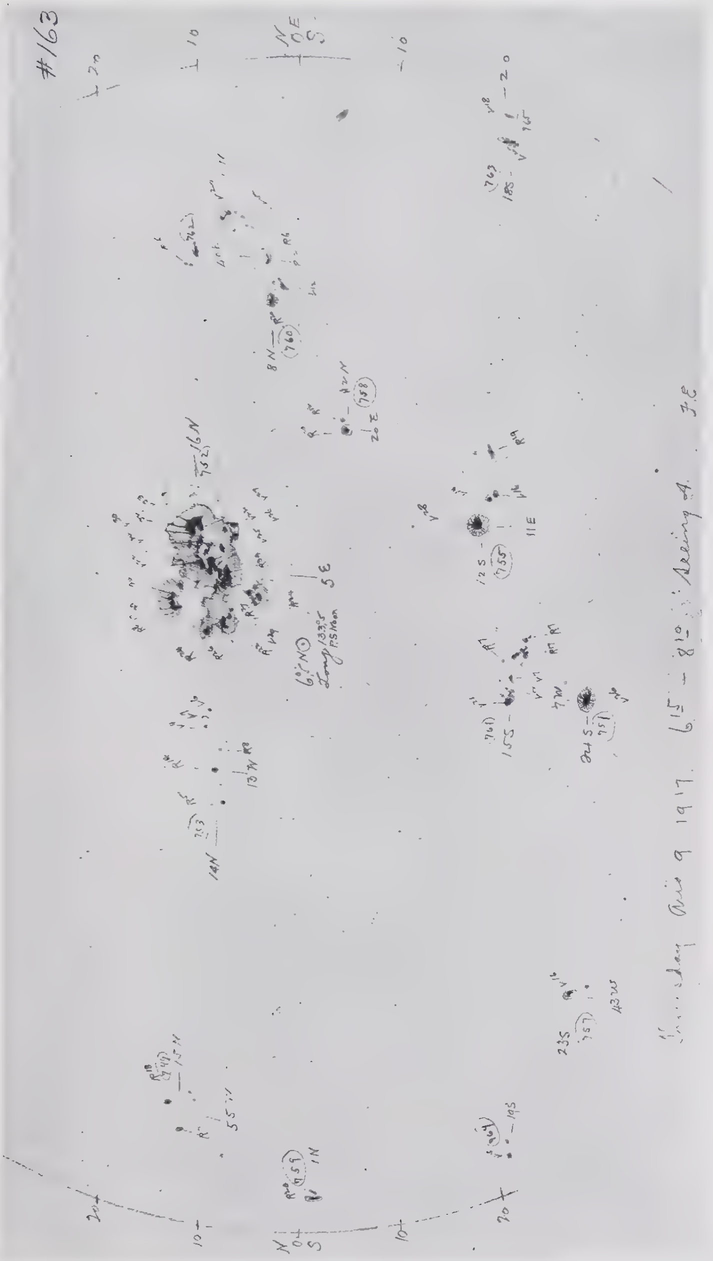


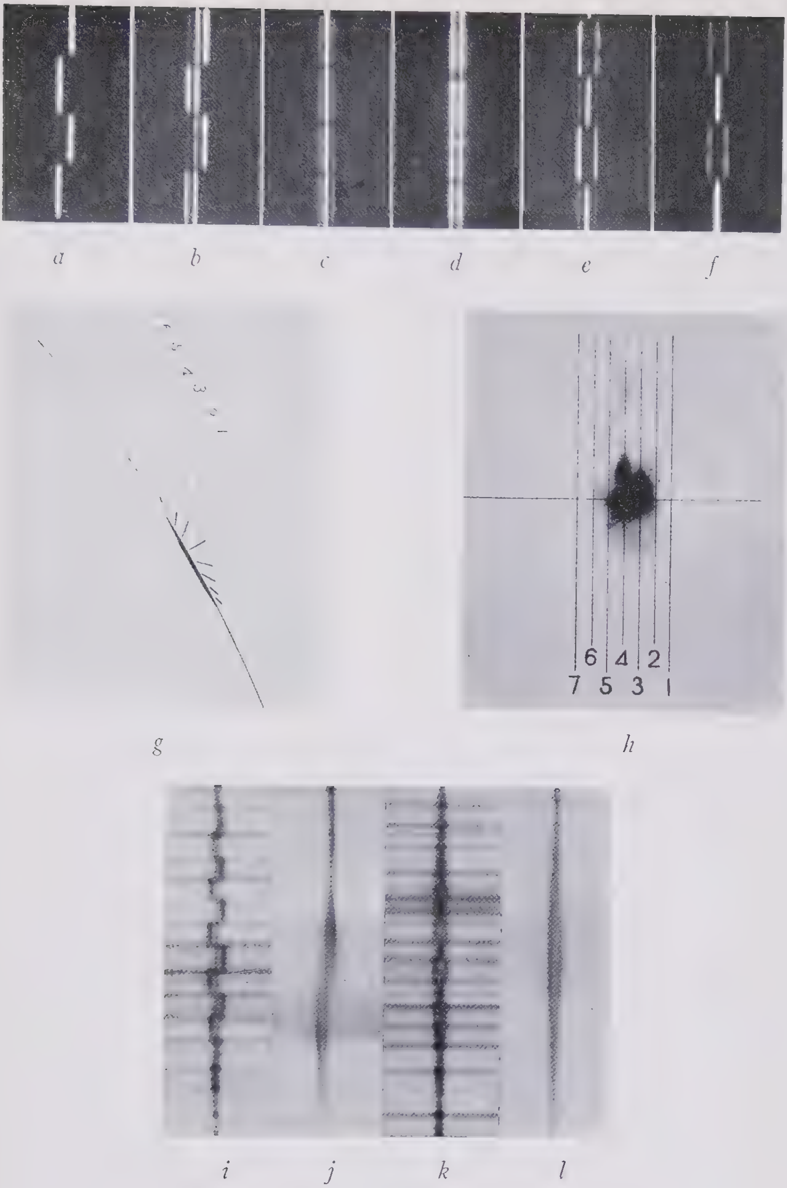
FIG 1.—Reproduction of sketch showing daily record of observations of sun-spot polarities

or red component of the Zeeman triplet is transmitted by a given strip of the quarter-wave plate. As the diameter of the focal image of the sun is 43 cm, it is possible in this way to observe very small spots. In addition to the record of polarities, the strength of the field in each spot is measured by means of a parallel-plate micrometer. The scale of this micrometer has been carefully calibrated by Ellerman and Nicholson, and when used to measure a triplet whose separation for a given field-strength has been determined in the laboratory its readings are readily convertible into gausses. In the case of the sharp triplet λ 6173, the *n*-components may still be distinguished separately in sun-spots when the field-strength is as low as 1000 gausses. For greater field-strengths the observed values are therefore fairly reliable, but below this limit they must be regarded as merely approximate. The readings with the parallel-plate micrometer are recorded in degrees; and since for λ 6173 one degree corresponds approximately to 100 gausses, the designation "V 28" (see Fig. 2) at a certain point on a sun-spot indicates that the polarity at this point corresponds to that of the north magnetic pole of the earth, and that the field-strength is about 2800 gausses. "R 7" would mean that the polarity corresponds to that of the south magnetic pole of the earth, while the field-strength is of the order of 700 gausses.

THE DETERMINATION OF SUN-SPOT POLARITIES; INCLINATION OF THE LINES OF FORCE

The determination of the magnetic polarity of a sun-spot depends upon an estimate of the relative intensities of the red and violet components of a Zeeman triplet. The zinc triplet λ 4680 in the spectrum of a spark between the poles of a large Weiss magnet, when photographed with a Nicol prism and compound quarter-wave plate at angles of 0°, 60°, and 90° with the lines of force, presents the appearance shown in Plate V*a*, *b*, and *c*. When the observation is made parallel to the lines of force, the *p*-component of the triplet is absent and one of the *n*-components is completely cut off. At right angles to the lines of force the *p*-component is twice as strong as the *n*-components (in the case of a normal triplet), while the two *n*-components are of equal intensity. At intermediate

PLATE V



a, *b*, *c*, Observations of the zinc triplet λ4680 in the physical laboratory, with compound quarter-wave plate in front of the slit of the spectrograph; inclinations of line of sight to lines of force are 0°, 60°, 90°, respectively; *d*, *e*, *f*, the same, with compound half-wave plate substituted for the quarter-wave plate; compare with Fig. 3 and *i* and *k* of this plate.

g, Lines of force in a sun-spot near the west limb on October 1, 1915; *h*, photograph of the same spot; the numbered lines indicate successive positions of the slit of the spectrograph used to determine the inclination of the lines of force.

i, *j*, *k*, *l*, Iron triplet λ6173 photographed in spectra of various sun-spots; *i*, *j*, circularly polarized light, spot near center of sun, observed with Nicol and (*i*, compound; *j*, single) quarter-wave plate; *k*, *l*, plane polarized light, spot near limb, with Nicol and (*k*, compound; *l*, single) half-wave plate; *j* shows reversal of polarity in two adjacent umbrae of large spot-group of August, 1917.

angles the plane polarized p-component is of intermediate intensity, while the elliptically polarized n-components are no longer completely cut off by the Nicol prism and quarter-wave plate. The relative intensities of the three components of a normal triplet for one set of alternate strips, when observed at any angle from 0° to 180° with the lines of force, are given by

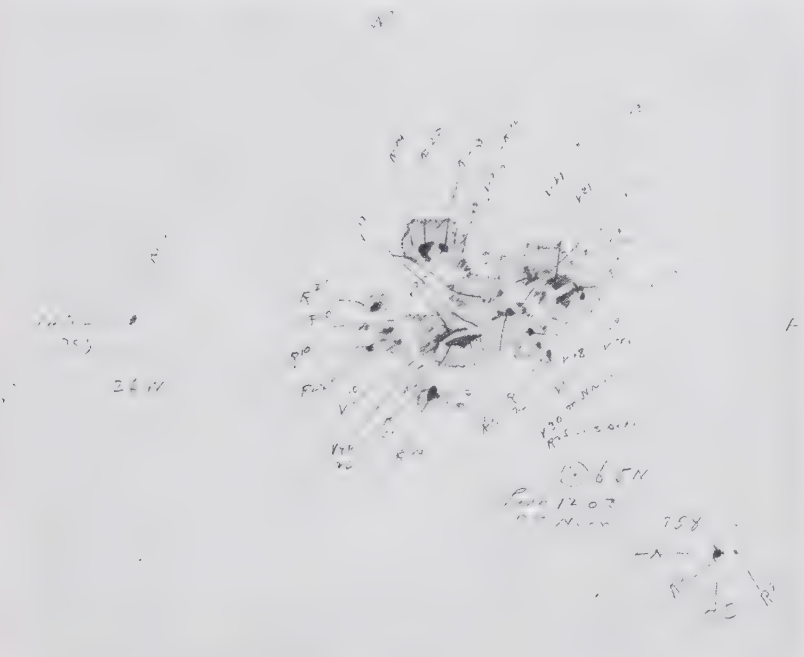


FIG. 2.—Sketch of large sun-spot group from the daily record for August 10, 1917, showing the manner of indicating the polarity and field-strength at different points in the group.

$$n_V=\tfrac{1}{4}\,(1-\cos\gamma)^2 \quad p=\tfrac{1}{2}\sin^2\gamma \quad n_R=\tfrac{1}{4}\,(1+\cos\gamma)^2$$

where n_V, p, and n_R are the intensities of the violet, central, and red components, respectively; γ is the angle between the line of sight and the lines of force, and the unit of intensity is such that $1=n_V+p+n_R$.[1] The variation of intensity in the three components with the angle γ is illustrated by Fig. 3. We thus have a means of determining the angle between the line of sight and the lines of force in observations of sun-spots. The important bearing of this on polarity determinations will shortly appear.

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[1] Seares, *Mt. Wilson Contr.* No. 72, p. 5; *Astrophysical Journal*, **38**, 99, 1913.

Repeating the foregoing observations, at the same angles of 0°, 60°, and 90°, but substituting the compound half-wave plate for the compound quarter-wave plate previously used with the Nicol, we have the appearances illustrated in Plate V*d*, *e*, and *f*. Parallel to the lines of force, when no plane polarized light is present, we naturally find that the half-wave plate and Nicol are without effect on the circularly polarized *n*-components. At 60°, where the plane polarized *p*-component is absent and the *n*-components are elliptically polarized, the appearance is as shown in Plate V*e*, the central component being completely cut off by alternate half-wave strips, while the *n*-components are materially reduced in intensity.

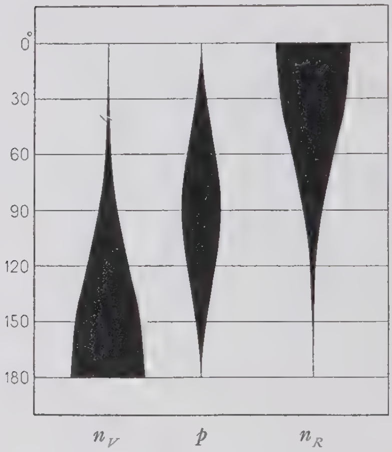

FIG. 3.—Relative intensities of the three components of a normal Zeeman triplet for inclinations of 0° to 180° between the line of sight and lines of force when observed with a Nicol prism and quarter-wave plate placed in front of the slit of the spectrograph. Thus for 0° the relative intensities are 0, 0, 1, respectively; for 90°: 0.25, 0.50, 0.25; for 180°: 1, 0, 0.

Consider now the bearing of these observations on the determination of the polarity of a sun-spot when close to the sun's limb. Let us assume that the lines of force at the center of the spot are parallel to a radius from the sun's center passing through this point, and that at other parts of the umbra and penumbra the lines of force are inclined approximately as indicated in Plate V*g*. This drawing relates to a spot photographed near the west limb of the sun on October 1, 1915, and reproduced in Plate *Vh*, where the scale is that of the image of the sun at the focus of the 150-foot tower telescope. The triplets λ 6302.709 and λ 6301.718 were observed at various parts of the spot, the successive positions of the slit on the image of the spot being indicated on the illustrations by the lines numbered 1 to 7. The observations were made visually by Hale in the second-order spectrum of the 75-foot spectrograph, using the Nicol and the compound quarter-wave plate over the slit. As these observations are typical of all fairly symmetrical spots when near the sun's limb, the phenomena observed at each position of the slit may be indicated here. Reference is made to the appearance of the triplet λ 6173.

1. Slight "zigzag" effect, red *n*-component (R) stronger than violet *n*-component (V). Diffuse.

2. Widened, diffuse. R stronger than V. Sharp at one point in spot, where *p* and R are separated. R stronger than *p*. V absent or very faint.

3. Wide, diffuse. R stronger than V. Three components of triplet sometimes visible.

Between (3) and (4). *p* visible but faint. V strong and separated from *p*. R apparently absent.

4. More diffuse. *p* hardly separated from V, which is very strong. Marked "zigzag."

5. Similar, but less "zigzag." Diffuse.

The chief point to be noticed is that, whereas the red *n*-component (R) was the stronger on the side of the spot toward the limb, the violet *n*-component (V) was the stronger on the opposite side of the spot. In other words, opposite polarities were indicated by the observations on the opposite sides of the spot. This is easily explained when the inclination of the lines of force at various points in the spot is considered.

As already stated, Plate V*h* represents the spot at the point of observation near the west limb of the sun. The direction of the lines of force corresponding to the various parts of the spot is hypothetically indicated in Plate V*g*. Thus when the slit occupied positions (2) and (3) the lines of force were directed away from the observer, and *R* was stronger than *V*.[1] Between (3) and (4) the *n*-components became of equal intensity, indicating that the line of sight was nearly at right angles to the lines of force. In subsequent positions of the slit the violet *n*-component, which had been weaker than the red component in positions (2) and (3), was invariably the stronger, showing that the lines of force from the inner side of the spot were directed toward the observer. Thus the true polarity of a sun-spot, corresponding with that given by observations made on the umbra when the spot is near the center of the sun, may also be determined near the limb by observing that portion of the penumbra which lies toward the center of the sun.

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[1] Assuming the lines of force at the center of the spot to be directed radially outward from the sun's center.

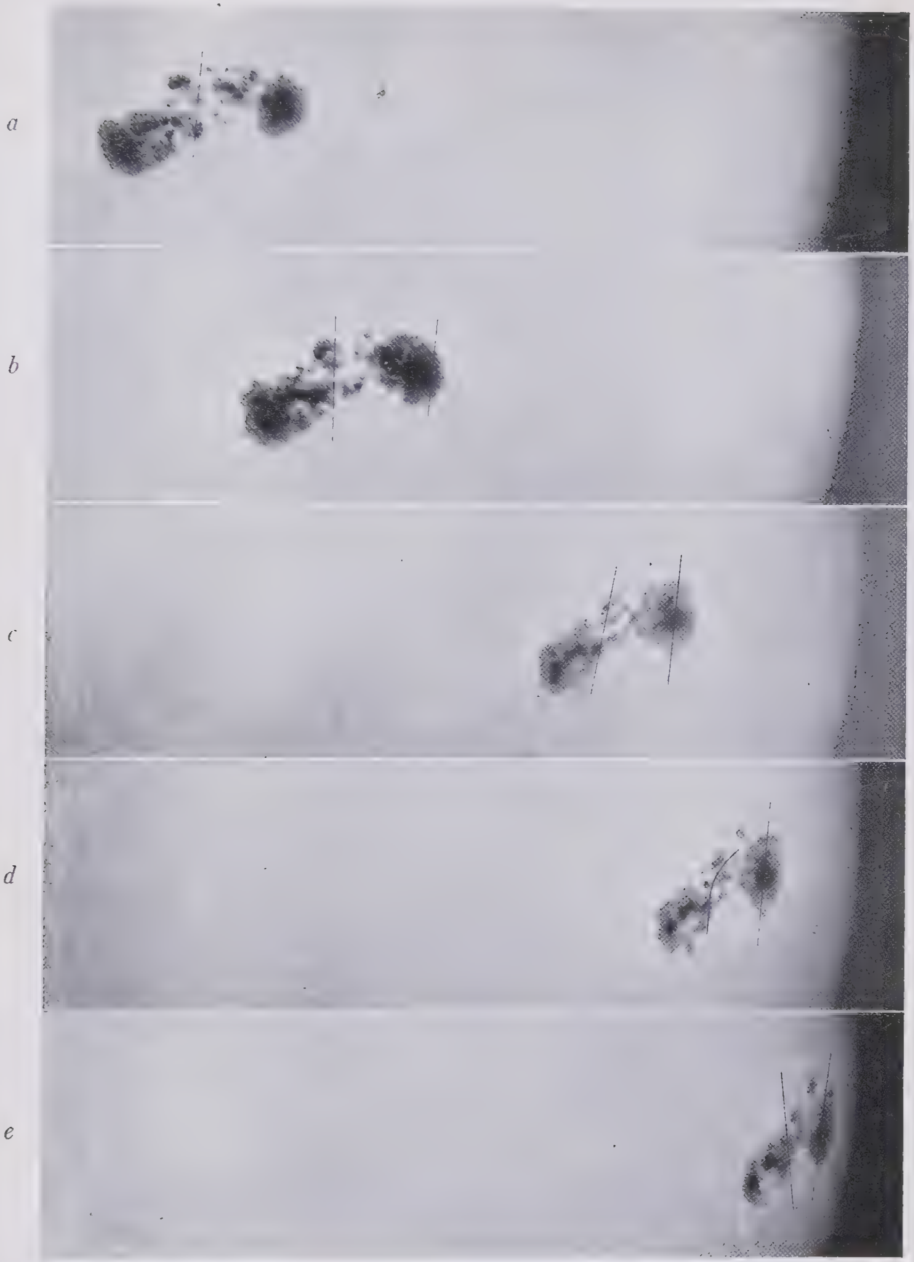
The observations indicate that at some position of the slit between (3) and (4) the lines of force were nearly at right angles to the line of sight. At this point the compound half-wave plate gave strong evidence of plane polarization (Plate V*k*), while the quarter-wave plate indicated the same result by showing that maximum intensity shifted from *R* to *V*.

The position on the spot occupied by the slit when the red and violet components of a Zeeman triplet are of equal intensity may be called the "neutral line" (see Fig. 3). By determining the position of this line in a large number of spots we may obtain data which will enable us to fix, with some precision, the average inclination of the lines of force in a plane passing through the center of the sun and the line of sight.

The photographs of a large bipolar spot group made on February 9, 10, 12, 13, 14, 1917, and reproduced in Plate VI, will serve to illustrate how the position of the neutral line changes with the longitude. Observations were made on both of the principal spots as the group was carried by the solar rotation toward the west limb from a point near the center of the sun. One day's observations are missing, because of clouds. The gradual displacement with longitude of the neutral line illustrated by these observations is perfectly typical, and may be seen in any spot of sufficient size as it is carried by the solar rotation away from the eastern or toward the western limb.

Irregularities in the form of spots, the presence of small components opposite in polarity to the chief umbra, and other magnetic peculiarities such as are noted below, frequently complicate the observations and affect the form and position of the neutral line. But its average shift with longitude, determined from observations of a considerable number of spots, gives the approximate inclination of the lines of force in various parts of the umbra and

PLATE VI



Photographs of Large Bipolar Sun-Spot Group

a, February 9; *b*, February 10; *c*, February 12; *d*, February 13; *e*, February 14, 1917, showing change in position of the neutral line with change in longitude of spot.

penumbra. Thus we find, from 105 observations of 61 spots made by Joy and Nicholson, the positions of the neutral line and the corresponding inclination of the lines of force (taken as normal to the lines of sight passing through the neutral line) given in Table I.

The position of the neutral line is given in tenths of the spot's radius, measured from the center of the spot toward the limb. In the average spot of moderate size the radius of the umbra is 0.4 that of the penumbra.

TABLE I

INCLINATION OF LINES OF FORCE

Longitude	Position of Neutral Line	Angle Between Lines of Force and Solar Radius
30°–40°	8.0	55°
40 –50	7.0	45
50 –60	4.7	35
60 –70	3.2	25
70 –90	2.0	10

These results relate to the lines of force that lie in a plane passing through the center of the spot, the center of the sun, and the line of sight. They do not show, however, the inclination of the lines of force in the plane at right angles. These can be very accurately determined by a method first applied by Nicholson, as follows:

When light from a Zeeman triplet is examined in a direction normal to the lines of force the vibrations producing the *p*-component are parallel to the lines of force. When a sun-spot is observed near the sun's limb the lines of force lying in a meridional plane through the center of the spot are nearly normal to the line of sight, and their inclination to the north or south can be found by determining the direction in which the light of the *p*-component is vibrating. The simplest way to do this would be to pass the light through a Nicol and observe in what part of the spot the *p*-component is completely extinguished for a given position of the Nicol. For that region the lines of force would be parallel to the long diagonal. A modification of this method has been used because the grating of the 75-foot spectrograph almost completely plane-polarizes the light in the region generally observed, with the result that for certain positions of the Nicol it would be almost completely extinguished. The difficulty could be avoided by leaving the Nicol fixed with reference to the slit and rotating the whole spectrograph. In practice the same result is accomplished by using a half-wave plate placed over a fixed Nicol. The half-wave plate is mounted on a scale calibrated in degrees so that the angle between its principal axis and that of the Nicol can be observed. By observing this angle the direction of vibration of the *p*-component projected on a plane perpendicular to the line of sight can be determined in all parts of the spot. Since the plane through the sun's axis and the center of the spot is nearly perpendicular to the line of sight, observations of points in the spot lying in the plane give directly, with good approximation, the inclination of the lines of force toward the north or south. The results of 24 observations by Nicholson on three typical unipolar spots are as follows, the unit of distance, as before, being one-tenth of the radius of the spot.

Distance from Center of Spot	Angle Between Lines of Force and Solar Radius
0	0°
3	18
6	49
9	73

The inclinations found by the two methods are shown in Fig. 4, where the ordinates are angles between the lines of force and the solar radius and the abscissae are distances from the center of the spot. The results of the first method are indicated by crosses and of the second by circles. Considering the difficulties and uncertainties of observation, the agreement is excellent.

Additional information can be obtained by determining the relative intensities of the *n*- and *p*-components of a Zeeman triplet in different parts of a spot observed near the center of the sun. The results of such observations vary among themselves, but in many spots the *p*-component is strong even at the center of the umbra when the spot is near the center of the sun (see Plate V*i, j*). This investigation, the results of which are not essential in the present study of polarities, is being continued by Nicholson.

By taking into account the position of the neutral line and avoiding the observation of spots at the extreme limb of the sun (where the lines of force along the line of sight are frequently not visible at the outer edge of the spot), the polarities of sun-spots may be determined with certainty. It is to be understood that for this purpose a quarter-wave plate is always used with a Nicol prism over the slit of the spectroscope. Serious errors in the determination of the relative intensities of the lines of spot triplets may enter, as Zeeman has pointed out, when the observations are made without polarizing apparatus. In the case of the 75-foot spectrograph, for example, the plane polarization due to the grating varies greatly for different wave-lengths and for different orders of the spectrum. It is therefore entirely insufficient to determine the relative intensities of the lines without complete knowledge of the polarization phenomena of the grating and also those of the tower telescope. With a Nicol and quarter-wave plate, however, the results of polarity observations are essentially unaffected by these causes.

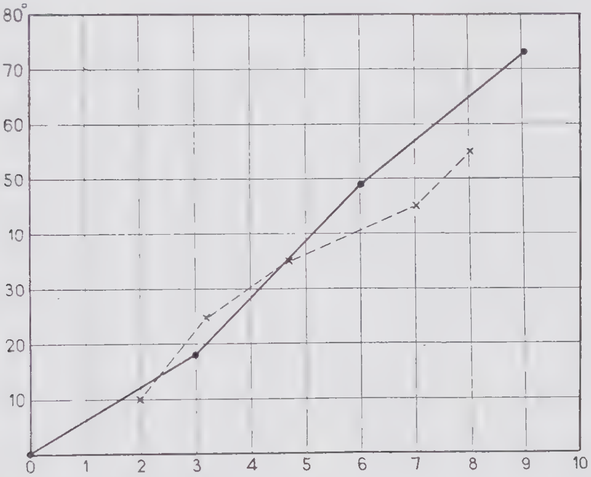


FIG. 4.—Inclination (ordinates) of lines of force to sun's radius at different distances from the center (abscissae, in tenths of radius of spot) of sun-spots. Crosses indicate results from observations of the position of the neutral line. Points are from measures of the orientation of the plane of polarization for the p-components in spots observed near the sun's limb.

BIPOLAR SUN-SPOTS

In Hale's first paper, "On the Probable Existence of a Magnetic Field in Sun-Spots,"[1] photographs were reproduced of two large spots, of opposite polarity, lying on opposite sides of the solar equator. Other cases which at first seemed to suggest a relationship between polarity and hemisphere were subsequently found. But it soon appeared that the simple law of terrestrial cyclones does not apply to the sun; for spots of opposite polarity were detected, not only in the same hemisphere, but also in the same spot-group.

Thus the large spot (Greenwich No. 6728) observed in the focal image of the 60-foot tower telescope on September 24, 1908, was seen to have at least three well-defined smaller umbrae, in addition to the principal umbra, within the boundary of the large penumbra. Photographs of the iron triplet λ 6303, made with the 30-foot spectrograph, showed the three small umbrae to have the same polarity, opposite to that of the principal umbra and penumbra. Many similar cases have been found, while cases of small companion spots, completely separated from a larger member of the group, and differing from it in polarity, are also common.

In one very remarkable instance, illustrated in Plate VII*a* and *b*, two large umbrae of the same spot, separated only by a bridge, were of opposite polarity, while the polarities of other members of the same group were as indicated in the sketch, Plate VII*c*, accompanying the photograph.

It fortunately happens, however, that cases of mixed polarity are not so common as to obscure our perception of a simple unifying principle leading directly to the detection of a general law of polarities. This has its origin in a well-known peculiarity of sun-spots, indicated in the records of the earliest observers, and commented upon by Carrington.

Sun-spots frequently occur in pairs, the principal members of which may be several degrees apart. The western or preceding member of such a group is often the first to be formed, but sooner or later a second spot, comparable with the first in size, but frequently smaller, or split into several components, is likely to appear behind it. Sometimes both members of the group appear simul-

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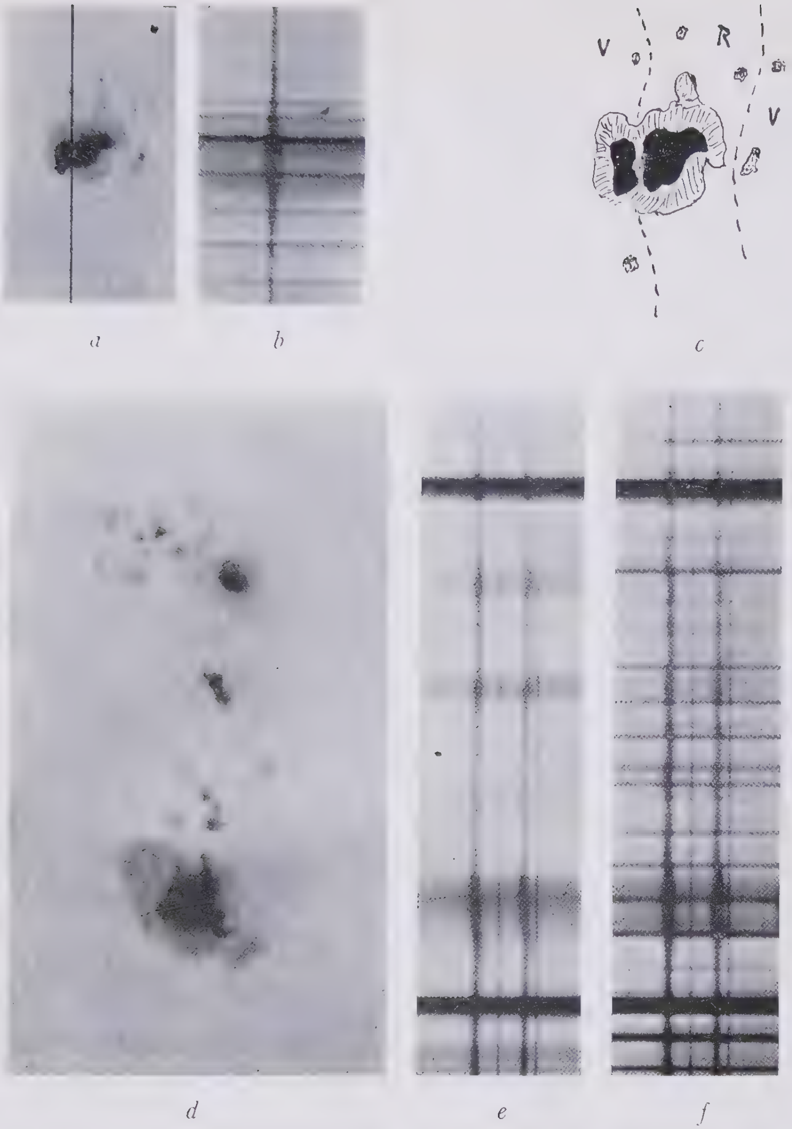
[1] *Mt. Wilson Contr.* No. 30; *Astrophysical Journal*, **28**, 315, 1908.

PLATE VII



a, Sun-spot photographed March 9, 1916, in longitude W. 23°, the umbra divided by a bridge; *b*, iron triplet λ6173 observed with Nicol and compound quarter-wave plate, the slit occupying the position shown by the line in *a;* the nearly equal intensities of the *n*-components in the *same* strip of spectrum indicate the presence of two fields of opposite polarities; compare with Plate V*i*, which shows the appearance when only a single field is present; *c*, sketch showing the distribution of polarities for other members of the group.

d, Bipolar sun-spot group photographed June 19, 1914; *e*, *f*, the iron lines λ 6301.7 and λ6302.7 as shown in the spectra of the three principal members of the group, with Nicol and (*e*, single; *f*, compound) quarter-wave plate; the configuration of the lines indicates that the two smaller spots are of the same polarity, but opposite to that of the largest spot.

taneously, and in other cases the following member is formed first. Many minor spots usually accompany the larger ones, either clustered about them or lying in the space between the principal spots. It is probably a significant fact that the axis of the group usually makes only a small angle with the equator.

TABLE II

INCLINATION OF AXIS OF SUN-SPOT GROUPS

Semicycle	Latitude						
	0°–4°	5°–9°	10°–14°	15°–19°	20°–24°	25°–29°	30°–34°
Min.–Max. 1856–1860	7°(3)*	5°(27)	10°(69)	7°(94)	9°(103)	8°(60)	13°(20)
Max.–Min. 1860–1867	1 (24)	3 (117)	4 (148)	6 (73)	9 (40)	8 (9)	14 (3)
Min.–Max. 1867–1871	4 (4)	5 (15)	5 (55)	6 (80)	6 (97)	9 (45)	8 (23)
Max.–Min. 1871–1879	7 (35)	3 (139)	6 (147)	6 (90)	8 (65)	10 (13)	10 (13)
Min.–Max. 1879–1884	7 (11)	2 (42)	4 (105)	3 (123)	9 (85)	9 (31)	14 (9)
Max.–Min. 1885–1889	3 (34)	−1 (72)	6 (110)	7 (43)	10 (7)	17 (2)	(0)
Min.–Max. 1889–1893	−2 (7)	0 (21)	6 (81)	6 (93)	10 (88)	10 (46)	10 (12)
Weighted mean inclination and number of spots	3.7 (118)	2.4 (433)	5.6 (715)	5.8 (596)	8.7 (485)	9.3 (206)	10.8 (80)

* Quantities in parentheses are numbers of spots.

A study by Joy of the sun-spot drawings of Carrington (1856–1861)[1] and Spörer (1861–1893)[2] shows that there is little change in this angle during the life of the group, but that in the mean the angle bears a definite relation to the latitude of the group. Twenty-six hundred and thirty-three bipolar and multiple groups, covering three and one-half sun-spot cycles, were examined. The following spot of the pair tends to appear farther from the equator than the preceding spot, and the higher the latitude the greater is the inclination of the axis to the equator. This relation holds for both hemispheres. The details are shown in Table II and the results are illustrated in Fig. 5.

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[1] *Observations of Spots on the Sun*, London, 1863.

[2] *Publicationen der Astronomischen Gesellschaft*, **13**, 1874; *Pub. Astrophys. Obs. zu Potsdam*, No. 1, 1878; No. 5, 1880; No. 17, 1886; No. 32, 1894.

In general, the angle of inclination was found to depend entirely on the latitude of the group, without reference to the number of the cycle or the time within the cycle. A knowledge of the polarities of the spots would have aided greatly in determining the position of the axes of the groups.

The most significant characteristic of these binary spot-groups lies in the fact that the two principal members, whether single or multiple, are almost invariably of opposite magnetic polarity. A photograph made in the second-order spectrum of the 75-foot spectrograph will serve to illustrate this point, and at the same time to indicate how these polarity phenomena may be recorded photographically. The long Nicol prism and compound quarter-wave plate are used over the slit and the spectrograph is rotated in position angle until the slit passes through the two principal members of the group. A single exposure on a wide Zeeman triplet such as λ 6173 or λ 6303 then gives the polarity of the two members, as shown in Plate VII. The spot-group reproduced in Plate VII*d* from a direct photograph made with the Snow telescope on June 19, 1914, consisted of a larger preceding spot and two smaller spots to the east of it, with various minor companions between them. Plate VII*e*, from a photograph of the spectrum made with the long Nicol and single quarter-wave plate, shows that the two smaller spots were of the same polarity, but opposite to that of the largest spot. Plate VII*f*, taken with the compound quarter-wave plate, not only

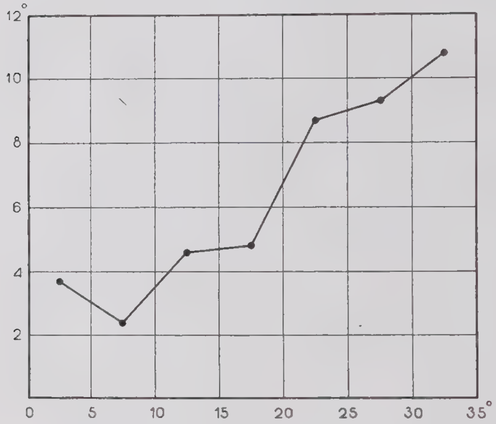

FIG. 5.—Summary of a statistical study of the sun-spot drawings of Carrington and Spörer showing the variation with latitude (abscissae) in the preferential inclination (ordinates) of the axis of bipolar sun-spot groups. In low latitudes the axes are nearly parallel to the sun's equator, but with increasing latitude the mean inclination increases to a maximum of about 11°.

confirms this result, but also serves to better advantage for the study of the weaker fields of the small companion spots lying between the preceding and following members of the group. It will be seen from this figure that magnetic displacements occur at points where no spots are shown, though faint indications of them appear in the original negative.

Analyzing these spectra, we find that there are apparently two "spheres of influence" of opposite polarity which meet near the center of the group. The first of these is dominated by the large preceding spot, while the other comprises the oppositely directed magnetic fields of the two following spots. Minor spots of opposite polarity sometimes occur, as already stated, within these "spheres of influence," but the generality of the bipolar effect is very strikingly shown by an examination of hundreds of spot-groups.

The preceding and following members of bipolar groups may be split into several components, often well separated, and small companion spots, of either polarity, may be present. Furthermore, the strength of the field and the area over which it can be detected may be very different for the preceding and following members of the group. But if magnetic fields of opposite polarity are distinctly shown by the preceding and following members of the pair, the essential characteristics of a bipolar group are present.

The tendency toward bipolar structure is so strongly marked that hardly more than 10 per cent of all spots observed are wholly free from it. In the case of spots which are apparently single, some traces of asymmetry, more or less suggestive of the structure of bipolar groups, can usually be detected. Sometimes such evidence of asymmetry is afforded by faculae following or preceding the spot. More commonly, however, especially in the central part of the sun, it is necessary to have recourse to spectroheliographic plates for the purpose of detecting the asymmetrical structure. In such photographs it is usually found that a single spot, or a group of small spots all having the same magnetic polarity, is near the preceding end of a mass of calcium flocculi elongated in a direction not greatly inclined to the solar equator. Much less frequently the spot occurs near the following end of such a group of flocculi. In about 10 per cent of all cases hitherto observed here the distribution of the flocculi is fairly symmetrical to the east and west of single spots.

Thus in the magnetic classification of sun-spots it is highly desirable to investigate the magnetic records in the light of information afforded by the distribution of the flocculi. The calcium flocculi serve admirably for the purpose just stated, but in order to study the characteristic vortex structure associated with bipolar spot groups we must have recourse to spectroheliograms of the hydrogen flocculi, taken under high dispersion with the light from the center of the Hα line.

In a later paper the characteristics of these bipolar groups will be discussed more in detail, and a variety of evidence bearing on their nature will be presented. Our immediate purpose is to present a scheme of magnetic classification and to bring out the fact that the recognition of this typical structure, even in the rudimentary form where the second spot is absent[1] and represented only by a train of flocculi, is essential to an intelligent discussion of the distribution of spots of different polarities in the northern and southern hemispheres of the sun.

The methods described above for recording polarities can of course be applied to the more complicated groups as well as to those of the simple bipolar type. A case of unusual complexity is illustrated and described in Plate VIII.

MAGNETIC CLASSIFICATION OF SUN-SPOTS

Our scheme of classifying sun-spots is based primarily upon the determination of their magnetic polarities. Supplementary evidence is frequently needed, however, and this is supplied, as already stated, by calcium and hydrogen spectroheliograms. Three classes of spots are included in the scheme: (α) unipolar, (β) bipolar, and (γ) multipolar. These may be subdivided as indicated below.

(α) *Unipolar spots.*—Single spots, or groups of small spots, having the same magnetic polarity. It should be noted, however, that

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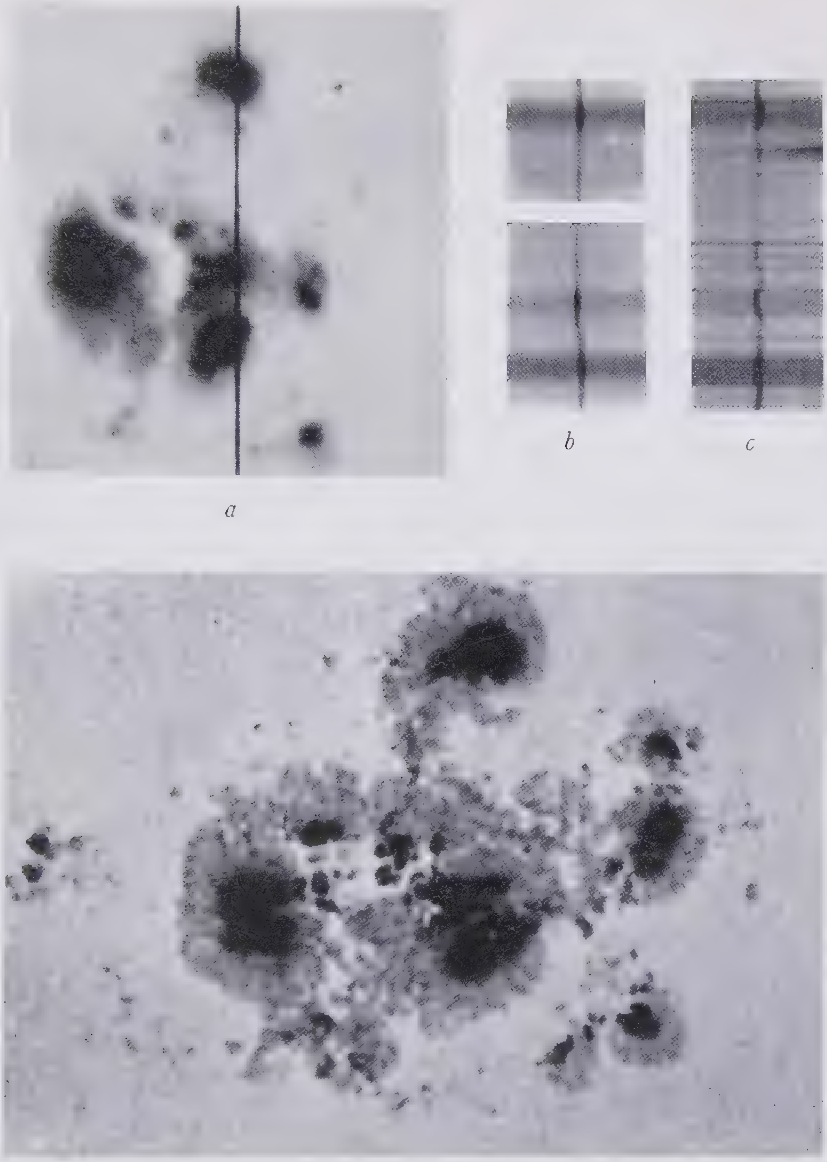
[1] In view of the fact that the general absorption in the spectrum of a spot increases greatly in passing from red to violet, it may perhaps become possible to photograph very faint spots, invisible to the eye, by using ultra-violet light of the shortest possible wave-length.

PLATE VIII



a

b

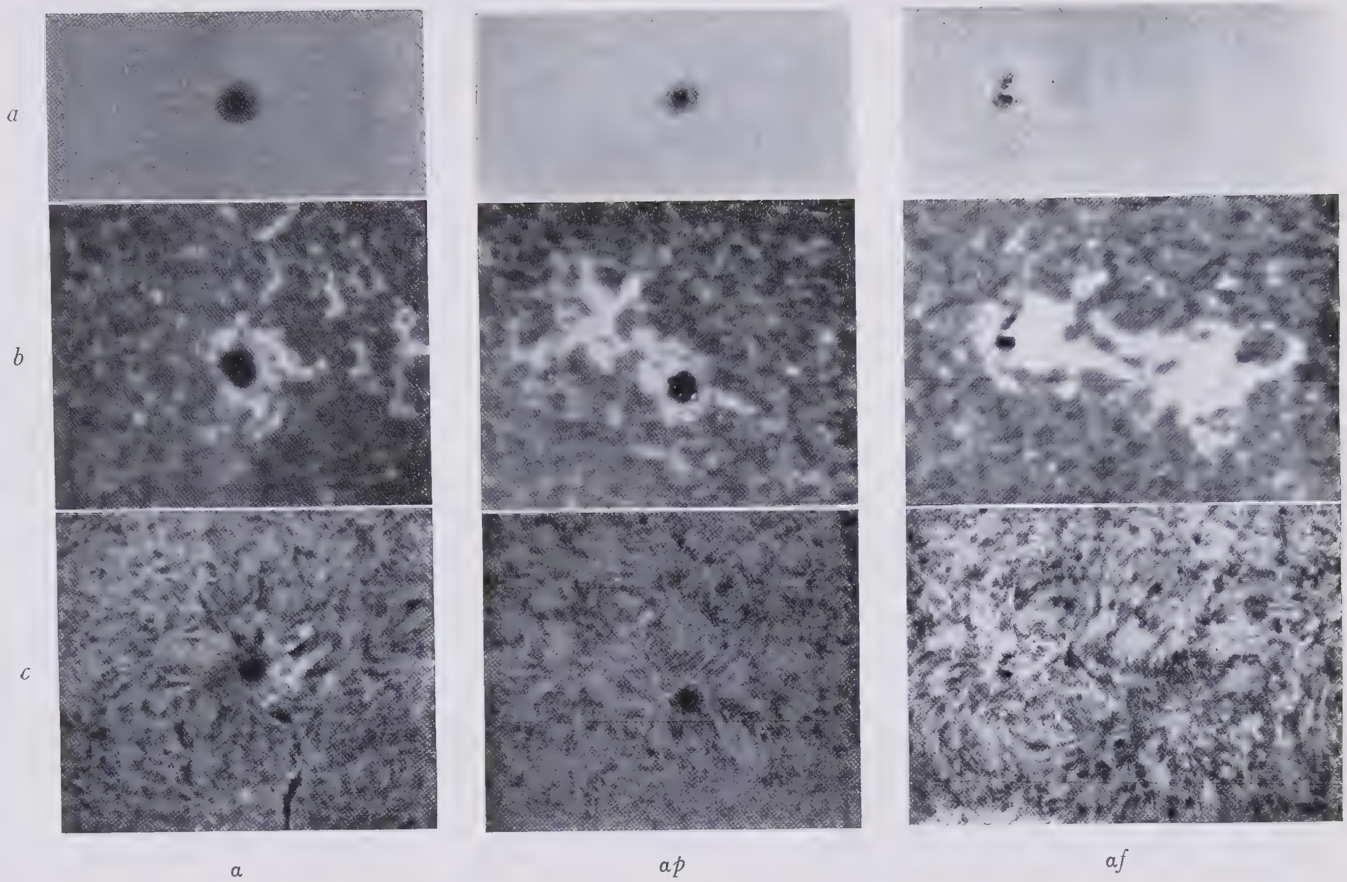
c

d

a, *b*, *c*, Photographic observations of a multipolar sun-spot group on August 8, 1917, similar to those described in Plate VII*d*, *e*, *f*, the spectral line in this case being the iron triplet λ6173; the lack of definition in *a* is due to poor seeing, together with the fact that it is a photograph of the image of the group formed by the 150-foot telescope, projected on a card mounted above the slit of the spectrograph.

d, An enlarged photoheliogram of the same group made with the 60-foot tower telescope on August 6, 1917, under good atmospheric conditions.

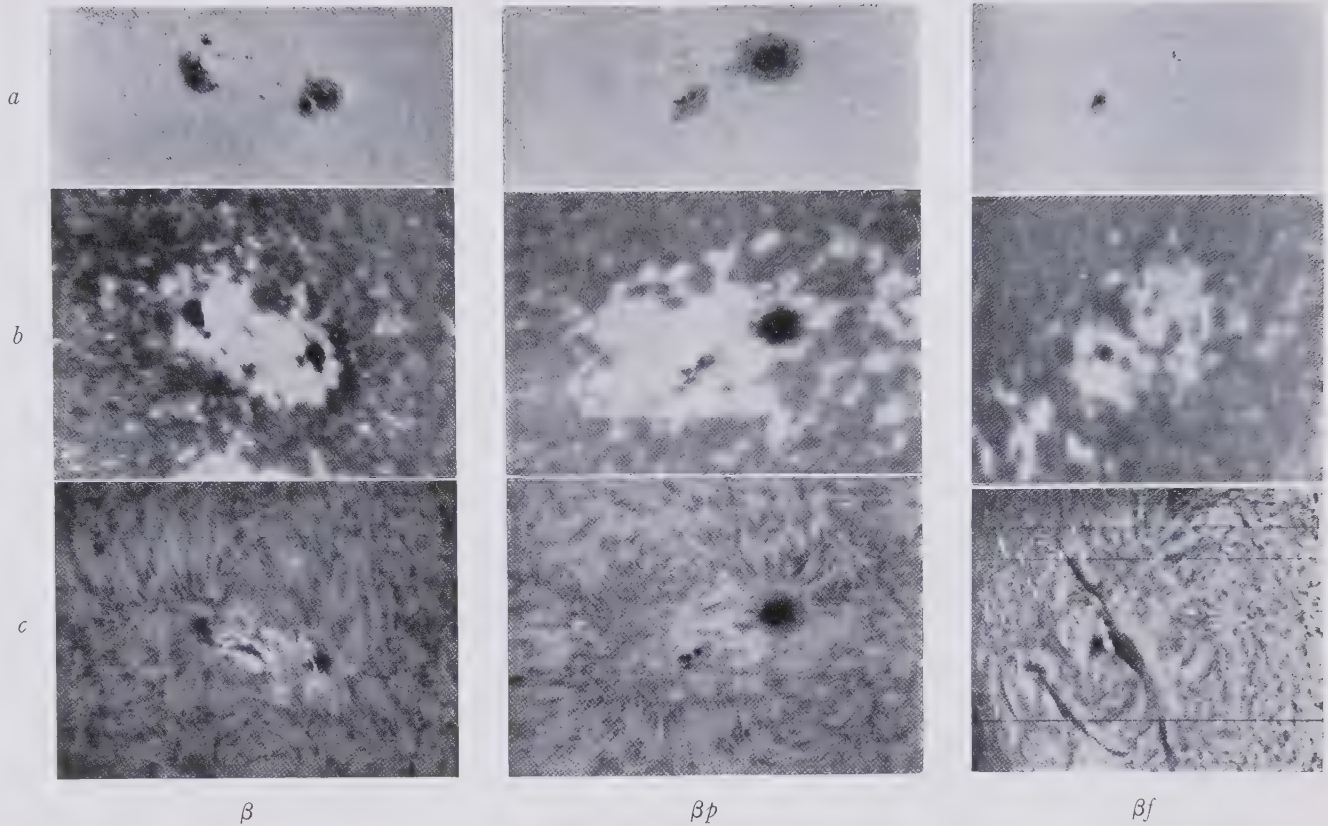
PLATE IX



Classification of Unipolar Sun-Spots

a, Photoheliograms; *b*, K_2 spectroheliograms; *c*, $H\alpha$ spectroheliograms; *a*, September 3, 1915; *ap*, September 15, 1914; *af*, September 15, 1914.

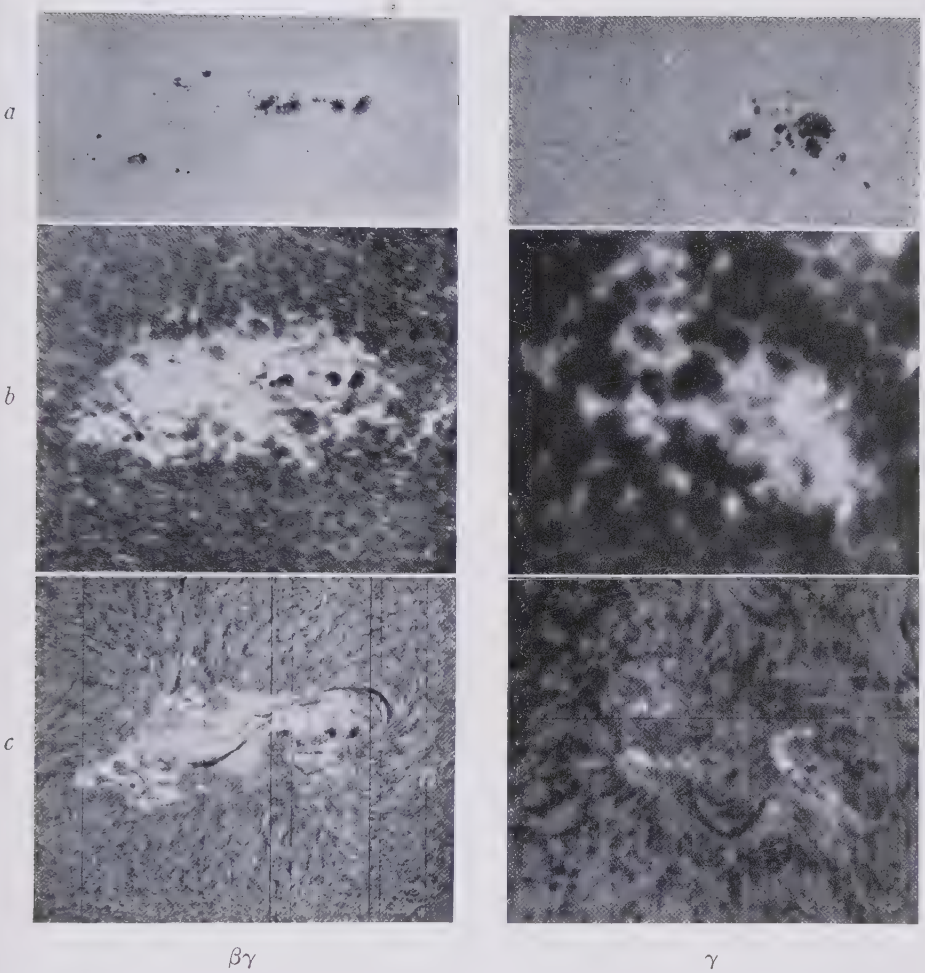
PLATE X



Classification of Bipolar Sun-Spots

a, Photoheliograms; *b*, K_2 spectroheliograms; *c*, Hα spectroheliograms; β, September 9, 10, 1909; β*p*, July 12, 1914; β*f*, April 22, 1916

PLATE XI



CLASSIFICATION OF MULTIPOLAR SUN-SPOTS

a, Photoheliograms; *b*, K_2 spectroheliograms; *c*, Hα spectroheliograms; βγ, an intermediate type, observed August 5, 1915; γ, July 25, 1917. Plate VIII*d* shows the photoheliogram of a large and unusually complicated γ group.

unipolar spots, and the chief components of bipolar groups, may occasionally have small companions of opposite polarity, which play such a minor and sporadic part in the group that they are disregarded in the classification.

Unipolar spots, whether single or multiple, may be divided into three groups, which are illustrated in Plate IX:

(α) Those in which the distribution of the calcium flocculi is fairly symmetrical preceding and following the center of the group.

(αp) Those in which the center of the spot-group precedes the center of the surrounding calcium flocculi.

(αf) Those in which the center of the spot-group follows the center of the surrounding calcium flocculi.

(β) *Bipolar spots.*—The simplest and most characteristic bipolar spot-group consists of two spots of opposite polarity. The line joining the two spots generally makes only a small angle with the solar equator. Each member of the group may be accompanied or replaced by many small spots, but the great majority of the spots constituting the preceding and following members of the group are of opposite magnetic polarity. One or more companion spots, of polarity opposite to that which characterizes the corresponding region of the group, sometimes occur in association with either the preceding or following member.

Bipolar spots may be divided into four groups, illustrated in Plates X and XI$\beta\gamma$:

(β) Those in which the leading and following members, whether single or multiple, are approximately equal in area.

(βp) Those in which the leading member is the principal member of the group.

(βf) Those in which the following member is the principal member of the group.

($\beta\gamma$) Those in which the preceding or following members are accompanied by minor companions of opposite polarity.

(γ) *Multipolar spots.* Groups of this character, comprising hardly more than 1 per cent of the total number of spots observed, contain spots of both polarities so irregularly distributed as to prevent classification as bipolar groups (see Plates VIII and XIγ).

MAGNETIC CLASSIFICATION OF 970 SUN-SPOTS OBSERVED DURING 1915–1917

While it is obvious that the changes that occur during the life of a spot must preclude a hard-and-fast determination of type, it may be interesting to give the preliminary results of a classification by Nicholson of the spots observed since our present method of recording was put into effect in 1915. In presenting the following figures it should be stated that they are provisional, and therefore subject to revision in the light of further study. Table III is self-explanatory, giving the number of spots observed in both hemispheres during 1915, 1916, and 1917, classified according to the system already explained.

Several interesting facts are brought out by this table. Most notable of these is the strong tendency toward the bipolar type, indicated not only by the large percentage of bipolar groups, but also by the small proportion of symmetrical unipolar spots. Another striking fact is the dominance of preceding spots, shown by the high percentage of βp spots as contrasted with βf spots, and also indicated by a similar preponderance of αp over αf spots. The constancy of the percentages for the successive years, and the very small number of multipolar (γ) spots, should also be noted.

The results of magnetic observation, revealing the great preponderance of bipolar groups, are closely in harmony with the conclusion reached by Father Cortie in his valuable paper "On the Types of Sun-Spot Disturbances."[1] After describing the various types he remarks:

> The chief type, however, of which the above mentioned are in most, possibly in all, cases but phases, is the double-spot formation, with a train of smaller spots between the two principal spots of the group, the whole group generally drifting into more or less parallelism with the solar equator. In this form the principal spot, which eventually becomes a normal spot of regular outline, is generally the leading spot, but in many cases it is the following spot, while sometimes the preponderance in area alternates between the two, as the group traverses the disk. In yet rarer instances both the chief spots develop as regular spots.

A discussion of the details of sun-spot formation and dissolution, and of the relationship between spot polarities and the structure of the accompanying hydrogen flocculi, must be reserved for another paper. Here it is our purpose to consider only such phases of these and other questions as bear directly upon the derivation of a law of sun-spot polarities.

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[1] *Astrophysical Journal*, **13**, 260, 1901.

TABLE III

MAGNETIC CLASSIFICATION OF 970 SPOTS

		α	αp	αf	β	βp	βf	$\beta\gamma$	γ	Unclassified	Total
1915	N.	10	17	5	17	29	3	4	3	11	99
	S.	7	16	4	21	22	6	4	0	3	83
	Total	17	33	9	38	51	9	8	3	14	182
	Percentages	9	18	5	21	28	5	4	2	8	
Total percentages			32			58					
1916	N.	22	26	4	49	63	12	7	0	8	191
	S.	31	28	8	17	37	13	6	0	7	147
	Total	53	54	12	66	100	25	13	0	15	338
	Percentages	16	16	4	20	30	7	4	0	4	
Total percentages			35			61					
1917	N.	25	54	2	66	62	17	9	3	3	241
	S.	16	41	7	53	69	11	7	0	5	209
	Total	41	95	9	119	131	28	16	3	8	450
	Percentages	9	21	2	26	29	6	4	1	2	
Total percentages			32			65					

SUMMARY (PERCENTAGES)

	α	αp	αf	β	βp	βf	$\beta\gamma$	γ	Unclassified
1915	9	18	5	21	28	5	4	2	8
1916	16	16	4	20	30	7	4	0	4
1917	9	21	2	26	29	6	4	1	2
Mean	11	18	4	22	29	6	4	1	5
Total number of spots	111	182	30	223	282	62	37	6	37

POLARITIES OF SPOTS IN THE NORTHERN AND SOUTHERN HEMISPHERES

For the purpose of deriving a law of sun-spot polarities we may regard unipolar spots (αp) followed by a train of flocculi as preceding spots of incomplete bipolar groups, and those (αf) which are preceded by a train of flocculi as following spots of such groups. During the last sun-spot cycle (observations from June 1908 to December 1912) it was found in the great majority of spots observed that the "marked strip" of the compound quarter-wave plate transmitted the violet component of preceding spots in the northern hemisphere and of following spots in the southern hemisphere.[1] It naturally follows, from the characteristics of bipolar groups, that during the same period the marked strip transmitted the red component of following spots in the northern hemisphere and of preceding spots in the southern hemisphere. The numbers of spots observed are given in Table IV.

TABLE IV

	REGULAR			IRREGULAR			RANGE IN LATITUDE	AVERAGE LATITUDE
	N V_p–R_f	S R_p–V_f	Total	N	S	Total		
Last cycle........... (June 1908 to December 1912)	7	17	24	0	2	2	18° to 3°	9°

Thus while the bipolar characteristic of sun-spots introduces an element not encountered in the case of terrestrial storms, the opposition in polarity north and south of the solar equator is analogous to the opposite direction of whirl, invariably observed in the case of northern and southern cyclones and tornadoes.

After the sun-spot minimum, which occurred in December, 1912, we found, to our surprise, that the polarity of the members of bipolar groups was opposite to that observed before the minimum. That is to say, the marked strip of the compound quarter-wave plate now transmits the red component of the preceding spots of bipolar groups in the northern hemisphere and the violet component of the preceding spots of bipolar groups in the southern hemisphere.

This sudden change was so remarkable that it was feared some observational error had been made. The results have been checked repeatedly by different observers, however, and in all cases the conclusion has been the same. The first observations of the former cycle were made with the 60-foot tower telescope and 30-foot spectrograph, but before the end of the cycle the 150-foot tower telescope and 75-foot spectrograph were put into commission, and this work was transferred from the old to the new tower. Thus the danger of any confusion due to the change of spectrographs and polarizing apparatus was eliminated, as the results obtained with the 150-foot tower telescope during the former cycle were in harmony with those secured with the 60-foot tower telescope during the same period. During the former cycle the observations were both visual and photographic, but since the beginning of the present cycle most of the polarities have been determined visually, though the results have frequently been checked photographically.

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[1] Few spots were observed during this cycle, because attention was then concentrated upon a small number of very large spots.

TABLE V

Present Cycle	Regular			Irregular			Range in Latitude	Average Latitude
	N R_p–V_f	S V_p–R_f	Total	N	S	Total		
1913, 1914.....	18	14	32	1	1	2	34° to 13°	22°
1915..........	73	68	141	2	0	2	29 " 2	19
1916..........	166	120	286	7	7	14	37 " 4	17
1917..........	221	193	414	8	6	14	30 " 1	14

The observations of the present cycle are given in Table V, which corresponds with Table IV except for the fact that in Table V the polarities are classed as "regular" when the marked strip transmits the red component in preceding spots of the northern hemisphere, whereas during the former cycle it transmitted the violet component in such spots.

In a preliminary paper describing the observed change of polarity at the last sun-spot minimum,[1] it was suggested that the explanation might be connected with the difference in latitude of the spots of the old and new cycles. The mean latitude of the spots observed during the old cycle was 9°, while that of the spots observed up to that date during the new cycle was 23°. It therefore seemed possible that there might be two zones on the sun, of low and high latitude, in which the polarities of spots were of opposite sign.

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[1] Hale, *Mount Wilson Communication*, No. 10; *Proceedings National Academy of Sciences*, 1, 385, 1915.

This view of the case, however, has not been borne out by time, as the results given in Table V indicate. The percentage of irregular spots has not increased materially during the present cycle, although their mean latitude has decreased from 22° in 1913–1914 to 14° in 1917. The maximum of solar activity has also definitely passed, so that it can hardly be supposed that any reversal of polarity will be observed before the close of the cycle, unless it should be found to lag considerably behind the maximum.

DISCUSSION OF RESULTS

The present investigation should ultimately lead to the formulation of a definite law of sun-spot polarities, from which the polarities of normal spots of any type can be predicted for either hemisphere and for any epoch in the sun-spot cycle. We already know that the preceding and following spots of binary groups, with few exceptions, are of opposite polarity, and that the corresponding spots of such groups in the northern and southern hemispheres are also opposite in sign. Furthermore, the spots of the present cycle are opposite in polarity to those of the last cycle (see Fig. 6, in which these results are expressed graphically). It is evident, however, that the formulation of a law of polarities cannot be undertaken until after the close of the present cycle, when it will be learned whether a reversal of sign is actually characteristic of the sun-spot minimum.

After an empirical law of spot polarities has been formulated, the problem of its interpretation will remain. The possibility of expressing such a law in the same terms that apply in the case of terrestrial storms depends upon the possession of data still beyond our reach. Thus the view that sun-spots are vortices depends upon the assumption that the magnetic field must be caused by electrically charged particles whirling in a vortex. Supposing that the existence of a true hydrodynamical vortex and the direction of the whirl within it can be determined by observations of the Evershed effect, or in some other way, the sign of the dominant charge will follow at once from our knowledge of the magnetic polarity of the spot in question. Or, if the sign of the charge can be independently found, the direction of the whirl will follow.

In Hale's first paper on the Zeeman effect in sun-spots, the sign of the dominant charge was found to be negative by proceeding on the assumption that the direction of whirl in the spot vortex coincides with that in the hydrogen vortex above it. A long series of observations of the hydrogen flocculi has since shown that this assumption is probably unwarranted. A good majority of the hydrogen whirls associated with preceding spots are right-handed in the southern hemisphere and left-handed in the northern hemisphere, as in the case of terrestrial storms, but there are many exceptions to this rule. Moreover, the hydrogen whirls showed no reversal of direction at the sun-spot minimum, and during the present cycle we have found cases in which either right-handed or left-handed hydrogen whirls are associated with spots of a given polarity. The whole subject is so complex that it will be advantageous to postpone further discussion until the great mass of observational material at our disposal can be more exhaustively studied.

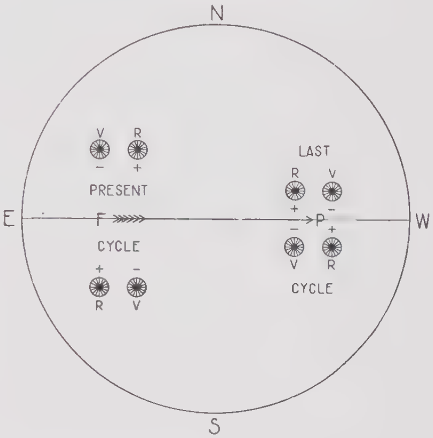

FIG. 6.—Diagram summarizing the results of polarity observations of sun-spots during the present and last cycles. The arrow indicates the direction of the sun's rotation; the letters *R* and *V*, the components of a normal triplet transmitted by the "marked strip" of the compound quarter-wave plate; and the algebraic signs, the distribution of the polarities between the preceding and following members of a bipolar group. Unipolar spots are normally of the same polarity as the preceding members of bipolar groups in the same hemisphere.

SUMMARY

1. The magnetic polarity of a sun-spot can be determined by observations, made with Nicol and quarter-wave plate, of the relative intensities of the *n*-components of a Zeeman triplet in its spectrum.

2. Such determinations of polarity can be made at almost any position on the sun's disk, but certain precautions must be taken to avoid error in the case of spots near the limb.

3. The inclination of the lines of force in sun-spots can be measured with considerable precision.

4. About 60 per cent of all sun-spots are binary groups, the single or multiple members of which are of opposite magnetic polarity.

5. Unipolar spots usually exhibit some of the characteristics of bipolar groups.

6. Before the last sun-spot minimum, the magnetic polarity of unipolar spots and of the preceding members of bipolar spots was positive in the southern and negative in the northern hemispheres of the sun.

7. Since the minimum these signs have been reversed.

8. The paper describes a scheme of classifying sun-spots on the basis of their magnetic properties.

9. The results of a magnetic classification of 970 spots observed during the years 1915–1917 are briefly summarized.

Mount Wilson Solar Observatory
November 11, 1918

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A FURTHER STUDY OF METALLIC SPECTRA PRODUCED IN HIGH VACUA

By EDNA CARTER and ARTHUR S. KING

Using the methods employed in a former investigation,[1] we have made a study of the spectra of manganese, titanium, magnesium, and cadmium, as produced by vaporizing the substance through the heating effect of a stream of cathode rays and exciting the vapor to luminescence by the bombardment of the cathode particles. The spectra thus obtained seem to be quite definite and characteristic of this mode of excitation. Additional observations of the calcium spectrum have confirmed the previous results, and for the iron spectrum a considerable number of lines has been obtained with a slightly different arrangement of circuit.

The form of the discharge-chamber is similar to that used in the previous work. In order to withstand the heat developed by the heavier discharge required to vaporize the more refractory metals, the parts formerly of glass were replaced by fused quartz or silica-ware. The glass bell-jar was superseded by an inverted crucible of silica-ware eight inches in diameter. Through the top a hole was bored into which a tube, *T*, Fig. 1, of opaque quartz was fitted by tapering it slightly at the end. Through this was passed another similar tube, which was fitted into a hole bored into a small quartz crucible, *B*, about two inches in maximum diameter. This flared around the cathode, *C*, and helped to concentrate the discharge. The aluminum cathode was melted by the heat, so it was replaced by a more massive one of copper faced with iron. This was connected by a copper rod with a copper receptacle, *E*, in which ice was placed. The joints were cemented with sealing-wax. In the large crucible two holes were bored opposite each other, into which tubes bearing quartz windows were set for viewing and photographing the luminescence spectrum just above the anticathode.

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[1] *Mt. Wilson Contr.* No. 125; *Astrophysical Journal*, **44**, 303, 1916.

The base-plate of $\frac{3}{4}$-inch glass was retained, protected from the heat by a $\frac{1}{4}$-inch aluminum plate, *D*, supported a short distance above it. An opening in the aluminum plate, covered by a quartz window, permitted a view of the discharge from below. The anode, *A*, was introduced through a quartz tube, which projected through the base-plate and the aluminum plate. The metal to be examined, *M*, was contained in a quartz tube supported in a base-plate, *F*, of the same material. The tube was slotted vertically at the upper end to expose the luminescent vapor, while also confining it. The metal was placed just below the slot, the quantity used depending upon the ease with which it could be vaporized.

The discharge was produced by a transformer with a rotary spark-rectifier. The evacuation was effected by two rotary oil pumps in series. In general, the metal, as soon as it began to vaporize, aided the evacuation by absorbing the gases in the chamber. Sometimes the absorption was so complete that it was necessary to leave a fine capillary tube open to the outside air in order to keep up the discharge. The iron vapor did not seem to have this absorbing power, so that it was difficult to obtain a satisfactory spectrogram of iron except by making the metal the anode. When a Fe-Mn alloy was used there was no trace of iron in the spectrum of manganese thus obtained, although a hole several millimeters deep was made in the metal at the focus of the rays.

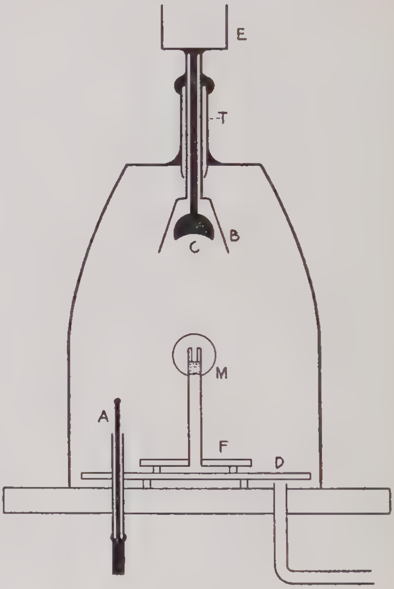


FIG. 1

The spectra were photographed for the most part in the first order of a 1-meter concave grating, the scale being 1 mm = 17 A. In order to detect any differences in the spectrum at different distances from the hot metal, photographs of the spectra of titanium, magnesium, and çalcium were also made with a quartz spectrograph, but no notable difference was found. The exposure time varied from fifteen minutes in the case of cadmium, with a discharge current of about 5 milliamperes and a parallel spark of 20–30 mm, to seventy minutes for titanium, the current being 9–12 milliamperes and the parallel spark 60–120 mm in length. It was imprudent to run this latter discharge more than ten minutes at a time on account of the warming of the sealing-wax joints.

MANGANESE

The number of manganese lines identified is the largest thus far photographed in luminescence spectra. The best spectrogram was obtained with a current of 8 milliamperes, the parallel spark-gap being about 5 cm. The lines which appear in the luminescence spectrum are listed in Table I, with their intensities in this source and for the furnace (at about 2200° C), arc, and spark. The intensities for the arc and spark are taken from the tables of Exner and Haschek, and, while on a different scale from the other estimates, they serve to show in which of these two sources a line is relatively strong.

A comparison of the luminescence lines with those of the three other sources serves to show the distinctive character of the luminescence emission. Among the lines relatively strong in the spark, only λ 4235 is notably strong in the luminescence. Of a group of nine enhanced lines from λ 3442 to λ 3498 only a trace of the strongest appears.

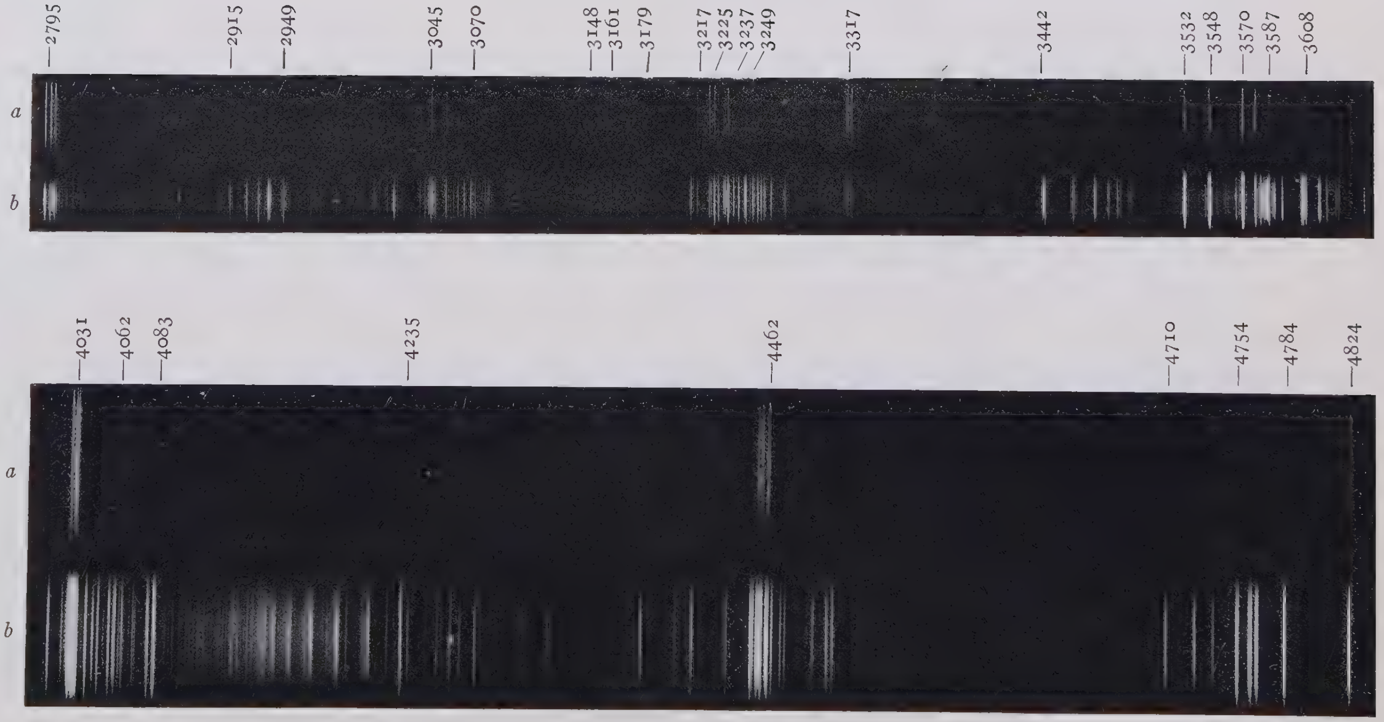
An inspection of Table I shows also many divergences of the luminescence spectrum from that of the furnace. A notable contrast is given by the moderately strong luminescence lines λλ 4058, 4060, and 4062, which do not appear in the furnace. The resemblance to the arc spectrum is closer, but this is found to result from the fact that the luminescence emission picks out certain arc lines and gives them with high intensity. That most of the luminescence lines appear also in the furnace is a consequence of the usual condition that most of the arc lines are given with greater or less intensity by the furnace.

TABLE I

MANGANESE

λ (Exner and Haschek)	Luminescence	Furnace	Arc	Spark
2576.20	6	....	4R	30R
2593.82	5	....	4R	15R
2605.78	3	....	4	10R
2704.00	2	....	5	....
2713.42	2	....	5	....
2726.22	2	....	5	....
2761.03	3	....	3	3
2776.25	1	....	2	....
2778.68	1	....	2	....
2782.85	1	....	2	....
2794.92	150	200R	50R	5R
2798.37	125	150R	50R	5R
2799.99	1	2	2	1
2801.20	100	100R	50R	5R
2802.90	1	1	2	1
2818.09	1	3	3	1
2858.85	1	1	2	....
2872.68	1	1	2	....
2914.71	3	5	8	1
2925.67	5	10	8	1
2930.35	1	1	2	....
2933.19	2	1	3	15
2939.40	3	1	3	20
2940.50	6	4	8	1
2949.31	5	1	3	30
3044.69	25	5	5	2
3054.53	8	8	4	2
3062.30	4	5	4	1
3066.19	5	5	4	1
3070.46	6	5	4	1
3073.33	5	5	4	1
3079.80	3	5	3	1
3081.52	1	4	3	1
3148.36	3	5	3	1
3161.19	4	6	3	1
3178.61	5	7	3	1
3212.98	1	10	2	2
3217.04	10	30R	1	1
3224.90	25	30R	2	1
3228.20	15	20r	3	3
3230.81	2	8	2	2
3234.0	2	1	2	....
3235.1	3n	1	2	....
3236.90	8	8	2	3
3237.4	4	3	2	....
3240.53	1	10	2	1
3240.75	1	7	2	1
3243.93	3	6	3	2
3247.80	1	....	1	....
3248.64	6	6	3	3
3253.09	2	7	3	2
3254.14	1	3	2	1
3256.25	3	6	3	2
3258.52	2	6	3	2
3260.40	1	6	3	2
3264.83	4	8	3	2
3278.65	1	4	2	1
3280.90	3	4	2	1
3297.01	2	4	2	1
3312.05	3	2	2	....
3313.41	5	1	2	....
3313.70	5	1	2	....
3314.59	5	2	2	....
3315.07	8	2	2	....
3316.61	4	1	1	....
3317.47	20	4	3	1
3320.82	4	4	2	1
3330.80	4	6	2	....
3442.13	1	....	2	30
3532.02	30	10	20	2
3532.15		15		3
3532.26		15		3
3547.93	40	15	10	4
3548.16		15	10	3
3548.30		10	3	3
3569.65	40	20	15	5
3569.96		15	10	4
3570.23		12	4	3
3576.25	5n	....	1	....
3577.99	20	15	10	5
3580.3	4n	....	1	....
3586.69	12	10	5	4
3595.29	3	7	4	2
3607.69	6	10	5	3
3608 66	8	10	5	3
3610.49	5	8	5	3
3619.42	5	6	4	3
3623.96	3	6	4	2
3629.89	1	4	3	1
3790.38	1	6	3	4
3800.70	1	....	2	2
3806.90	15	12	10	8
3809.75	2	8	5	6
3823.64	10	10	5	6
3824.03	1	6	4	4
3829.80	3	3	3	3
3834.01	2	6	4	4
3834.50	7	10	8	8
3839.90	1	5	4	5
3841.23	3	7	5	6
3844.12	1	5	3	4
3926.63	2	....	3	4
3943.05	1	....	2	2
3982.73	1	5	2	3
3985.40	1	1	2	3
3986.98	4	1	2	4
3987.26		1	2	4
4018.28	3	20	10	8
4030.92	150	200R	100R	20R
4033.21	100	150R	100R	20R
4034.62	100	150R	100R	10R
4035.88	1	15	5	8
4041.53	10	40	20	10
4048.90	3	15	8	8
4055 70	4	20	4	8
4058.10	6	....	3	2
4059.54	8	....	3	2
4061.90	10	....	4	3
4063.70	1	20	5	6
4079.38	3	10	3	5
4079.61		10	3	5
4083.11	2	10	4	6
4083.82	2	12	3	6
4147.71	1	....	2	2
4235.41	10	4	10	20
4239.90	1	1	3	5
4257.83	1	1	3	4
4266.10	2	1	3	5
4281.30	2	2	3	5
4436.52	1	1	8	5
4451.78	2	8	8	10

PLATE XII



(*a*) Luminescence and (*b*) arc spectra of manganese

TABLE I—*Continued*

λ (Exner and Haschek)	Luminescence	Furnace	Arc	Spark	λ (Exner and Haschek)	Luminescence	Furnace	Arc	Spark
4455.20	15	2	5	3	4766.63....	1	6	8	3
4455.51....			5	3	4783.62....	15	50	30	8
4456.02.....	5	1	5	3	4823.71....	15	50	30	10
4457.76.....	8	1	5	4	5341.25....	15	50	15	8
4458.48.....	15	2	6	5	5394.89....	1	40	8	1
4461.30.....	8	1	5	4	5420.61....	6	25	10	3
4462.20.....	40	3	10	8	5470.88....	5	15	10	2
4464.88.....	1	4	8	5	5481.61....	2	8	8	1
4754.24.....	10	50	30	8	5517.00....	2	10	10	2
4762.60.....	2	10	10	4	5537.99....	2	6	10	1
4766.08.....	1	4	6	3					

Two sections of the manganese luminescence spectrum, with that of the arc for comparison, are reproduced in Plate XII. While many of the luminescence lines are too faint to appear in the copy, the general effect of certain arc lines being brought out while other strong ones are absent may be clearly seen.

TITANIUM

The titanium luminescence lines are given in Table II, with the arc and spark intensities of Exner and Haschek for comparison. The results, considering the high melting-point of this element, show the possibility of studying the more refractory substances by this method. It will perhaps be sufficient to note briefly the points of difference from the furnace, vacuum arc, and the arc and spark in air, photographs of which were available for comparison. As for manganese, a decided selective action by the luminescence is apparent, many strong arc lines being absent or very faint. A comparison with the furnace spectrum, however, shows many resemblances. The prominent luminescence lines are strong also in the furnace and reverse at high temperature, their relative strength in a given region often being not greatly different in the two sources. There are exceptions, however, to this rule, and strong furnace lines are frequently lacking in the luminescence spectrum. There is an absence of even a slight approach to the spark spectrum, since the enhanced lines of titanium, many of which appear with considerable strength in the arc and can be obtained faintly even in the furnace, are either absent in the luminescence or among the fainter lines of this spectrum. The

TABLE II

Titanium

λ (Exner and Haschek)	Luminescence	Arc	Spark	λ (Exner and Haschek)	Luminescence	Arc	Spark
2641.20	1	2	2	3753.77	3	3	3
2644.35	1	2	3	3759.48	1	10	20
2646.71	1	2	2	3761.50	1	10	10
2942.12	4	3	4	3771.84	10	4	3
2948.37	4	3	2	3786.20	1	3	3
2956.27	4	2	2	3789.43	tr	2	2
2983.41	tr	2	2	3796.02	tr	2	2
3000.99	1	2	2	3798.44	tr	2	2
3186.58	10	8	3	3818.35	tr	2	1
3192.14	12	8	1	3836.90	1	2	....
3200.08	20	8	3	3846.54	1	2	2
3203.55	1	2	2	3921.61	2	5	2
3214.38	1	2	1	3924.71	10	8	3
3234.68	1	8	15R	3930.04	8	8	3
3236.72	1	5	6R	3947.98	10	10	3
3342.01	50	4	10R	3948.87	40	12	4
3342.30		3		3956.50	40	15	4
3349.17	6	3	8R	3958.39	60	15	5
3349.56		8	10R	3963.05	6	8	3
3354.80	60	8	3	3964.48	6	8	3
3358.56	1	2	....	3981.95	50	15	3
3361.41	8	10	30R	3982.63	2	8	3
3370.61	40	5	2	3989.94	60	10	6
3371.62	70	10	2	3998.80	70	20	6
3373.03	1	3	20R	4009.12	15	8	4
3377.70	15	10	3	4024.76	20	10	3
3379.35	10	3	1	4078.62	1	8	4
3382.47	3	3	1	4112.87	6	5	2
3383.90	3	3	20R	4122.30	1	3	2
3385.80	20	2	....	4123.70	1	3	2
3386.10		5	....	4127.67	2	3	3
3390.84	tr	2	2	4137.46	tr	3	2
3394.74	tr	2	10	4151.12	1	3	3
3444.49	tr	4	10	4171.20	tr	3	2
3505.06	1	3	30	4200.90	tr	2	2
3610.35	1	4	2	4282.87	2	5	3
3635.61	50	15	3	4286.19	1	10	4
3642.81	70	15	3	4289.26	1	15	4
3653.66	100	15	4	4291.19	1	10	2
3654.74	2	3	2	4295.93	1	10	4
3658.26	12	4	3	4298.89	1	12	4
3660.80	2	3	2	4300.73	4	15	2
3669.15	3	3	2	4301.24	5	15	3
3671.85	10	4	3	4306.09	6	20	8
3685.37	2	8	100	4314.96	10	5	3
3690.09	2	3	2	4318.85	1	10	3
3717.52	5	5	2	4321.89	tr	8	3
3722.73	7	3	3	4325.36	tr	8	3
3729.97	50	8	4	4326.54	tr	4	2
3741.25	70	15	2	4427.28	tr	10	4
3748.22	1	2	6	4430.52	tr	3	2
3753.00	100	15	5	4434.16	tr	5	3

TABLE II—*Continued*

λ (Exner and Haschek)	Luminescence	Arc	Spark	λ (Exner and Haschek)	Luminescence	Arc	Spark
4449.35	1	10	5	4599.40	tr	3	1
4451.13	1	8	3	4617.40	tr	10	8
4453.52	12	8	3	4656.63	25	8	3
4455.50	12	12	4	4667.77	25	10	5
4457.61	15	15	5	4682.10	30	10	6
4496.35	tr	10	3	4981.93	6	20	10
4512.90	1	15	4	4991.24	5	20	10
4518.19	2	15	4	4999.68	4	20	10
4522.98	2	15	4	5007.35	2	20	10
4527.47	1	15	4	5014.39	8	20	8
4533.40	20	20	5	5036.08	2	10	8
4534.95	10	15	4	5036.65		10	8
4535.71	40	8	3	5038.59	1	10	8
4536.12		10	2	5040.14	3	10	3
4536.25		10	2	5064.79	3	10	5
4544.88	1	10	3	5173.92	1	15	5
4548.93	1	8	3	5193.12	1	20	8
4552.70	1	10	4	5210.59	2	20	10
4555.70	1	10	3				

titanium vacuum arc also does not show differences from the ordinary arc in the direction of a resemblance to the luminescence spectrum. The effects point to a distinctive character in the emission resulting from the cathode bombardment.

MAGNESIUM

Magnesium metal was used as the anticathode, and spectra were obtained both for the usual arrangement with an insulated target and when it was connected as anode. The intensities of the lines under these conditions are given in Table III. The difference found was chiefly in the general intensity of the light, the anode connection giving a much stronger spectrum when the same length of parallel spark was used.

An interesting mixture of arc and spark lines is presented by these high-vacuum spectra. Six ultra-violet enhanced lines, λλ 2791, 2796, 2798, 2803, 2929, and 2937, are strong for both arrangements of the anticathode. These are placed by Fowler[1] in a series of "wide doublets." The much-studied enhanced line λ 4481, belonging to a different series, is just visible in the luminescence spectrum, and in the anode spectrum is much weaker than the adjacent arc lines. The ultra-violet doublets appear in the ordinary arc in air, while λ 4481 does not. It appears that in the high-vacuum spectra for magnesium we have a slight approach to the spark spectrum, but not as pronounced as can be obtained in the arc under special conditions, either in air at a few millimeters pressure, in hydrogen, or in the "tube-arc" formed when the furnace tube is burned apart at a low voltage. In these sources λ 4481 appears with great intensity. Without further data on the state of the electric field producing the high-vacuum spectrum, it is perhaps useless to speculate on what features may bring about this difference.

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[1] *Philosophical Transactions of the Royal Society*, A, **214**, 225, 1914.

TABLE III

MAGNESIUM

λ (Exner and Haschek)	Luminescence	Anticathode as Anode	λ (Exner and Haschek)	Luminescence	Anticathode as Anode
2630.52		1	2936.99	15	30
2633.13		2	2938.67	4	8
2646.61		tr	2942.21	6	15
2649.30		tr	3091.18	8	30
2659.5		1	3093.14	20	50
2668.26		2	3097.06	30	60
2669.84		6	3106.5		3
2672.90		8	3330.08	5	10
2695.53		1	3332.28	10	20
2698.44		2	3336.83	15	40
2732.35	1	5	3829.51	15	50
2733.80	5	10	3832.46	40	80
2736.84	8	12	3838.44	60	100
2765.47		2	3904.0	1	2
2768.57	1	5	3938.6	2	3
2776.80	3	12	3987.08	3	5
2778.36	2	10	4058.45	5	8
2779.94	10	15	4167.81	8	12
2781.53	3	10	4352.18	12	20
2783.08	2	10	4481.3	1	6
2790.88	10	20	4571.33	10	25
2795.63	80	80	4703.33	10	35
2798.07	15	20	4730.42	1	4
2802.80	60	60	5167.55	2	15
2846.91	1	8	5172.87		15
2848.53	2	10	5183.84	2	30
2852.22	500	500	5528.75		25
2915.57		tr	5711.38		2
2928.74	8	20			

The line λ 4571, faint in both arc and spark, is strong in the luminescence and in the anode spectra. This furnished the only point of similarity to the furnace spectrum, in which at low temperature λ 4571 dominates this region of the spectrum.

A high intensity of the Rydberg series of single lines is a notable feature of both of the high-vacuum spectra. The members in this list are λλ 5529, 4703, and λλ 4352–3987. Two other lines of this series appear, which have been observed by Fowler and Payn[1] in the vacuum arc. We have obtained these two lines also by prolonged exposure of the magnesium arc in air, the spectrum being formed by the 15-foot concave grating. Their wave-lengths were then measured as 3938.6 and 3904.0, close to the calculated values of the tenth and eleventh members of the Rydberg series.

Janicki and Seeliger[2] obtained several metallic spectra in a discharge at low pressure and observed the lines occurring near the anode and cathode respectively. Those in the positive column were the stronger arc lines, while in the cathode glow the spectrum contained also enhanced lines. Comparing their results for magnesium with ours, it is evident that the presence of enhanced lines in the luminescence spectrum and in that obtained with the anticathode connected as anode corresponds most nearly with conditions in the cathode glow spectrum of Janicki and Seeliger, even when we observed the vapor close to the anode. The vacuum employed by us was probably much higher than the vacuum they used, and the similarity to the cathode glow spectrum indicates that, with a high vacuum and the concentration of cathode particles which we have employed, the cathode emission is transferred to the focus of the stream at the anticathode. With magnesium the effect of the cathode bombardment at this point appears to be practically the same when the anticathode is also the anode.

CADMIUM

The cadmium luminescence spectrum shows a much closer similarity to that of the spark than was found for magnesium. The lines obtained are listed in Table IV, with their intensities, and also the arc and spark intensities given by Exner and Haschek. Three enhanced lines, λλ 3250.51, 3535.83, and 4415.89, are among the stronger lines of the luminescence spectrum. Certain strong arc lines are seen by the table to be greatly weakened in the luminescence.

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[1] *Proceedings of the Royal Society*, **72**, 253, 1904.

[2] *Annalen der Physik* (4), **44**, 1151, 1914.

The vacuum arc spectrum of cadmium was photographed for comparison, but no large difference from that of the arc in air was observed, the effects offering a decided contrast with those for magnesium, for which the spectrum of the arc in vacuum is more like that of the spark than is the luminescence spectrum.

TABLE IV

CADMIUM

λ (Exner and Haschek)	Luminescence	Arc	Spark	λ (Exner and Haschek)	Luminescence	Arc	Spark
2265.11	6	2	10	3467.76	3	50	15
2288.09*	50	500	10	3535.83	4	1	4
2880.89	2	10	3	3610.72	40	500	100
2980.80	3	30	3	3613.04	5	50	15
3250.51	30	......	10	4415.89	100	1	20
3252.86	1	20	3	4662.8	4	5	2
3261.23	100	20	5	4678.38	6	50	50
3403.86	5	100	30	4800.14	10	100	100
3466.33	15	100	30	5086.06	5	100	50

*Reverses widely in arc, with low photographic density. Not given in arc tables of Exner and Haschek.

A comparison with the results of Janicki and Seeliger[1] shows, as with magnesium, a qualitative correspondence with the spectrum of the negative glow, the lines strengthened in the luminescence spectrum being absent in that of the positive column.

CALCIUM

Two successful spectrograms for calcium with the quartz spectrograph confirmed the effects described in the former paper, the distinctive features being great intensity of λ 4227 and relatively high strength of the spark line-pairs and members of the single-line series. The position of the luminescence spectrum of calcium with reference to the arc and spark spectra is about the same as in the case of magnesium.

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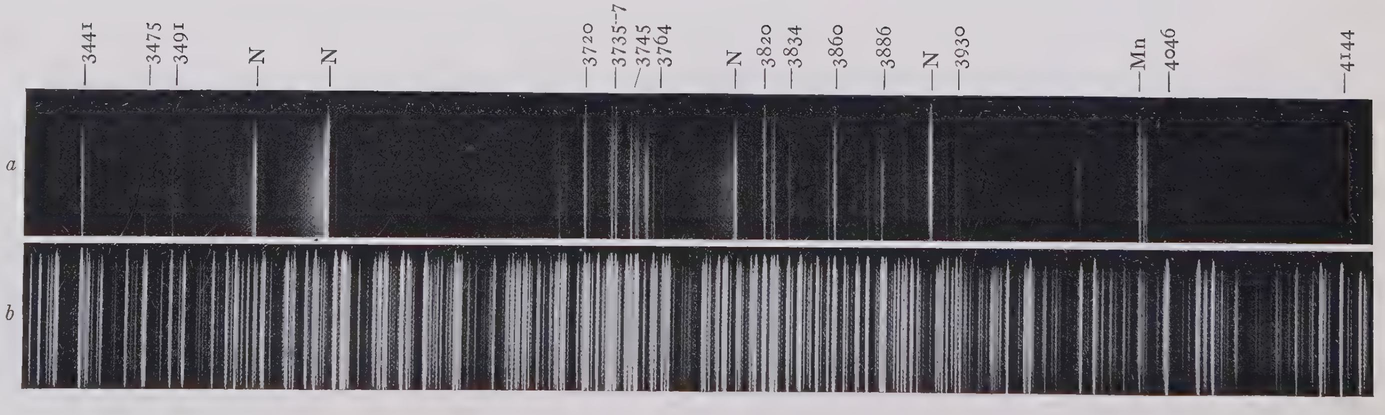
[1] *Loc. cit.*

PLATE XIII



(*a*) Luminescence and (*b*) arc spectra of iron

ANODE SPECTRUM OF IRON

Several attempts to obtain the luminescence spectrum of iron yielded no better results than those reported in the former paper. An experiment in which the anticathode was made the anode, however, gave a spectrum of such distinctive character that it seems worth while to print a list of the lines which appeared and the results of a brief comparison with other sources. The vapor a few millimeters above the anticathode was photographed in the same way as when the anticathode was without anode connection. A current of 15 milliamperes was used, with a parallel spark-gap in air varying from 22 to 38 mm.

The lines of the spectrum thus obtained, with their intensities, are given in Table V. The effect is that of a picking out of certain arc lines, but this selection results in a spectrum apparently unique. Among its many differences from the ordinary arc or spark spectrum may be mentioned the extreme faintness of the triplets, whose strongest lines are at λ 4045 and λ 4383, and the absence of enhanced lines. Certain low-temperature lines are strong, λ 4376 being twice as strong as λ 4383, while λλ 4427, 4461, and 4482 are relatively prominent. This feature can be reproduced at low temperatures of the electric furnace, from 1500° to 1600° C., and in the low-temperature flames. In the furnace at this temperature, however, λ 4260 does not appear, and the very strong anode line λ 4227.45 belongs also to higher furnace temperatures. The vacuum arc spectrum of iron shows no approach to this high-vacuum spectrum. In the ultra-violet, while many of the strong lines are prominent also in arc, spark, and furnace, others are abnormally strong in the anode spectrum, and many arc lines of high intensity do not appear. The extension toward shorter waves is much beyond that reached by the high-temperature furnace, though the spectrum of the latter is richer in the region which it covers.

Plate XIII illustrates the strongly selective action of the anode discharge as compared with the arc in air. The lines not belonging to iron are the manganese triplet at λ 4030 and several nitrogen bands. The latter occur in both the negative and the positive pole groups. The presence of the negative bands is another indication of cathode conditions due to the pencil of rays focused here. Both kinds of bands were found also when the anticathode was insulated.

TABLE V

Anode Spectrum of Iron

λ (I A)	Intensity	λ (I A)	Intensity	λ (I A)	Intensity
2462.65	1	3465.86	3	3859.91	100
2472.91	1	3475.45	8	3865.53	1
2479.78	1	3476.71	1	3872.51	2
2483.28	10	3490.58	15	3878.02	2
2488.15	7	3497.11	tr	3878.58	5
2490.66	5	3497.84	1	3886.29	30
2491.16	2	3581.20	8	3895.66	2
2522.86	6	3608.86	1	3899.71	3
2527.44	3	3610.15	1	3902.95	2
2545.98	1	3618.77	1	3920.26	1
2549.62	2	3631.46	2	3922.92	6
2719.04	5	3647.85	1	3927.93	5
2720.91	2	3679.92	7	3930.30	5
2741.83	2	3687.46	2	4045.82	2
2749.49	4	3705.57	12	4062.45	1
2966.90	2	3709.25	3	4063.60	2
2981.45	tr	3719.94	150	4066.98	2
2983.57	3	3722.57	10	4067.99	1
2994.43	4	3727.62	1	4071.75	1
2999.52	35	3733.32	4	4084.51	1
3000.95	2	3734.87	100	4198.31	1
3008.14	1	3737.14	100	4216.19	2
3009.58	4	3745.56	80	4227.45	15
3018.99	1	3748.26	20	4238.83	1
3020.50		3749.49	40	4247.44	tr
3020.64	40	3758.23	15	4258.39	1
3021.08		3763.79	5	4260.49	2
3024.04	1	3767.19	2	4271.76	3
3025.85	1	3795.00	2	4282.41	1
3037.39	3	3812.97	2	4307.91	tr
3040.43	2	3815.84	2	4325.77	tr
3047.61	4	3820.43	100	4375.93	4
3057.45	20	3824.44	10	4383.55	2
3059.09	5	3825.89	50	4404.75	1
3067.25	5	3827.83	tr	4427.31	3
3286.76	1	3834.23	15	4461.66	2
3440.61	100	3840.44	4	4482.26	1
3440.99	50	3849.97	1	4528.62	2
3443.88	1	3856.37	10		

SUMMARY OF RESULTS

Fairly rich spectra of manganese, titanium, iron, magnesium, calcium, and cadmium have been obtained by vaporizing the metal at the focus of a beam of cathode rays in a high vacuum and photographing the spectrum of the vapor in the path of the rays. The spectrum of iron was obtained by making the anticathode the anode, but photographs of the magnesium spectrum made in both ways indicated that, although the spectrum was more intense when the metal acted as anode, it did not differ in character from that produced with a separate anode so placed that the anticathode was out of the path of the current.

Comparison of these luminescence spectra with the arc, spark, and furnace spectra of the same elements shows that, although in general the majority of the lines are those easily excited in the arc and furnace, the following peculiarities mark this as a source having special characteristics:

1. Certain arc and furnace lines are intensified, while other lines, strong in these sources, are greatly weakened or are absent.

2. The resemblance to the spark spectrum, as measured by the tendency to give enhanced lines, differs greatly for different elements. The production of enhanced lines is very marked in the case of cadmium; they are present also to a considerable extent with magnesium and calcium, but to a very slight degree for titanium, manganese, and iron.

3. Where series relations are known, "single-line" series are likely to show a relatively greater intensity than other series lines.

4. There is a relatively high intensity of lines in the ultra-violet as compared with the arc and furnace spectra.

Mount Wilson Solar Observatory
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Contributions from the Mount Wilson Solar Observatory, No. 167

INVESTIGATIONS ON PROPER MOTION

FIRST PAPER: THE MOTIONS OF 85 STARS IN THE NEIGHBORHOOD OF ATLAS AND PLEIONE

By ADRIAAN VAN MAANEN

In 1913, soon after the 80-foot equivalent focus of the 60-inch reflector had been adapted for direct photographic work, a photograph of thirty minutes' exposure was secured of a portion of the Pleiades. The center of the plate is halfway between Atlas and Pleione; and in a field about 24′ by 30′ are shown 85 stars which have measurable images, the faintest being of magnitude 15.7 on the scale published by Hertzsprung.[1]

In 1918 several more exposures were made and two of these recent photographs have been compared with the 1913 plate. In the interval of five years the motion of the stars belonging to the Pleiades group surpasses a quarter of a second of arc, large enough to be easily detected on plates of our scale (1 mm = 8″.227).

Apart from the possibility of detecting some fainter members of the group, a discussion of this material should afford evidence as to what accuracy can be reached with this instrument in analogous researches. In the parallax work I have restricted my measures to stars whose distances from the center of the plate are usually less than 8′ and always less than 10′; for other lines of work it might be worth while to use larger fields, if the accuracy would permit. The results of this paper will show that we can advantageously use all the stars on plates 8×10 inches for which the available field is approximately that indicated above.

As several succeeding papers will deal with similar material, I shall give here a full account of the procedure followed; in the future it will be necessary to give only the details in which we deviate from this scheme.

The data for the plates used in the present discussion are collected in Table I, which, in the successive columns, gives the current plate number, the date of exposure, the observer, the quality, and the hour angle at the middle of the exposure.

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[1] *Astronomische Nachrichten*, **199**, 247, 1914.

TABLE I

Plate No.	Date	Obs.	Qy	H.A.
55	1913 Nov. 22	C	f	− 6°7
2014	1918 Oct. 29	H	fg	− 1.0
2029	1918 Nov. 10	H	fg	−13.2

THE MEASURES

The plates were measured differentially with the monocular arrangement of the stereocomparator in four positions, with east, west, north, and south, respectively, in the direction of the increasing readings of the micrometer screw; they were oriented by means of a trail, inserted on plate 2029.

In order to eliminate any relative displacements due to the measuring instrument, plate 55 was placed in the right-hand plate-carrier while compared with 2014 and in the left when compared with plate 2029.

The measurement of all the stars in one position of the plates requires several hours, distributed over a whole day; I therefore followed a slightly different procedure from that used in the parallax work, where the measures of a pair of plates in one position take only a short time. Three settings were made on each image of stars 1, 2 . . . . 85, after which two more settings were made in the reverse order. Any relative shift of the two plate-carriers during a series of measures can thus be detected. When the discrepancy between the two sets of measures on one star was larger than would be expected from the internal consistency of successive settings, that star was measured again five times. The total number of cases where remeasuring seemed necessary was 65 out of 652 cases, or 10 per cent. No systematic differences could be detected, and in practically all cases the discordance can be explained by the quality of the images; 44 of the 65 stars remeasured are at distances from the center greater than 10′, where the images are likely to be somewhat distorted, while most of the other cases belong to the very faintest or the very brightest stars on the plates, both of which are more difficult to measure. The results are therefore a strong recommendation for the usefulness of the stereocomparator, a point also corroborated by the small probable error in the final proper motions.

It goes without saying that care was taken to keep the temperature of the room as constant as possible; in no series of measures did the range exceed 1° C.

THE REDUCTIONS

In the reductions the two measures of each pair of plates in right ascension were combined into one set; the same was done for the measures in declination. The means M_α and M_δ, with indices 1 and 2 for the two pairs of plates, were then used in equations of the form:

$$\begin{aligned} a+bx+cy+m_\alpha&=M_\alpha \\ a'+b'x+c'y+m_\delta&=M_\delta, \end{aligned} \tag{1}$$

in which a, b, c, a', b', and c' are plate constants, x and y the coordinates of the star in decimeters, and m_α and m_δ the components of the proper motion of the star in the interval expressed in divisions of the micrometer screw. To derive the plate constants, we make the usual supposition that m_α and m_δ in the mean are equal to zero and are not functions of x and y. We can then solve equations (1) for a, b, c, a', b', c' by least squares. For the determination of the constants the three stars, Atlas, Pleione, and Bessel 33, were excluded, both on account of their brightness and because they were known to share the motion of the Pleiades group. The solution was made separately for both pairs of plates in α and δ. The substitution of the values for the constants into equations (1) then gives $m_{\alpha 1}$, $m_{\alpha 2}$, $m_{\delta 1}$, and $m_{\delta 2}$. Finally, to derive the annual proper motions in 0.″001 as a unit these values must be multiplied by:

$$q=\frac{\text{value of a division of the micrometer screw in } 0.''001}{\text{interval in years between the plates}} \tag{2}$$

For pair 2014–55, $q=0.330$; for pair 2029–55, $q=0.328$.

From the mean annual proper motions μ'_α and μ'_δ so derived it was seen, when they were plotted, that the influence due to the neglect of the quadratic terms of x and y in the reductions could not be very large; for several reasons, however, it seemed best to repeat the reductions with the inclusion of these terms.

It is easier to use the values μ'_α and μ'_δ than M_α and M_δ since we can then deal with smaller numbers. For the further reductions it is, moreover, advisable to exclude those stars which, in the first solution, have been shown to have considerable motions or to belong to the Pleiades group. For these reasons stars 5, 14, 22, 24, 26, 28, 29, 34, 41, 42, 55, 66, 74, 75, 78, and 80 have been excluded; and in order to be able to use unchanged the left-hand members of the equations of condition, and hence also of the normal equations in the solutions for both pairs of plates, stars 2, 4, 8, 9, 30, and 69, which had been measured on only one of the pairs, were also omitted. The remaining 63 stars, which with the exception of star 72 are all fainter than the eleventh magnitude, give equations of condition of the form

$$\begin{aligned} \mu_\alpha+a+bx+cy+dx^2+exy+fy^2&=\mu'_\alpha \\ \mu_\delta+a'+b'x+c'y+d'x^2+e'xy+f'y^2&=\mu'_\delta . \end{aligned} \qquad (3)$$

Supposing, as before, that μ_α and μ_δ are in the mean zero and are not functions of x and y, we can derive the unknown quantities $a, b, c, d, e, f, a', b', c', d', e', f'$.

With x and y expressed in decimeters, the resulting normal equations are:

$$\begin{aligned}
+\ 6.613f+1.196e+\ 5.240d+\ 0.236c+\ 0.595b+15.162a&=[\mu'_\alpha y^2]\\
+\ 1.196f+5.240e+\ 0.757d+\ 0.595c+\ 1.568b+\ 3.027a&=[\mu'_\alpha xy]\\
+\ 5.240f+0.757e+17.137d+\ 1.568c+\ 1.858b+23.453a&=[\mu'_\alpha x^2]\\
+\ 0.236f+0.595e+\ 1.568d+15.162c+\ 3.027b+\ 1.830a&=[\mu'_\alpha y]\\
+\ 0.595f+1.568e+\ 1.858d+\ 3.027c+23.453b+\ 4.000a&=[\mu'_\alpha x]\\
+15.162f+3.027e+23.453d+\ 1.830c+\ 4.000b+63.000a&=[\mu'_\alpha]
\end{aligned}$$

with a similar set for μ'_δ involving the constants a', b', c', d', e', and f'. The right-hand members for the four cases are given in Table II.

The resulting constants are summarized in Table III. The substitution of these quantities into equations (3) for each of the 85 stars gives us the final values for μ_α and μ_δ derived from each pair of plates.

TABLE II

2014–55 $\mu_{\alpha 1}$	2029–55 $\mu'_{\alpha 2}$	2014–55 $\mu'_{\delta 1}$	2029–55 $\mu'_{\delta 2}$
− 46	−13	+101	+ 90
+ 59	+21	+109	+ 71
− 24	−48	+144	− 9
+ 13	+54	+ 8	− 18
+ 38	+13	− 20	− 5
−115	−89	+263	+192

TABLE III

Constant	2014–55 $\mu'_{\alpha 1}$	2029–55 $\mu'_{\alpha 2}$	Constant	2014–55 $\mu'_{\delta 1}$	2029–55 $\mu'_{\delta 2}$
a	−0″00155	−0″00149	*a′*	−0″00166	+0″00292
b	+ .00099	+ .00013	*b′*	− .00259	− .00093
c	+ .00022	+ .00370	*c′*	− .00040	− .00111
d	+ .00239	− .00195	*d′*	+ .00696	− .00837
e	+ .01322	+ .00412	*e′*	+ .01927	+ .01087
f	− .00777	+ .00210	*f′*	+ .01033	+ .01173

THE RESULTS

These values, $\mu_{\alpha 1}$, $\mu_{\alpha 2}$, $\mu_{\delta 1}$, and $\mu_{\delta 2}$, are given in Table IV, columns 5, 6, 7, and 8; columns 9 and 10 give the mean relative motions in α and δ. The mean probable error of these values is 0″0036 for μ_α and 0″0034 for μ_δ. The other columns need little explanation. The first gives the current numbers of the stars, the second the approximate photographic magnitudes. The magnitudes were derived in the following way: twenty-seven of the stars on my plates appear in Hertzsprung's publication, "Photographische Sterngrössen schwacher Zentralplejaden,"[1] while four are given in his later article, "Prüfung der photographischen Grössen Skala der hellen Plejadensterne."[2]

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[1] *Astronomische Nachrichten*, **199**, 247, 1914.

[2] *Ibid.*, **200**, 137, 1915.

TABLE IV

No.	Mag.	x	y	$\mu_{\alpha 1}$	$\mu_{\alpha 2}$	$\mu_{\delta 1}$	$\mu_{\delta 2}$	μ_{α}	μ_{δ}
1...	15.10	−15′.2	+ 4′.7	+0″.044	+0″.009	+0″.012	+0″.002	+0″.026	+0″.007
2...	15.42	15.1	− 6.6	+ 9	....	− 4	....	+ 9	− 4
3...	13.33	14.7	− 1.6	− 14	− 13	+ 1	− 2	− 14	0
4...	15.15	14.4	+11.4	....	− 17	....	+ 14	− 17	+ 14
5...	9.41	14.1	+ 2.7	+ 6	+ 24	− 41	− 15	+ 15	− 28
6...	13.28	13.7	+ 8.9	− 17	− 10	− 7	− 9	− 14	− 8
7...	13.70	12.9	− 6.0	− 16	+ 23	− 13	+ 1	+ 3	− 6
8...	15.42	12.3	+11.1	....	− 24	....	+ 4	− 24	+ 4
9...	13.70	12.1	+11.6	....	− 12	....	− 4	− 12	− 4
10...	14.70	11.9	+ 4.8	+ 24	− 2	+ 12	+ 2	+ 11	+ 7
11...	14.56	11.6	− 0.1	+ 5	+ 6	− 5	− 10	+ 6	− 8
12...	13.20	11.4	− 2.2	− 10	− 2	− 21	+ 1	− 6	− 10
13...	14.75	10.3	+ 6.5	− 13	− 26	+ 2	+ 33	− 20	+ 18
14...	10.22	9.7	− 2.9	+ 12	+ 30	− 56	− 52	+ 21	− 54
15...	13.95	9.3	− 0.3	− 5	− 17	− 27	− 9	− 11	− 18
16...	13.95	8.9	−11.0	− 1	− 16	+ 8	+ 5	− 8	+ 6
17...	14.95	8.8	− 8.8	− 2	+ 7	+ 27	− 13	+ 2	+ 7
18...	12.55	8.8	+10.5	− 4	+ 19	+ 8	− 12	+ 7	− 2
19...	14.81	8.6	−11.2	+ 16	+ 10	+ 8	+ 20	+ 13	+ 14
20...	13.52	8.2	−11.2	+ 4	+ 4	− 2	− 5	+ 4	− 4
21...	14.12	7.8	− 1.1	+ 9	+ 3	+ 10	+ 13	+ 6	+ 12
22...	9.41	7.7	+ 5.1	+ 31	+ 29	− 28	− 25	+ 30	− 26
23...	13.82	7.4	+ 7.3	+ 14	+ 10	+ 9	+ 15	+ 12	+ 12
24...	14.50	5.4	+ 3.0	+ 11	+ 9	+ 21	+ 31	+ 10	+ 26
25...	15.51	5.2	− 5.6	− 12	− 11	− 12	− 33	− 12	− 22
26...	15.76	5.2	+ 6.6	+ 1	+ 11	− 44	− 28	+ 6	− 36
27...	14.75	4.9	− 8.8	− 13	− 25	+ 12	+ 12	− 19	+ 12
28...	11.88	4.7	+10.3	+ 14	+ 17	− 27	− 37	+ 16	− 32
29...	13.70	4.5	− 7.1	+ 27	+ 21	− 42	− 49	+ 24	− 46
30...	14.12	4.1	+11.6	....	− 10	....	+ 11	− 10	+ 11
31...	12.18	3.3	+ 2.7	− 27	+ 5	− 14	− 23	− 11	− 18
32...	12.79	3.0	− 4.4	+ 13	+ 25	− 9	+ 4	+ 19	− 2
33...	14.89	2.5	+10.3	− 19	− 1	+ 8	− 12	− 10	− 2
34...	14.81	1.9	− 8.4	+ 40	+ 26	+ 28	+ 31	+ 33	+ 30
35...	15.42	1.5	+ 0.4	+ 1	− 8	− 5	+ 2	− 4	− 2
36...	14.81	1.5	−11.1	+ 8	− 6	+ 2	− 9	+ 1	− 4
37...	13.89	1.4	− 2.2	+ 9	+ 8	− 2	0	+ 8	− 1
38...	15.51	0.5	− 3.6	− 10	− 8	+ 2	+ 12	− 9	+ 7
39...	14.18	0.4	− 7.7	+ 6	+ 9	+ 9	− 27	+ 8	− 9
40...	14.00	− 0.4	− 5.2	− 3	+ 8	− 2	+ 21	+ 2	+ 10
41...	3.77	+ 0.1	− 2.5	+ 50	− 2	− 53	− 73	+ 24	− 63
42...	4.95	0.4	+ 2.5	+ 21	+ 29	− 38	− 22	+ 25	− 30
43...	12.97	0.4	+ 8.4	+ 17	+ 9	− 23	− 19	+ 13	− 21
44...	14.56	0.5	− 1.9	− 9	+ 10	− 4	+ 13	0	+ 4
45...	15.66	1.8	+ 5.2	+ 9	− 4	+ 7	+ 1	+ 2	+ 4
46...	15.02	1.8	− 1.6	+ 7	− 3	+ 9	− 2	+ 2	+ 4
47...	15.42	2.1	+ 2.7	− 1	− 8	+ 3	− 4	− 4	0
48...	14.70	2.2	−10.1	+ 13	− 12	− 12	− 1	0	− 6
49...	13.28	2.2	+ 9.3	+ 6	− 5	+ 15	+ 11	0	+ 13
50...	14.95	2.2	− 6.5	+ 12	+ 1	+ 3	+ 11	+ 6	+ 7
51...	12.90	2.5	− 4.3	− 16	− 3	− 12	− 15	− 9	− 14
52...	14.18	2.6	+10.1	+ 5	+ 6	− 16	+ 2	+ 6	− 7
53...	15.35	2.6	+ 3.4	− 13	− 9	+ 5	− 10	− 11	− 2
54...	15.51	3.2	+ 7.1	+ 2	− 11	+ 9	+ 5	− 4	+ 7

TABLE IV—*Continued*

No.	Mag.	x	y	$\mu_{\alpha 1}$	$\mu_{\alpha 2}$	$\mu_{\delta 1}$	$\mu_{\delta 2}$	μ_{α}	μ_{δ}
55...	8.19	3′.7	+ 9′.2	+0″.017	+0″.016	−0″.053	−0″.057	+0″.016	−0″.055
56...	15.51	4.1	+ 7.3	+ 3	− 20	+ 3	− 5	− 8	− 1
57...	14.38	4.7	+10.4	− 1	− 2	+ 16	+ 5	− 2	+ 10
58...	13.70	5.6	− 4.0	+ 11	− 3	+ 9	− 2	+ 4	+ 4
59...	13.95	5.9	− 1.0	+ 7	+ 1	+ 6	− 3	+ 4	+ 2
60...	14.70	6.0	+ 0.3	− 10	− 27	+ 21	+ 16	− 18	+ 18
61...	13.89	6.0	+ 9.7	+ 4	0	+ 14	− 3	+ 2	+ 6
62...	13.39	6.9	+ 7.7	+ 11	+ 13	− 9	− 10	+ 12	− 10
63...	15.15	6.9	− 1.1	+ 3	− 7	− 5	− 1	− 2	− 3
64...	13.64	6.9	− 8.4	+ 10	+ 7	+ 1	− 3	+ 8	− 1
65...	11.80	7.3	− 3.6	− 12	+ 19	0	+ 1	+ 4	0
66...	15.15	7.5	− 6.0	− 24	− 23	− 25	− 8	− 24	− 16
67...	11.16	8.4	+ 8.9	− 14	+ 5	− 21	+ 10	− 4	− 6
68...	14.08	8.8	+ 2.2	+ 19	+ 5	+ 10	+ 13	+ 12	+ 12
69...	14.95	9.6	− 2.6	+ 7	....	− 8	....	+ 7	− 8
70...	13.09	10.1	− 4.0	+ 4	+ 21	− 1	− 11	+ 12	− 6
71...	14.89	10.3	+ 3.2	+ 10	+ 8	+ 3	+ 8	+ 9	+ 6
72...	9.32	10.3	+ 7.4	− 1	+ 3	− 11	+ 5	+ 1	− 3
73...	15.15	10.8	+ 9.2	+ 16	+ 15	− 12	+ 3	+ 16	− 4
74...	10.12	11.1	− 8.5	0	+ 14	− 26	− 26	+ 7	− 26
75...	14.12	11.1	− 8.6	+ 1	− 4	− 23	− 24	− 2	− 24
76...	13.82	11.8	−10.8	+ 3	+ 1	− 10	+ 16	+ 2	+ 3
77...	14.00	11.8	+ 5.6	+ 29	− 7	+ 7	+ 6	+ 11	+ 6
78...	14.95	12.9	+ 0.4	+ 17	− 11	− 24	− 16	+ 3	− 20
79...	11.94	12.9	+ 7.3	− 18	− 16	− 11	+ 5	− 17	− 3
80...	13.46	13.2	− 4.5	+ 36	+ 49	+ 12	+ 10	+ 42	+ 11
81...	12.30	13.4	+ 1.6	− 9	0	+ 3	− 12	− 4	− 4
82...	14.44	13.8	+ 3.0	− 4	− 4	+ 7	+ 11	− 4	+ 9
83...	12.55	13.8	+ 8.0	− 10	− 6	− 7	− 14	− 8	− 10
84...	14.30	14.0	− 9.0	− 23	− 11	− 11	− 6	− 17	− 8
85...	15.15	14.8	− 0.8	+ 9	+ 2	+ 27	− 9	+ 5	+ 9

For these stars Hertzsprung's magnitudes were plotted against the values read from plate 2014 with the help of a scale of images made with exposures proportional to 1, 2, 4, etc. A smooth curve was drawn through the plotted points; and with the scale values as argument the magnitudes for all the stars, except 41 and 42, were read from the curve. The errors may be 0.2 or 0.3 of a magnitude, but this is accurate enough for the work in hand. For the two very bright stars, 41 and 42, no magnitudes could be derived in this way; the values given for them are those published by Schwarzschild.[1] Columns 3 and 4 contain the co-ordinates x and y in minutes of arc; these are only for identification purposes and accordingly only one decimal is given.

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[1] *Publikationen der v. Kuffner'schen Sternwarte*, **5**, 1897.

In Fig. 1 the 85 stars are shown with their relative proper motions, the scale being 1 mm=0″008. In the upper right-hand corner the mean proper motion derived from Boss's *Preliminary General Catalogue*, for 12 stars belonging to the Pleiades, is indicated on the same scale by a broken arrow.

Three stars, 41, 42, and 55, were already known through other researches to share in the cluster motion; stars 14 and 29 have motions so nearly the same that it seems safe to include them as members. The magnitude of star 29 is 13.7; unless for other reasons (parallax, radial velocity, or spectrum) this object is dropped as a member of the group, it would indicate that the dispersion of the group-stars in brightness is at least ten magnitudes.

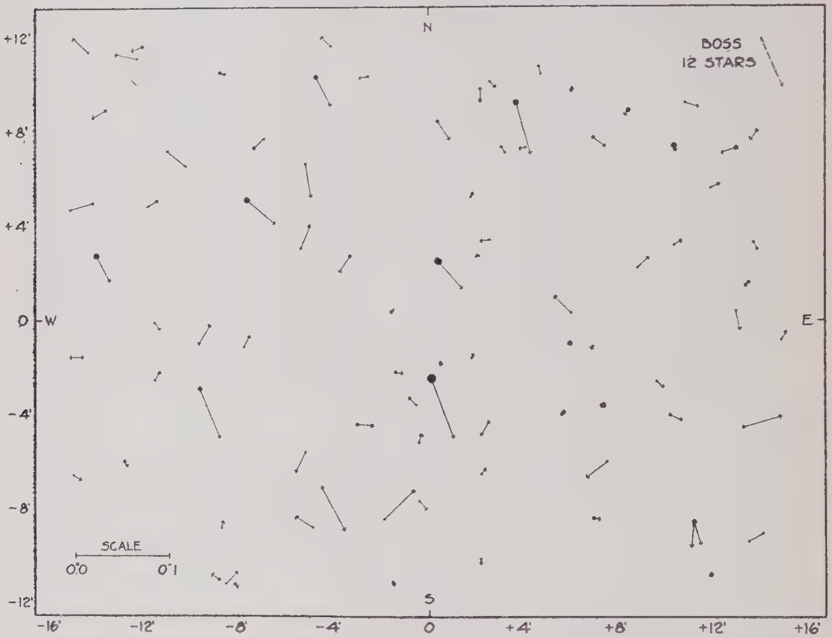

Fig. 1.—Relative proper motions of 85 stars in the neighborhood of Atlas and Pleione. The absolute motion of the cluster as determined from 12 stars in Boss's *Catalogue* is indicated by the arrow in the corner.

Stars 5, 22, 26, 28, 43, 74, 75, and 78 are also moving in the same direction; the amount of motion, however, is considerably smaller. Moreover, since the position angle of the parallactic motion in this region of the sky is 146°, while that of the motion of the cluster is 158°, it will be difficult to decide whether these stars belong to the group or not. As probable members of the group I have therefore used only Nos. 14, 29, 41, 42, and 55.

Doubts may be felt about the inclusion of a star as faint as No. 29 in the group. From Trümpler's discussion[1] we know that up to magnitude 9.5 the number of group-stars is increasing with each following magnitude, although the increase is much less rapid than the average for stars in that part of the sky. For stars fainter than 9.5 the existing material is as yet extremely scarce. Supposing that No. 29 belongs to the group we can make the following deduction for the range in absolute magnitude of the stars in the cluster. From the parallactic motion and radial velocities Hartmann[2] derives a parallax of $0\overset{''}{.}014$, Hayn[3] $0\overset{''}{.}013$ to $0\overset{''}{.}021$, Plummer[4] $0\overset{''}{.}024$, and Kapteyn[5] $0\overset{''}{.}018$. For only one of the group-stars has a parallax been determined by Elkin[6]—its absolute value is $-0\overset{''}{.}016$. Adopting $0\overset{''}{.}015$ we shall not be far from the truth; this gives for the brightest member of the group absolute magnitude -0.9, for the faintest $+9.6$.

REDUCTION FROM RELATIVE TO ABSOLUTE MOTION

The motions μ_α and μ_δ of Table IV are relative with respect to the mean motion of the 63 stars used for computing the plate constants. The reduction to absolute motions is always a difficult point. For small fields, such as are usually discussed in photographic work, the number of stars for which absolute motions can be derived from meridian work is always extremely small. The large number of brighter stars in the region of the Pleiades and the many researches concerning the motion of the cluster might lead to the expectation that here the necessary material is available.

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[1] *Publications of the American Astronomical Society*, **3**, 333, 1918.

[2] *Astronomische Nachrichten*, **199**, 305, 1914.

[3] *Ibid.*, **198**, 147, 1914.

[4] *Monthly Notices*, **73**, 492, 1913.

[5] *Proc. Kon. Akad. van Wet. te Amsterdam*, **14**, 911, 1912.

[6] *Transactions of the Astronomical Observatory of Yale University*, **2**, 320, 1912.

The fact is, however, that practically all the researches on the proper motions of the Pleiades give relative motions with respect to one or more members of the group. These investigations have certainly established the fact of the common motion of many stars of the group, but for our purpose they do not give the required data. Only Kromm has derived the absolute motions, of 160 stars, in the neighborhood of the Pleiades.[1] Seven of these stars appear on my plates; for these we find a mean difference Kromm—van Maanen of $-0''.019$ in μ_α and $+0''.007$ in μ_δ. This difference is much larger than would be expected for the mean motion of the 63 stars used in the reductions. I have therefore compared Kromm with Boss's *Preliminary General Catalogue.*[2] The 14 stars appearing in both give the differences Kromm—Boss of $-0''.020$ in μ_α and $+0''.009$ in μ_δ; these are very close to the differences Kromm—van Maanen.

Boss's proper motions certainly are among the most reliable that we have. I have therefore made an attempt to reduce my relative motions to the system of Boss's *Catalogue;* since we have only two stars in common, I have used a roundabout way. I have assumed that there is no relative motion in the various parts of the cluster and have used the mean motion of the 12 cluster stars which appear in Boss's *Catalogue* as the absolute motion of the cluster. The result is $\mu_\alpha = +0''.023$, $\mu_\delta = -0''.051$. For the three stars on my plates which were already known as members of the group, my measures give: $\mu_\alpha = +0''.022$, $\mu_\delta = -0''.049$; including also stars 14 and 29, these figures become $+0''.021$ and $-0''.050$, respectively. In both cases the correction from my motions to Boss's system is exceedingly small. To reduce my relative motions to absolute motions, I have adopted the corrections $+0''.002$ in μ_α and $-0''.001$ in μ_δ.

That these corrections cannot be far from the truth can also be seen from the distribution of the τ components (at right angles to the parallactic motion). Calling the component of proper motion in the direction of 56°, $+\tau$, those in 236°, $-\tau$, I find 38 stars with τ positive, two with $\tau=0$, and 34 with τ negative. The mean values are $+\tau=0\overset{''}{.}0109$, $-\tau=0\overset{''}{.}0098$. For this calculation the cluster stars and those which were measured on one plate only were excluded. Although the number of objects then available is only 74, I have given in Table V the mean absolute motion in α and δ for the different magnitudes. In the last two columns are the components of the parallactic motions, computed from Kapteyn's formula[1] with the solar motion $=21.5$ km/sec. and the right ascension and declination of the apex $270\overset{\circ}{.}9$ and $+29\overset{\circ}{.}2$,[2] respectively; except for the slight systematic difference in δ for the brighter stars, where n is small, the agreement with the observed quantities is as good as can be expected from the small number of stars available. Finally, Table VI shows the distribution according to the amount of the total proper motion.

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[1] *Annales de l'Observatoire de Bordeaux,* **14,** 77, 1911.

[2] The proper motions in Boss's *Catalogue* were corrected according to the statement on p. xxviii.

TABLE V

Limits of Mag.	Mag.	μ_α	μ_δ	n	μ_α Par.	μ_δ Par.
9–10	9.38	$+0\overset{''}{.}017$	$-0\overset{''}{.}020$	3	$+0\overset{''}{.}010$	$-0\overset{''}{.}014^5$
10–11	10.12	+0.009	−0.027	1	+0.008	-0.011^5
11–12	11.70	+0.002	−0.011	4	+0.005	-0.007^5
12–13	12.61	+0.003	−0.011	7	+0.004	−0.006
13–14	13.64	+0.006	−0.001	20	+0.003	-0.004^5
14–15	14.63	+0.003	+0.003	24	$+0.002^5$	-0.003^5
>15	15.38	+0.001	−0.005	15	+0.002	-0.002^5

TABLE VI

Limits of μ	n Observed	n Theory
$<0\overset{''}{.}010$	28	61.4
0.010–0.019	25	12.6
0.020–0.029	15	0.0
0.030–0.039	3	0.0
0.040–0.049	3	0.0

In a previous publication on the proper motions of the stars in and near the double cluster in Perseus,[3] attention was called to the

[1] *Publications of the Astronomical Laboratory at Groningen*, **8**, 1901.

[2] These are the values derived by Strömberg in *Contributions from the Mount Wilson Solar Observatory*, No. 144; *Astrophysical Journal*, **47**, 7, 1918.

[3] *Recherches Astronomiques de l'Observatoire d'Utrecht*, **5**, 44, 1911.

fact that there most of the proper motions do not fall between 0″.000 and 0″.009, but between 0″.010 and 0″.019; a re-reduction of the correction from relative to absolute motions[1] changed the original figures, but not the fact that the maximum frequency falls between 0″.010 and 0″.019. In *Publications of the Astronomical Laboratory at Groningen*, No. 11, the same phenomenon was noticed. In the present investigation we find, as indicated in Table VI, that only a relatively small number fall under the smallest motion—a fact that can be explained partly by the influence of the errors in the proper motions, but one which may also be partly due to real motions. If the proper motions were really zero, the distribution of μ would follow the combined probability curve; the larger the probable error the flatter the curve would be. If the motions are not really zero, we should find a different skew curve, which would also be flatter for large probable errors.

TABLE VII

Source	P.E.	<0″.030	0″.030–0″.049	0″.050–0″.079	0″.080–0″.099	n
		Per Cent	Per Cent	Per Cent	Per Cent	
van Maanen.......	0″.008	87.1	11.5	0.7	0.6	706
Rambaut.........	0.012	79.8	15.8	4.6	0.0	241
Groningen No. 25*a*.	0.015	73.3	20.5	5.7	0.7	1026
Groningen No. 19..	0.017	51.0	31.5	14.9	2.9	3161
Groningen No. 25*b* .	0.038	25.3	28.5	35.2	11.0	2257

This fact can be well illustrated by a summary of the distribution of proper motions collected by Schouten.[2] In discussing his data I have, in order to use fairly homogeneous material, restricted myself to stars of magnitude 8 to 13 and of proper motion less than 0″.100, derived by means of photographic plates. This leaves us five sources of data, viz.: van Maanen (Perseus Cluster), Rambaut, *Groningen* No. 25*a*, *Groningen* No. 19, and *Groningen* No. 25*b*, for which the probable errors of the total motion are, respectively, 0″.008, 0″.012, 0″.015, 0″.017, and 0″.038. In Table VII are given the percentages of the motions between certain limits for these five sources of data.

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[1] *Publications of the American Astronomical Society*, **3**, 227, 1918.

[2] "On the Determination of the Principal Laws of Statistical Astronomy," Dissertation, Amsterdam, 1918.

For our purpose it is to be regretted that the limits used by Schouten are not smaller, but even so the effect of the probable error is clearly visible. The table is arranged in order of increasing probable error for the different sources, and shows a strong decrease in the percentage of the smallest and an increase in that of the larger motions. In *Groningen* No. 25*b* we even find the maximum frequency displaced to stars with motions between 0″.050 and 0″.079. Objection may be raised to the fact that in Table VII we have included stars of a wide range of magnitudes and of different galactic latitudes; but from the detailed results given in Schouten's paper (pp. 28–34) we can see that the same tendency in the distribution holds for practically every case.

THE QUADRATIC TERMS

From the values of the constants a, b, c, d, e, f, a', b', c', d', e', and f' given in Table III the influence of the quadratic terms on the proper motions can be deduced. Since the zero point for the co-ordinates, which were expressed in decimeters, was chosen at the center of the plates, the necessary corrections for stars at the center are equal to a and a'. These are very small, especially in right ascension, which corroborates the statement made in my parallax papers, that for the central (=parallax) stars the corrections due to the neglect of the quadratic terms are negligible. In Table VIII the number of mean corrections between various limits is given for stars at different distances from the center of the plates.

TABLE VIII

Distance from Center	Corrections						
	<0″.003	0″.003–0″.005	0″.006–0″.008	0″.009–0″.011	0″.012–0″.014	>0″.014	n
<8′......	64	27	5	0	0	0	96
8′–10′.....	21	20	11	2	0	0	54
>10′....	54	62	27	23	6	8	180

For stars more distant than 10′ there is a considerable number of cases for which the correction is so large that for the most accurate work it cannot be safely neglected; in one case it amounts to not less than 0".021. That the magnitude of these corrections is due to the use of a reflecting telescope for securing the plates seems extremely doubtful. A subsequent paper on the motion of stars in the neighborhood of the Orion nebula contains a re-reduction of the motions already published.[1] The corrections to the annual motion arising from the quadratic terms amount in some cases to 0".017. A re-reduction of the proper motions of the stars near Messier 37, which were derived by Joy,[2] shows that the correction to the annual motion may there reach 0".012. In both cases the data were derived from 8×10-inch plates, secured with the Yerkes 40-inch refractor. Since in these cases the time-interval was ten and eleven years, respectively, it is clear that for the same interval the necessary corrections are not less than those to be expected for plates taken with the 60-inch reflector. In neither case can the quadratic terms be safely neglected if the best results are sought.

In his article, "The Influence of Quadratic Terms in Reductions of Stellar Parallaxes,"[3] Lee, after the reduction with and without the quadratic terms of several plates, draws the conclusion: "The reduction of the quantity $[vv]$ obtained by including the quadratic terms in the solution is in most cases negligible, and will in no case justify the additional labor involved in deriving it."

In my opinion this statement is not free from objections. The values v are not a measure of the errors, as they include the real proper motions or parallaxes of the stars. If these were really zero, or even small as compared with the errors, a small reduction in $[vv]$ might justify his statement. In cases where the probable errors are small, $[vv]$ is largely due to the real motions or parallaxes; and the inclusion of the quadratic terms, although their amount may be large enough to vitiate the results, will then give only a small percentage change in $[vv]$.

In our case, for instance, we can compute the number of stars which, according to the theory of errors, should have apparent motions between certain limits, if the real motions are assumed to be zero; the values are given in Table VI, under the heading "Theory." There is scarcely any doubt that the difference between the actual numbers and the theoretical values is due to the existence of appreciable motions. The reductions in [*vv*] produced by including the quadratic terms are here respectively 8, 0, 22, and 18 per cent—in the mean 12.0 per cent as against 11.5 per cent in Lee's discussion; yet we have seen that the corrections sometimes amount to quantities which cannot be neglected.

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[1] *Astronomical Journal*, **27**, 139, 1912.

[2] *Ibid.*, **29**, 101, 1916.

[3] *Ibid.*, **29**, 74, 1916.

SUMMARY

1. At the 80-foot focus of the 60-inch reflector fields of 24′ by 30′ can be measured successfully. To obtain the best results the reductions should include the quadratic terms in x and y. The results from two pairs of plates may be estimated to have a probable error in each co-ordinate of 0″018 divided by the number of years in the interval.

2. Of the 85 stars, down to magnitude 15.7, which were measured in a region of the Pleiades, five show proper motions equal to those of the group. The faintest of these stars is of photographic magnitude 13.7; with 0″015 as the adopted parallax of the group, its absolute magnitude is +9.6, while that of the brightest star of the Pleiades would be −0.9.

3. The lack of a predominant maximum frequency for the smallest motions is partly caused by the errors in the measurements, but is undoubtedly largely due to the existence of real motions among the background stars.

Mount Wilson Solar Observatory
January 1919

Contributions from the Mount Wilson Solar Observatory, No. 168

INVESTIGATIONS ON PROPER MOTION

SECOND PAPER: THE MOTIONS OF 162 STARS IN THE NEIGHBORHOOD OF THE ORION NEBULA

By ADRIAAN VAN MAANEN

In the *Astronomical Journal*, No. 642, I have published the relative proper motions of 162 stars in the neighborhood of the great nebula in Orion, the results being derived from plates taken with the 40-inch Yerkes refractor; in obtaining these proper motions, however, quadratic terms of the co-ordinates were not used in the reductions. Later, evidence was found that these terms cannot be neglected when the highest accuracy is required; accordingly a new reduction was made, in which the quadratic terms were included. A few other improvements were also made, and the new results seem to justify the publication of the revised motions.

The computations were made in great part by Miss Coral Wolfe, of the Computing Division, to whom I wish to express my acknowledgment.

THE REDUCTIONS

For each of the two pairs of plates the reductions were made separately in both α and δ in exactly the same way as fully described in *Mt. Wilson Contribution* No. 167. The equations of condition take the form:

$$\begin{aligned} \mu_\alpha+a+bx+cy+dx^2+exy+fy^2&=\mu'_\alpha \\ \mu_\delta+a'+b'x+c'y+d'x^2+e'xy+f'y^2&=\mu'_\delta \end{aligned} \qquad (1)$$

where μ_α and μ_δ are the new values of the proper motions; the values $\mu''_{\alpha 1}$, $\mu''_{\alpha 2}$, $\mu''_{\delta 1}$, and $\mu''_{\delta 2}$, respectively, of the original paper were used as the second members of the equations of condition.

As can be seen from their excessive concentration near the Trapezium, most of the stars near the center of the plates must belong to the system of the Orion nebula; we have therefore to exclude the central stars as reference objects in the computation of

TABLE I

No. Bond	Mag.	Group	$\mu_{\alpha 1}$	$\mu_{\alpha 2}$	$\mu_{\delta 1}$	$\mu_{\delta 2}$	μ_{α} abs.	μ_{δ} abs.
303....	10.9	*b*	+0.″024	+0.″005	+0.″001	−0.″007	+0.″018	−0.″002
314....	12.9	*b*	− 19	+ 5	+ 4	+ 10	− 3	+ 3
315....	10.9	*b*	+ 14	+ 6	+ 1	+ 5	+ 14	+ 4
311....	10.9	*v*	+ 11	+ 9	− 11	− 1	+ 14	− 5
323....	12.3	*b*	− 8	− 8	+ 2	+ 6	− 4	+ 2
326....	11.8	*b*	− 4	+ 6	+ 1	− 15	+ 5	− 8
335....	11.9	*b*	+ 18	− 5	− 7	− 20	+ 16	− 15
363....	11.5	*b*	+ 9	− 1	+ 4	− 6	+ 8	− 1
370....	14.0	*b*	− 15	− 7	+ 5	+ 6	− 7	0
373....	12.8	*b*	− 10	− 2	+ 4	+ 1	− 2	− 1
377....	13.2	*b*	− 10	+ 2	+ 5	− 1	0	− 2
378....	14.7	*b*	+ 3	− 2	+ 10	− 1	+ 4	− 4
382....	12.6	*b*	+ 2	+ 2	+ 2	+ 10	+ 6	+ 3
387....	11.5	*b*	+ 22	+ 7	− 13	− 32	+ 18	− 22
399....	13.9	*b*	+ 48	+ 51	− 75	− 74	+ 54	− 80
402....	12.8	*b*	− 7	− 5	+ 9	− 1	− 2	+ 1
409....	14.5	*b*	− 16	− 1	+ 4	0	− 4	− 5
423....	10.9	*b*	+ 6	− 2	− 13	+ 10	+ 6	− 1
427....	11.8	*b*	− 2	− 15	+ 1	+ 4	− 4	+ 1
430....	12.7	*b*	− 3	− 7	+ 3	+ 7	− 1	+ 2
435....	14.4	*v*	− 7	+ 1	+ 10	+ 7	+ 1	+ 1
438....	9.8	*b*	+ 11	− 13	− 32	− 11	+ 3	− 18
443....	14.6	*b*	− 7	+ 5	+ 12	+ 20	+ 3	+ 8
449....	11.5	*b*	+ 9	− 4	− 14	− 10	+ 6	− 12
458....	12.8	*b*	− 6	0	+ 6	− 3	+ 1	− 1
467....	8.5	*n*	+ 7	− 19	− 8	− 26	− 2	− 10
479....	11.3	*v*	+ 21	0	− 13	+ 1	+ 14	− 6
490....	14.2	*b*	− 2	− 2	+ 7	+ 1	+ 2	− 3
497....	10.7	*b*	+ 21	+ 10	− 8	− 4	+ 20	− 4
505....	9.6	*b*	+ 18	− 5	− 6	− 15	+ 10	− 6
506....	12.0	*b*	0	− 2	+ 2	− 1	+ 3	− 1
508....	13.7	*b*	− 4	− 4	+ 8	+ 12	0	+ 5
510....	13.9	*b*	− 2	− 17	+ 3	+ 1	− 6	− 4
516....	13.8	*b*	− 7	− 4	+ 6	− 5	− 1	− 6
521....	11.2	*b*	− 2	− 4	− 12	− 9	+ 1	− 9
524....	13.9	*v*	+ 2	+ 2	+ 18	+ 16	+ 6	+ 11
523....	10.5	*b*	+ 20	+ 8	− 8	+ 3	+ 18	0
532....	14.4	*b*	− 18	− 5	+ 2	+ 13	− 8	+ 1
534....	10.6	*b*	− 7	− 4	− 20	− 11	− 2	− 14
545....	14.4	*b*	− 11	+ 7	− 8	+ 3	+ 2	− 9
543....	10.4	*n*	+ 1	− 12	− 14	− 8	− 2	− 9
551....	11.6	*b*	+ 7	− 4	− 12	− 9	+ 6	− 10
β......	14.3	*b*	+ 4	+ 6	+ 9	− 7	+ 9	− 6
558....	12.0	*v*	+ 3	− 10	0	+ 6	0	+ 2
554....	8.3	*b*	+ 36	− 11	− 36	− 9	+ 16	− 15
566....	13.9	*b*	− 6	− 5	− 2	+ 6	− 2	− 4
567....	14.0	*v*	− 14	− 1	+ 15	+ 19	− 4	+ 11
570....	9.0	*n*	+ 26	− 14	− 20	− 11	+ 10	− 10
α......	14.6	*c*	− 22	+ 6	+ 13	− 9	− 4	− 6
573....	13.3	*b*	− 12	− 5	− 1	− 4	− 4	− 6
575....	12.7	*c*	+ 9	+ 3	+ 2	+ 13	+ 10	+ 5
580....	13.6	*b*	− 1	− 2	+ 9	+ 8	+ 2	+ 3
581....	13.9	*b*	+ 1	− 2	+ 15	+ 4	+ 4	+ 4
a......	13.9	*c*	− 7	+ 33	+ 27	− 12	+ 17	+ 2

TABLE I—*Continued*

No. Bond	Mag.	Group	$\mu_{\alpha 1}$	$\mu_{\alpha 2}$	$\mu_{\delta 1}$	$\mu_{\delta 2}$	μ_α abs.	μ_δ abs.
589....	12.7	*v*	+0″.010	+0″.009	+0″.020	+0″.009	+0″.014	+0″.011
595....	13.8	*c*	− 4	− 18	+ 17	− 4	− 7	0
γ......	14.3	*c*	− 6	+ 4	+ 7	+ 12	+ 3	+ 3
598....	14.1	*b*	− 3	− 10	+ 2	− 3	− 2	− 6
602....	13.7	*c*	+ 23	+ 10	− 10	− 10	+ 20	− 15
l......	13.7	*c*	+ 4	− 9	− 14	− 17	+ 2	− 21
δ......	14.2	*c*	− 6	+ 21	+ 23	+ 12	+ 12	+ 11
612....	12.8	*c*	+ 23	− 8	− 11	+ 13	+ 12	− 2
618....	12.7	*c*	+ 4	+ 13	− 9	− 2	+ 12	− 9
621....	14.2	*c*	+ 1	+ 11	− 20	− 6	+ 10	− 20
619....	7.2	*n*	....	....	....	....	+ 10	+ 1
622....	12.7	*c*	+ 10	+ 18	+ 15	− 21	+ 18	− 6
624....	8.4	*n*	....	....	....	....	+ 15	− 4
e......	13.7	*c*	− 9	+ 20	− 17	+ 29	+ 10	+ 1
628....	5.8	*n*	....	....	....	....	− 12	+ 7
631....	13.7	*c*	0	− 8	+ 7	+ 7	0	+ 2
k......	13.7	*c*	− 5	− 14	+ 6	− 3	− 6	− 3
635....	10.8	*c*	+ 7	+ 6	− 17	− 6	+ 10	− 11
h......	13.9	*c*	+ 51	+ 25	+ 3	+ 14	+ 42	+ 2
636....	12.8	*c*	− 45	+ 22	+ 11	+ 9	− 8	+ 7
641....	13.5	*v*	− 2	+ 13	− 7	− 1	+ 10	− 9
640....	7.3	*n*	....	....	....	....	− 2	− 3
642....	13.9	*v*	+ 54	+ 50	+ 30	+ 16	+ 56	+ 17
647....	11.7	*v*	− 2	+ 10	+ 16	− 2	+ 8	+ 6
648....	14.1	*c*	− 24	+ 6	+ 20	+ 12	− 5	+ 10
650....	13.6	*b*	− 4	− 2	+ 4	+ 8	+ 1	+ 1
651....	13.2	*v*	+ 9	− 8	+ 8	+ 18	+ 4	+ 9
652....	12.9	*c*	+ 5	− 1	+ 1	+ 9	+ 6	+ 1
653....	14.3	*b*	− 5	− 11	0	+ 13	− 4	− 1
654....	14.3	*v*	− 1	+ 1	0	+ 5	+ 4	− 5
ϵ......	14.7	*c*	− 23	− 7	+ 15	+ 13	− 11	+ 6
657....	12.9	*c*	+ 1	− 12	0	+ 3	− 2	− 2
663....	13.3	*v*	− 10	0	+ 6	+ 13	− 1	+ 6
666....	13.8	*b*	+ 9	− 8	+ 12	+ 2	+ 4	+ 1
669....	10.1	*c*	+ 15	0	− 23	− 7	+ 12	− 12
667....	10.1	*b*	+ 13	− 20	− 29	− 20	0	− 21
670....	11.2	*v*	− 2	− 8	− 12	− 14	− 1	− 12
671....	12.2	*c*	− 10	− 3	+ 5	+ 17	− 2	+ 9
675....	13.8	*v*	− 14	+ 7	+ 17	+ 11	0	+ 8
677....	14.4	*b*	+ 3	− 16	+ 2	− 14	− 2	− 13
676....	13.2	*v*	+ 3	+ 12	+ 6	− 3	+ 12	− 2
678....	14.5	*b*	+ 5	− 10	+ 7	− 9	+ 2	− 8
680....	14.4	*v*	− 2	− 4	+ 13	+ 16	+ 1	+ 7
681....	13.7	*c*	− 1	0	0	+ 4	+ 4	− 3
686....	14.3	*c*	+ 7	+ 18	+ 11	+ 14	+ 16	+ 5
685....	5.6	*n*	− 32	+ 5	+ 30	+ 12	− 10	+ 35
684....	14.7	*b*	+ 17	+ 6	+ 12	+ 2	+ 16	− 1
690....	12.5	*b*	+ 8	+ 9	0	− 23	+ 12	− 15
ζ......	14.7	*c*	− 15	− 8	+ 1	+ 8	− 8	− 4
η......	14.7	*c*	+ 39	+ 1	+ 22	+ 10	+ 24	+ 8
693....	14.5	*b*	− 4	− 5	+ 9	− 2	0	− 3
688....	14.5	*c*	+ 20	− 24	− 9	− 5	+ 2	− 14
695....	13.8	*v*	− 4	− 4	+ 9	+ 8	0	+ 2
696....	12.2	*b*	+ 1	− 3	− 5	− 7	+ 3	− 8

TABLE I—*Continued*

No. Bond	Mag.	Group	$\mu_{\alpha 1}$	$\mu_{\alpha 2}$	$\mu_{\delta 1}$	$\mu_{\delta 2}$	μ_{α} abs.	μ_{δ} abs.
700....	11.9	*b*	+0″.003	−0″.014	+0″.013	+0″.014	−0″.002	+0″.013
707....	12.4	*b*	+ 3	+ 4	+ 5	− 7	+ 8	− 3
709....	12.5	*v*	+ 6	0	+ 7	− 12	+ 7	− 5
703....	13.8	*v*	+ 3	− 21	+ 8	0	− 5	− 2
708....	6.8	*n*	+ 31	+ 8	− 19	− 18	+ 24	− 7
705....	12.1	*b*	+ 24	+ 12	− 4	− 1	+ 22	− 4
ι......	14.8	*c*	− 12	+ 2	+ 22	− 17	− 1	− 6
722....	14.6	*b*	− 7	− 10	− 6	+ 2	− 4	− 10
k......	14.6	*c*	+ 5	− 12	+ 23	+ 2	0	+ 4
724....	10.1	*c*	+ 10	− 17	− 7	− 15	0	− 8
732....	12.9	*b*	+ 2	− 1	+ 8	− 9	+ 4	− 4
730....	14.1	*b*	+ 11	+ 24	+ 1	− 3	+ 21	− 7
λ......	14.7	*b*	+ 6	− 2	+ 10	+ 2	+ 6	− 2
μ......	14.3	*b*	− 7	− 8	+ 4	− 2	− 4	− 6
734....	7.1	*n*	+ 31	− 14	− 45	− 9	+ 12	− 17
741....	8.5	*n*	+ 23	− 8	− 17	− 13	+ 12	− 8
740....	13.3	*b*	0	− 2	+ 9	+ 1	+ 3	+ 1
746....	10.6	*b*	+ 14	− 11	− 15	− 25	+ 6	− 18
750....	12.2	*b*	+ 10	0	− 2	− 16	+ 9	− 11
752....	9.5	*b*	− 10	− 24	− 24	− 13	− 13	− 14
757....	10.1	*b*	+ 17	− 6	− 25	− 25	+ 10	− 22
767....	14.1	*b*	+ 3	0	− 41	− 47	+ 6	− 50
772....	14.2	*b*	+ 5	− 10	+ 6	− 12	+ 2	− 10
778....	13.8	*b*	+ 6	− 1	+ 15	+ 2	+ 6	+ 2
783....	14.2	*b*	− 29	+ 6	− 217	− 201	− 8	− 216
781....	11.6	*b*	+ 181	+ 163	− 11	− 28	+ 176	− 20
784....	12.0	*v*	+ 8	− 4	+ 1	− 13	+ 6	− 7
789....	14.7	*b*	− 7	− 21	+ 7	− 4	− 10	− 6
785....	11.6	*b*	+ 31	+ 12	− 37	− 39	+ 26	− 38
787....	14.3	*b*	− 3	− 21	− 1	− 6	− 8	− 11
795....	14.0	*b*	− 2	− 3	− 13	+ 10	+ 2	− 8
794....	13.1	*b*	− 5	− 6	+ 5	+ 1	− 2	− 1
793....	13.4	*b*	+ 66	+ 68	− 88	− 86	+ 71	− 92
805....	14.0	*b*	− 7	+ 5	+ 5	− 1	+ 3	− 4
808....	12.9	*b*	− 3	− 5	− 1	+ 8	0	0
806....	12.5	*b*	0	+ 1	− 9	+ 1	+ 4	− 7
822....	var.	*n*	+ 6	+ 11	− 11	− 9	+ 12	− 11
824....	12.6	*b*	− 13	− 6	+ 6	+ 4	− 6	+ 2
830....	11.9	*b*	− 6	− 14	− 6	− 10	− 6	− 9
832....	14.7	*b*	+ 4	+ 27	− 15	+ 3	+ 20	− 14
847....	14.0	*b*	− 10	− 2	+ 27	0	− 2	+ 8
848....	9.4	*b*	+ 12	0	− 12	− 13	+ 10	− 7
863....	14.0	*b*	+ 5	+ 7	+ 13	+ 13	+ 10	+ 7
873....	13.1	*b*	− 4	+ 7	− 12	− 15	+ 6	− 18
889....	13.3	*b*	+ 6	+ 10	− 17	− 2	+ 12	− 14
893....	14.8	*b*	+ 11	+ 7	+ 11	+ 6	+ 13	0
908....	12.8	*v*	+ 3	+ 3	+ 1	+ 11	+ 7	+ 3
924....	9.5	*b*	0	+ 9	− 7	− 7	+ 8	− 2
938....	11.6	*b*	+ 1	0	− 2	− 7	+ 4	− 4
953....	9.3	*b*	− 30	− 19	− 43	− 33	− 20	− 33
955....	11.1	*n*	− 3	− 5	+ 3	+ 6	0	+ 5
956....	10.8	*b*	− 14	− 15	− 3	− 4	− 10	− 3
959....	13.9	*b*	− 16	− 19	− 48	− 50	− 14	− 55
966....	14.6	....	+ 13	....	− 36	....	+ 17	− 44

the constants, which involves a considerable reduction in the weight of the determination. In fields such as this the difficulty can be avoided only when we are able to decide which stars belong to the nebula (or cluster) and which do not. The results will show that for the individual stars we cannot decide this with certainty, but even in such an unfavorable case the introduction of the quadratic terms can be performed with success if we take care that the distribution of the reference stars is as uniform as possible over the rest of the plate.

For the calculation of the unknown quantities $a, b, c, d, e, f, a', b', c', d', e'$, and f', I have used the same 47 stars as in the original paper. The values $a, b, \ldots, e', f'$ thus found were then substituted into the equation of condition (1) for the 162 stars measured; the resulting values, $\mu_{\alpha 1}$, $\mu_{\alpha 2}$, $\mu_{\delta 1}$, and $\mu_{\delta 2}$, are given in Table I.

TABLE II

Distance from Center	Corrections						
	<0″.003	0″.003–0″.005	0″.006–0″.008	0″.009–0″.011	0″.012–0″.014	>0″.014	*n*
<8′	107	138	51	0	0	0	296
8–10	39	22	7	0	0	0	68
10–18.5....	106	94	37	5	1	1	244
>18.5.....	7	16	6	4	2	3	38

To what degree neglect of the quadratic terms vitiated the former values of the proper motions can be seen from Table II, which shows the numbers of corrections between certain limits to be applied to stars at different distances from the center of the plates. In order to compare the corrections to be applied in the case of plates taken with the Yerkes refractor with those necessary for the plates secured with the Mount Wilson reflector, the same zones were used as in Table VIII of *Contribution* No. 167; the last group with $r > 18'.5$ contains stars outside the boundaries of 8×10-inch Mount Wilson plates.

For a better comparison Table III was constructed, giving the percentages of the necessary corrections for the three zones contained on plates secured with both instruments.

In making this comparison we must bear in mind that the interval for the Yerkes plates is ten years as against five for the Mount Wilson plates; on this account, since we are dealing with annual motions, we might expect in the latter case corrections twice as large. If we take this into consideration it is clear that under equal conditions the reflector results will not be likely to need any corrections larger than those obtained with the refractor.

TABLE III

Distance from Center	Series	Corrections					
		<0″003	0″003–0″005	0″006–0″008	0″009–0″011	0″012–0″014	>0″014
		Per Cent	Per Cent	Per Cent	Per Cent	Per Cent	Per Cent
<8′	Yerkes....	36	47	17	0	0	0
	Mt. Wilson	67	28	5	0	0	0
8–10.....	Yerkes....	58	32	10	0	0	0
	Mt. Wilson	39	37	20	4	0	0
10–18.5...	Yerkes....	43^5	38^5	15	2	0^5	0^5
	Mt. Wilson	30	34^5	15	13	3	4^5

The reduction in the quantities $[vv]$, due to the introduction of the quadratic terms, is 12.5 per cent, which is practically identical with that found for the Pleiades measures[1] and that obtained by Lee[2] for Yerkes plates, where the derivation of the constants was not hampered by the difficulty mentioned on page 1 of this paper. The probable errors in the proper motions have in consequence changed but little and are for a mean μ_α, 0″0039, for a mean μ_δ, 0″0030. The small reduction in $[vv]$ does not mean that the quadratic terms can safely be neglected, as was pointed out in *Contribution* No. 167, page 1.

The values of the brightness given in Table I are photographic magnitudes derived from the photovisual magnitudes in *Astronomical Journal*, No. 642, with the aid of the corrections given by van Rhijn in Table 33 of *Groningen Publications* No. 27. Although for individual stars these corrections may be in error on account of the deviation of the individual color-index from the mean color, the conversion was made in order to secure a homogeneous system of magnitudes for different fields; other papers on the distribution of proper motions with magnitude, now in preparation, will deal with photographic magnitudes.

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[1] *Mt. Wilson Contr.* No. 167, 1919.

[2] *Astronomical Journal*, **29**, 74, 1916.

THE DIVISION OF THE MATERIAL INTO GROUPS

For the discussion of the resulting proper motions the 162 stars were divided into four groups:

1. *Nebulous stars.*—On plates of different exposure-times 13 stars were found which seem clearly to be connected with the nebulosity; their magnitudes range from 5.6 to 12.00; in Table I they are indicated by *n* in the third column.

2. *Central stars.*—Near the Trapezium we find such an abundance of faint stars, from magnitude 10.1 to 14.8, that we can hardly avoid the conclusion that most or all of them must be connected with the nebula. They are not uniformly distributed around the Trapezium but seem to be grouped in three directions with respect to it, viz., SE, in the direction of θ_2 Orionis; NNE, in the direction of Bond 734, a star which is the center of much nebulosity; and SW. The 32 stars indicated by *c* in Table I have been included in this group.

3. *Variable stars.*—On the Yerkes plates, which cover a field only 40′ by 32′, not less than 23 stars are visible which are either known or suspected to be variables. It is of course possible that the observed changes are partly due to the difficulty of securing reliable magnitudes for stars involved in nebulosity, yet we must accept the variability of most of these stars as well established. This may be caused by the motion of the nebula across the stars in the background but may also be due to a physical connection between them and the nebula. Since the cause cannot be decided a priori, it seems safe to exclude these stars from the discussion of both the nebula stars and the background stars. Bond 822, however, although a variable, was included among the nebulous stars because it coincides with a center of nebulosity; Bond 642, on the other hand, was included among the background stars on account of its proper motion, which is irreconcilable with its connection with the nebula. The resulting 21 stars are indicated by *v* in Table I.

4. *Background stars.*—This group, whose members are indicated by *b*, includes all stars not mentioned under 1, 2, or 3, with the exception of Bond 966, which was measured on one pair of plates only. They number 95, some of which may be connected with the nebula. Although it is impossible to designate the individual stars, the results show that we seem to have been successful in eliminating most of those belonging to the nebula.

REDUCTION FROM RELATIVE TO ABSOLUTE MOTION

As practically all the stars contained in 1 and 2 must belong to the system of the Orion nebula, they enable us to derive the reduction from relative to absolute proper motion. For this purpose we make the following supposition: All stars belonging to the nebula have the same motion, independent of their magnitude and position. That internal motions in the nebula do exist was first shown by the interferometer measures of Buisson, Fabry, and Bourget,[1] and their results have been confirmed by the spectroscopic work of Frost and Maney;[2] yet, as a first approximation, it may be accepted that the motion of the nebula as a whole is fairly well represented by the mean motion of a number of the stars, provided they are distributed around the Trapezium region with some approximation to uniformity.

Later we can investigate whether the motions at right angles to the line of sight reveal any internal motion in the different parts of the nebula.

In order to derive the absolute motion of the system we must use stars whose proper motions are based on meridian-circle observations. As in *Contribution* No. 167, I shall try to reduce the relative motions derived from my measures to the system of Boss's *Preliminary General Catalogue*. Only two stars listed in that catalogue, θ_1 and θ_2, appear on the plates; from other catalogues we might add one or two with more or less reliable motions; but even then the material would be hardly sufficient to derive the necessary corrections. I have therefore employed a roundabout method analogous to that described in the derivation of the proper motions of the Pleiades. The stars used are those in Boss's *Catalogue* between 5^h and 6^h in right ascension and $+10°$ and $-10°$ in declination that seem to be connected with the nebulosity in Orion, which, as well known, extends considerably beyond the Huyghenian region. Adopting for these stars also the supposition stated above, we can compare their mean motion as derived from Boss's *Catalogue* with that found from the plates.

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[1] *Astrophysical Journal*, **40**, 241, 1914.

[2] *Popular Astronomy*, **23**, 485, 1915.

In order to find which Boss stars seem to be connected with the nebula, Mr. Humason, of this Observatory, kindly took several plates for me with the 10-inch photographic refractor, the exposures being 15^m and 30^m. I wish to express to him my sincere thanks for the trouble he took in securing the necessary photographs.

For information given by longer exposures I have used the Franklin-Adams charts and those of Wolf-Palisa and also the reproduction of the region in question published in Volume XI of the *Publications of the Lick Observatory;* finally Mr. Barnard, who secured this last plate, has given me his valuable advice about certain other stars which seem to be involved in the nebulosity.

From this material it seems probable that the following stars in Boss's *Catalogue* are connected with the nebula: 1262, 1301, 1303, 1363, 1364, 1365, 1366, 1368, 1389, 1391, 1392, 1398, and possibly 1339 and 1370. All these stars are of spectral type Oe or B, with the exception of 1368 and 1392, which are, respectively, F and A3; 1392 has, moreover, in contrast with the other stars, a considerable proper motion. Both these stars were accordingly excluded. The mean motion of the remaining 12 stars from Boss's *Catalogue* is $\mu_\alpha=+0\overset{''}{.}0024$, $\mu_\delta=-0\overset{''}{.}0018$, which corrected, in accordance with the remark on page xxviii of that *Catalogue*, gives for the absolute motion $\mu_\alpha=+0\overset{''}{.}0058$, $\mu_\delta=-0\overset{''}{.}0021$. Excluding the doubtful stars, 1339 and 1370, the result is $\mu_\alpha=+0\overset{''}{.}0061$, $\mu_\delta=-0\overset{''}{.}0019$. As the difference between the two values is negligible I have finally adopted as the absolute motion of the nebular system

$$\mu_\alpha=+0\overset{''}{.}006 \qquad \mu_\delta=-0\overset{''}{.}002$$

From my measures we find as the mean motion of the stars of the nebuous and central groups

$$\mu_\alpha=+0\overset{''}{.}002 \qquad \mu_\delta=-0\overset{''}{.}001$$

Dividing these 45 stars into nine groups of five stars each according to magnitude, we find that μ_δ is clearly a function of the magnitude, as may be seen from Table IV. The values for μ_α show no such systematic change; the correction from $\mu_{\alpha\ \text{rel.}}$ to $\mu_{\alpha\ \text{abs.}}$ was accordingly assumed to be constant and equal to $+0\rlap{.}''004$. If we accept the change of μ_δ with magnitude as due to a magnitude-error in the motions derived from the plates, a simple least square solution of the form

$$\mu_{\delta\ \text{rel.}}+a+b\ \text{mag.}=\mu_{\delta\ \text{abs.}}=-0\rlap{.}''002 \qquad (2)$$

applied to the nine equations of condition derived from Table IV gives as the necessary correction to the observed values of μ_δ

$$+0\rlap{.}''0273-0\rlap{.}''00239\,m.$$

The absolute motions in α and δ thus derived are given in the last two columns of Table I.

TABLE IV

Mean Magnitude	Mean μ_α	Mean μ_δ
6.5	$+0\rlap{.}''001$	$-0\rlap{.}''008$
8.3	+0.003	−0.014
10.5	0.000	−0.009
12.5	+0.006	0.000
13.0	0.000	+0.005
13.7	0.000	−0.003
14.0	+0.008	+0.011
14.4	+0.001	+0.001
14.7	−0.003	+0.010

THE RESULTS

The results for the four groups of stars become:

1. *Nebulous stars:*
$\mu_\alpha=+0\rlap{.}''0052\pm0\rlap{.}''0020$
$\mu_\delta=-0\rlap{.}''0024\pm0\rlap{.}''0024$
$n=13$

2. *Central stars:*
$\mu_\alpha=+0\rlap{.}''0059\pm0\rlap{.}''0013$
$\mu_\delta=-0\rlap{.}''0021\pm0\rlap{.}''0010$
$n=32$

3. *Variable stars:* $\mu_\alpha = +0\overset{\prime\prime}{.}0046 \pm 0\overset{\prime\prime}{.}0009$
$\mu_\delta = +0\overset{\prime\prime}{.}0011 \pm 0\overset{\prime\prime}{.}0011$
$n = 21$

4. *Background stars:* $\mu_\alpha = +0\overset{\prime\prime}{.}0029$
$\mu_\delta = -0\overset{\prime\prime}{.}0103$
$n = 95$

As the background stars certainly do not form a group of stars having equal motions, a probable error for their mean motion would have no meaning. The position angle of the mean motion of this group is 165°, exactly the same as that of the parallactic motion for this part of the sky. To a great extent this is due to the influence of the largest motions. Excluding the seven stars for which $\mu > 0\overset{\prime\prime}{.}050$, we have:

$$\mu_\alpha = +0\overset{\prime\prime}{.}0035, \qquad \mu_\delta = -0\overset{\prime\prime}{.}0055, \qquad p = 147^\circ.$$

The mean motions for the different magnitudes of the remaining 88 stars are given in Table V. The last two columns show the parallactic motions, which were computed with the help of Kapteyn's formula, adopting for the solar motion 21.5 km/sec. and using Strömberg's data for the position of the apex. The agreement between the parallactic and the observed motions is as good as can be expected from the small number of stars included.

TABLE V

Limits of Magnitude	Magnitude	μ_α	μ_δ	n	μ_α par.	μ_δ par.
8– 9	8.3	$+0\overset{\prime\prime}{.}016$	$-0\overset{\prime\prime}{.}015$	1	$+0\overset{\prime\prime}{.}003$	$-0\overset{\prime\prime}{.}010^5$
9–10	9.5	0.000	−0.013	6	$+0.002^5$	-0.007^5
10–11	10.6	+0.008	−0.008	10	$+0.001^5$	-0.005^5
11–12	11.6	+0.007	−0.009	12	+0.001	−0.004
12–13	12.5	+0.003	−0.002	17	+0.001	−0.003
13–14	13.6	+0.002	−0.003	15	$+0.000^5$	-0.002^5
> 14	14.4	+0.002	−0.004	27	$+0.000^5$	-0.001^5

Finally Table VI shows the distribution of total motion for the 95 background stars.

TABLE VI

Limits of μ	n
$<0''.010$	50
$0''.010-0''.019$	27
$0.020-0.029$	9
$0.030-0.039$	1
$0.040-0.049$	1
$0.050-0.059$	3
$0.060-0.099$	1
$0.100-0.199$	2
$0.200-0.299$	1

The large number of stars having $\mu<0''.010$ shows that the correction from relative to absolute motion cannot be far from the truth. That this number is relatively higher than in the case of the Pleiades measures, where the probable error in the motion is the same, is undoubtedly due to the fact that the region discussed here is so near the apex of the solar motion ($\lambda=26°$) so that the motions at right angles to the line of sight must be small. That it is probably not caused by the inclusion of many stars which belong to the nebula will be shown below.

CONCLUSIONS

1. The agreement of the mean motion found for the nebulous and central stars is so close that we may accept practically all stars included in both groups as belonging to the system of the Orion nebula; the adopted motion of the nebula system ($\mu_\alpha=+0''.0057$, $\mu_\delta=-0''.0021$, $p=110°$) is of course practically the same as that of the 12 Boss stars mentioned on page 9. Several values of the parallax of the nebula have been derived: Kapteyn[1] gives $+0''.0054$, W. H. Pickering[2] $+0''.0005$, Coeburgh[3] $+0''.0260$, and Russell[4] $+0''.0050$. For nine stars included in the nebula system by either Kapteyn or the writer, directly measured parallaxes have been published by Flint or by the Yale or Yerkes observatories. After applying the necessary systematic corrections, the weighted

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[1] *Mt. Wilson Contr.* No. 147, 1918.

[2] *Harvard College Observatory, Circular*, **205**, 1917.

[3] *Hemel en Dampkring*, **16**, 105, 1918.

[4] *Astronomical and Astrophysical Society of America*, **1**, 268, 1910.

mean absolute parallax is negative, viz., $-0''.009$. Accepting Kapteyn's value, which is undoubtedly the most reliable, we find the resulting motion at right angles to the line of sight to be 5 km/sec., while, in the line of sight, the nebula is receding with a velocity of 16 km/sec.[1]

2. From the work of Buisson, Fabry, and Bourget, mentioned above, we know that there is some evidence that the nebula rotates about an axis NW–SE, the NE portion receding, the SW approaching. As their measures are confined to the region within 2′ from the Trapezium, I have investigated whether the stars measured within the same area show the effect of such a rotation. As most of the stars seen in the densest part are likely to be on this side of the nebula, they ought to show a preponderance of motion from SW toward NE. The mean motion in this direction (27 stars) was found to be $0''.0010 \pm 0''.0012$, while the mean motion at right angles is $0''.0000 \pm 0''.0015$. This small systematic motion is well within the limits of the probable error.

3. The mean motion of the 21 variable stars of the third group is in fair agreement with that of the nebula, although the difference in μ_δ is larger than might be expected from the probable errors. Excluding, however, the three stars, Bond 567, 589, and 675, whose variability, suspected by Parkhurst,[2] needs further confirmation, the mean motion is $\mu_\alpha = +0''.0049$, $\mu_\delta = -0''.0003$; this value agrees so well with the motion of the nebulous and central stars that it may safely be accepted that most of these variables form a part of the system of the Orion nebula.

4. For the background stars the change in the mean values of μ_α and μ_δ with magnitude, which may be seen from Table V, and the fair agreement of these values with the parallactic motion, show that we have been successful in excluding from the fourth group most of the stars which are connected with the nebulosity. As a further test I have constructed the diagrams in Fig. 1. The number of stars and the total motion were determined for sectors of 15°; then, to smooth the curves, overlapping means for three successive sectors were formed. In order to avoid the influence of

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[1] *Astrophysical Journal*, **40**, 253, 1914.

[2] *Ibid.*, **20**, 137, 1904.

the large proper motions, stars with $\mu > 0''.050$ were excluded. The smaller diagram shows the results for the number of stars, the larger for the total motion in the different directions. If among the background stars we have included a considerable number of nebula stars, we should find a preponderance of both numbers and motion in the direction of $p = 110°$, in which the nebula is moving; the fact that no such preponderance appears is another proof that only a few stars of the nebula system can have been included among the background stars.

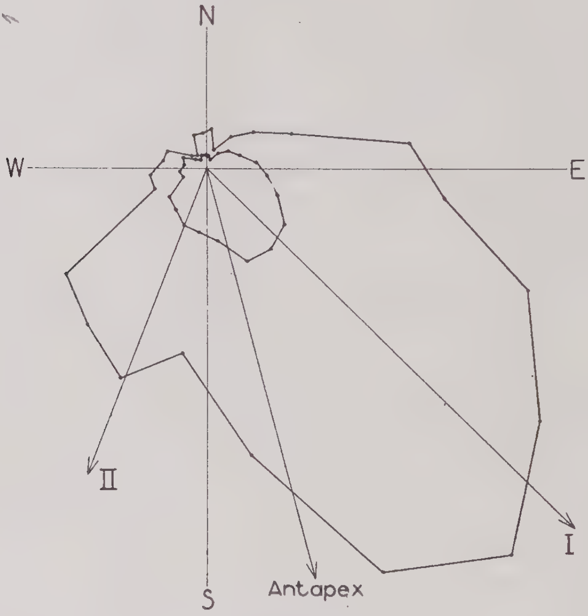

FIG. 1

5. The same diagram shows that neither the maximum frequency nor the maximum motion coincides with the direction of the parallactic motion. To investigate whether this fact can be accounted for by stream-motion, I computed the direction of both streams, adopting as co-ordinates of the vertices:[1]

Stream I,	$\alpha = 91°$,	$\delta = -15°$
Stream II,	$\alpha = 288°$,	$\delta = -64°$

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[1] A. S. Eddington, *Stellar Movements*, 1914, p. 100.

We find for

Stream I, $p=139°$, $\lambda=13°$
Stream II, $p=191°$, $\lambda=108°$

The agreement of the maximum frequency, as well as that of the maximum motion, with Stream I is very remarkable when we take into account the small number of stars available (88), the more so as λ, the distance from the vertex, is only 13°; the influence of Stream II, although not so strong, can also be plainly seen.

More material for different parts of the sky will of course be necessary before we can conclude with any certainty that stars as faint as the fourteenth and fifteenth magnitude show the stream-motion.

Mount Wilson Solar Observatory
March 1919

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